Higgs pair production at LHC

Go Mishima (Tohoku Univ.)

[arXiv:2110.03697]
with J. Davies, F. Herren, M. Steinhauser

JHEP 05 (2019) 157
with J. Davies, F. Herren, M. Steinhauser

JHEP 11 (2019) 024
with J. Davies, G. Heinrich, S. P. Jones, M. Kerner, M. Steinhauser, D. Wellmann

JHEP 01 (2019) 176, JHEP 03 (2018) 048
with J. Davies, M. Steinhauser, D. Wellmann

Motivation: triple Higgs coupling

λ_{HHH} in the Standard Model

Higgs potential $$V(H) = \frac{1}{2} m_H^2 H^2 + \lambda_{HHH} v H^3 + \frac{1}{4} \lambda_{HHHH} H^4$$

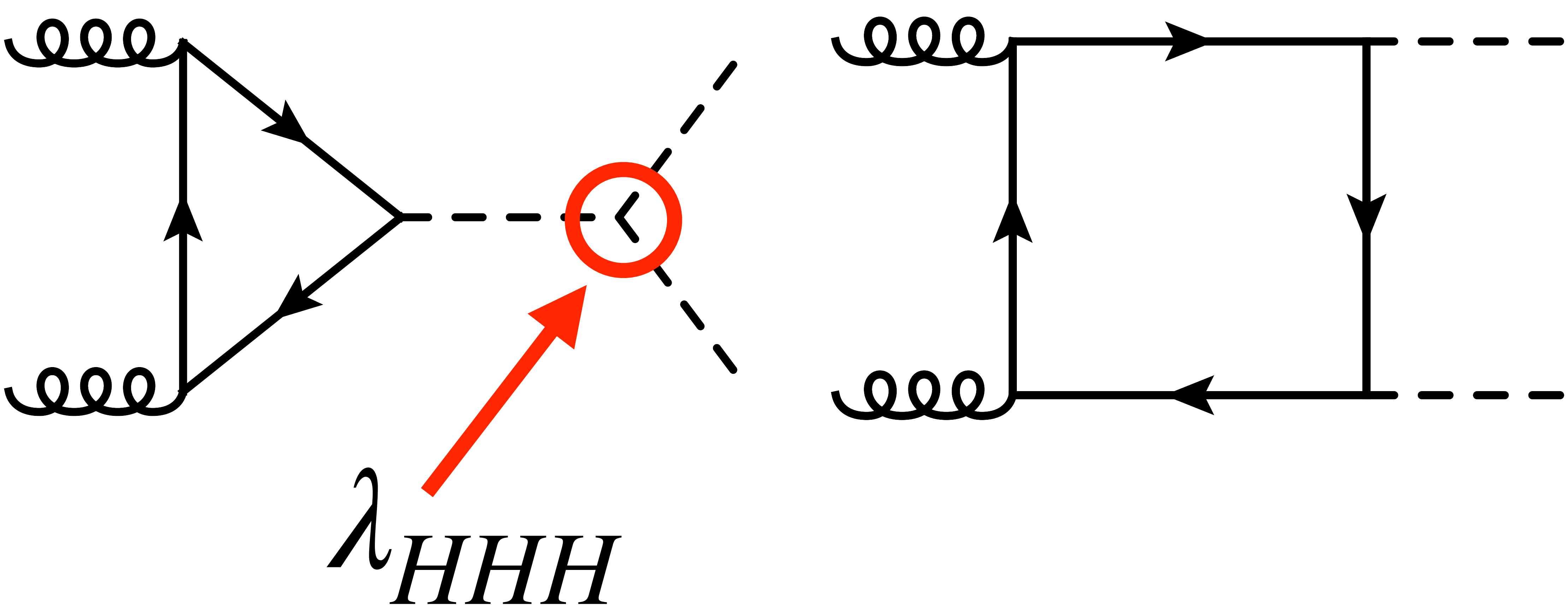
in SM: $$\lambda_{HHH} = \frac{m_H^2}{2v^2} = 0.13\ldots$$ (not directly measured)

[CMS: arXiv:2011.12373]: $$-3.3 < \lambda/\lambda_{\mathrm{SM}} < 8.5$$

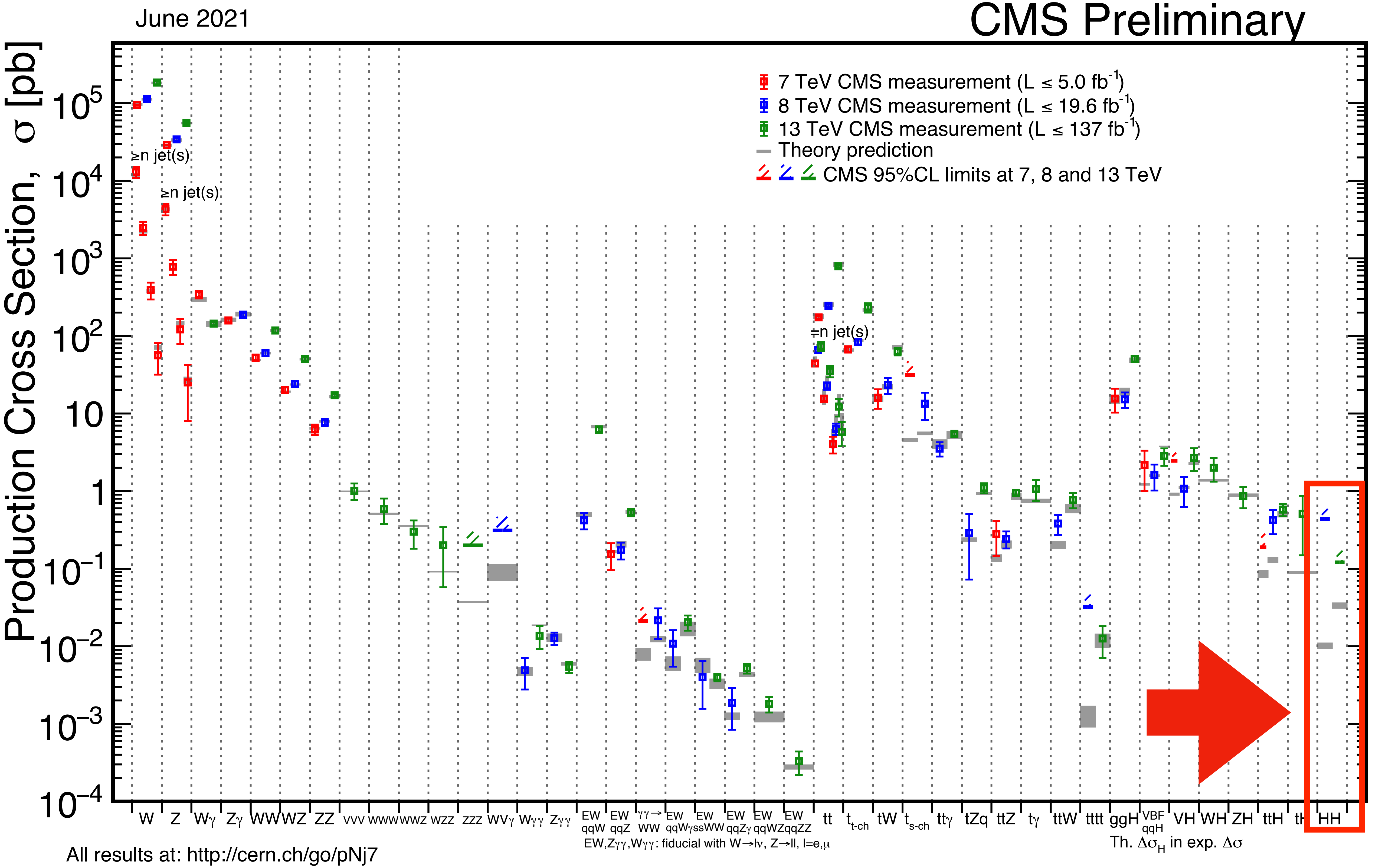
Motivation: triple Higgs coupling

λ_{HHH} in the Standard Model

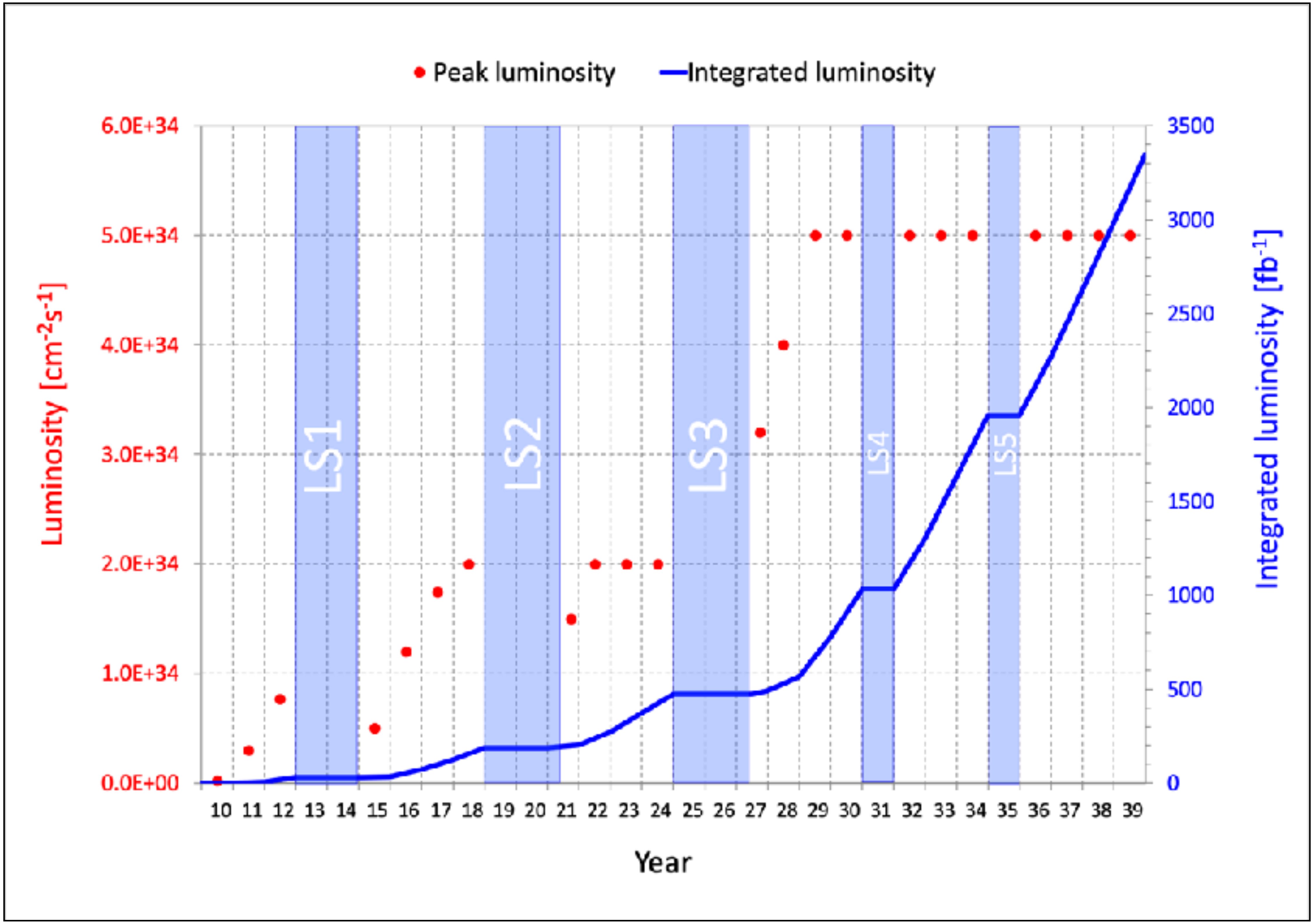
The simplest process is Higgs pair production.



Challenging channel but reachable in the future



more data will be available!



How to get the LHC theoretical prediction

	initial state	final state
1. Partonic Cross Section	gluon-gluon gluon-quark quark-quark	Higgs pair (gluon/quark radiation)
2. Hadronic Cross Section	proton-proton	Higgs pair (gluon/quark radiation)
3. Hadronization (Jets)	proton-proton	Higgs pair + Jets

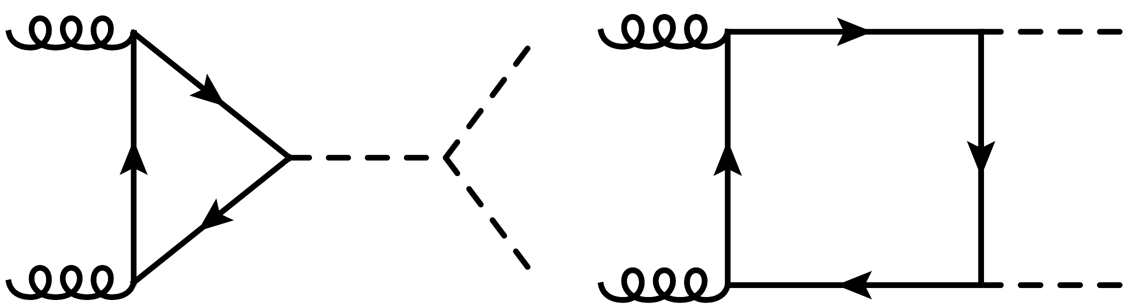
How to get the LHC theoretical prediction

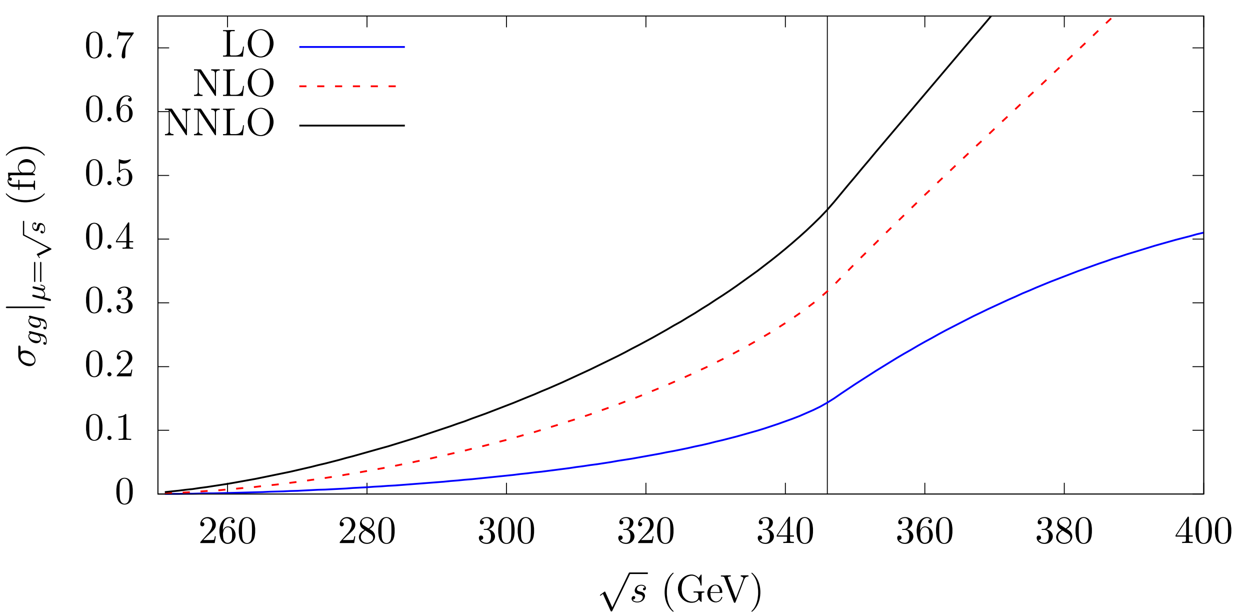
We focus here

	initial state	final state
1. Partonic Cross Section	gluon-gluon gluon-quark quark-quark	Higgs pair (gluon/quark radiation)
2. Hadronic Cross Section	proton-proton	Higgs pair (gluon/quark radiation)
3. Hadronization (Jets)	proton-proton	Higgs pair + Jets

partonic -> hadronic is well established

partonic cross section

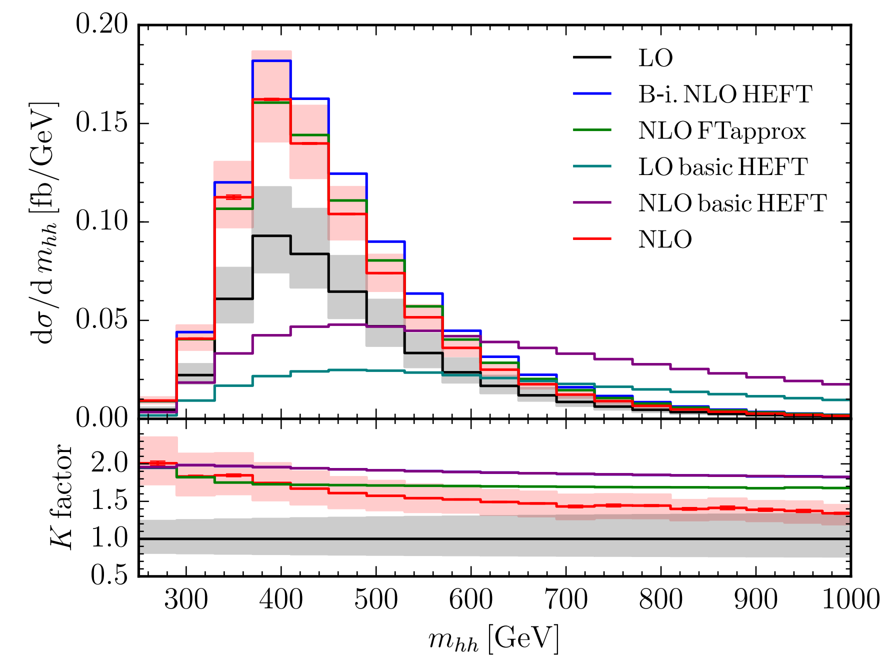




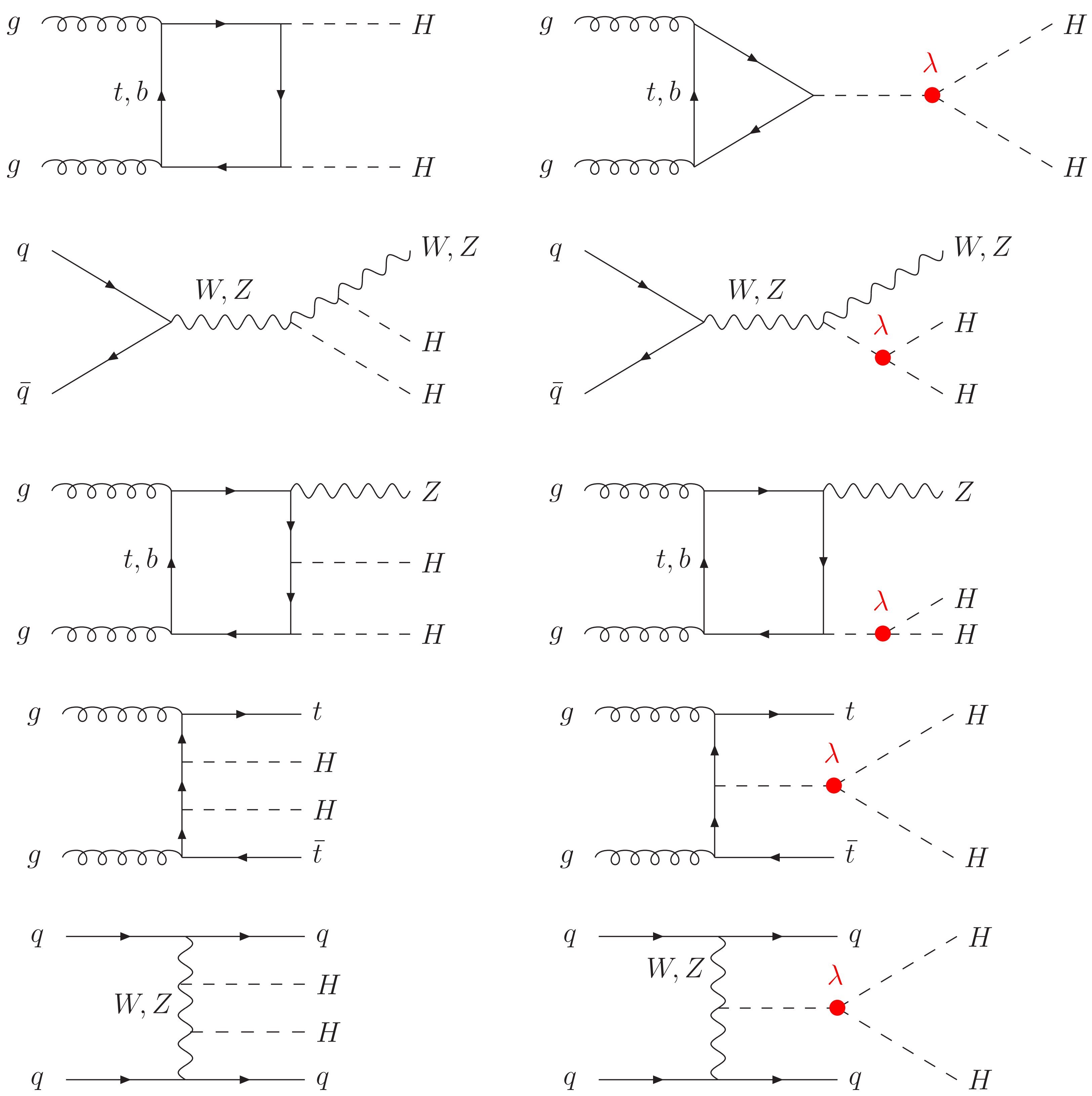
hadronic cross section (including hadronization)

$$\sigma_{pp\to HHX}(s) = \sum_{i,j=g,q,\bar{q}} \int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2) \sigma_{ij\to HH}(x_1 x_2 s)$$

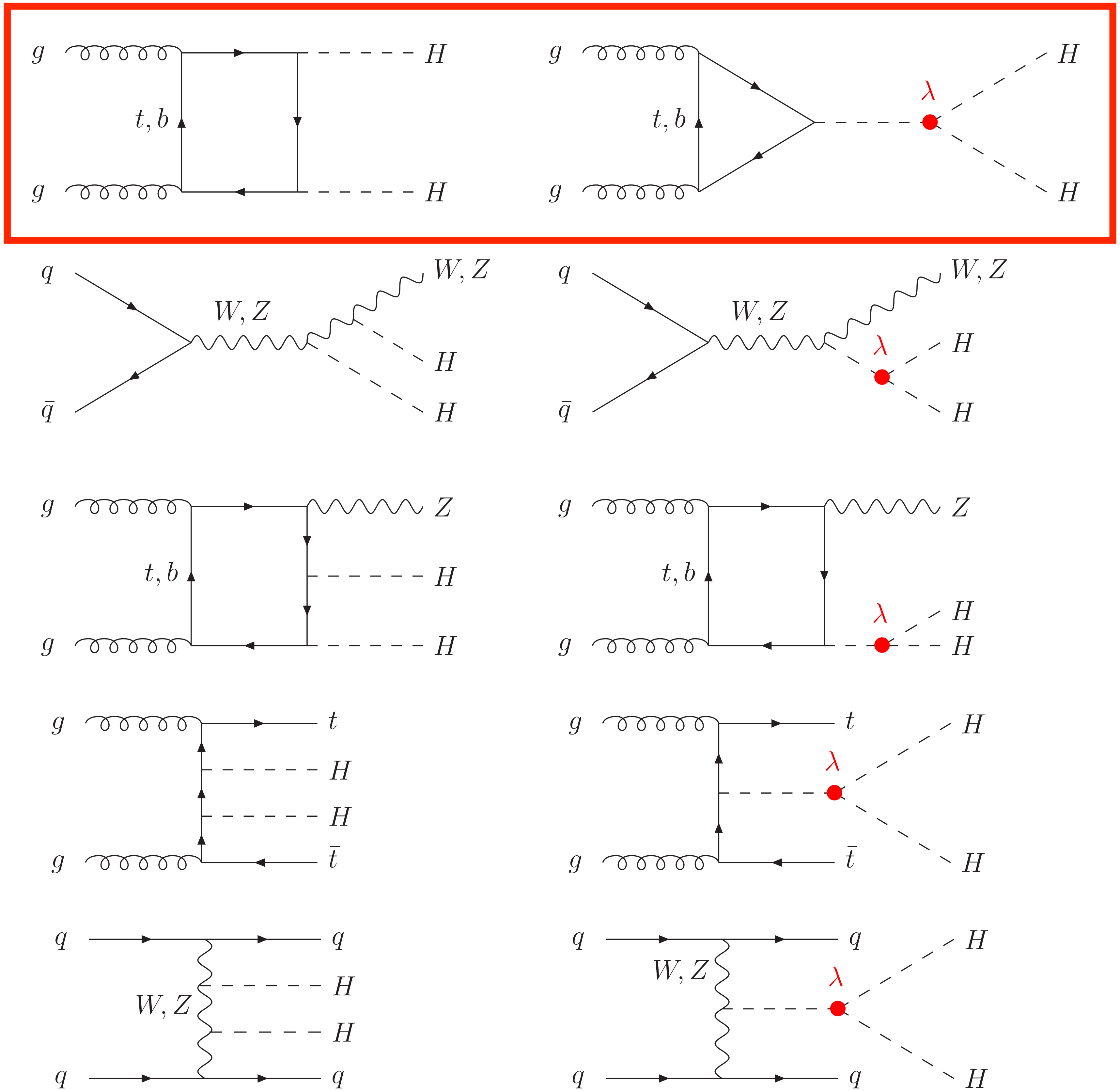
convoluting parton distribution function (PDF)
taking into account parton shower
(POWHEG, MG5_aMC@NLO, Sherpa, Pythia, Herwig,...)



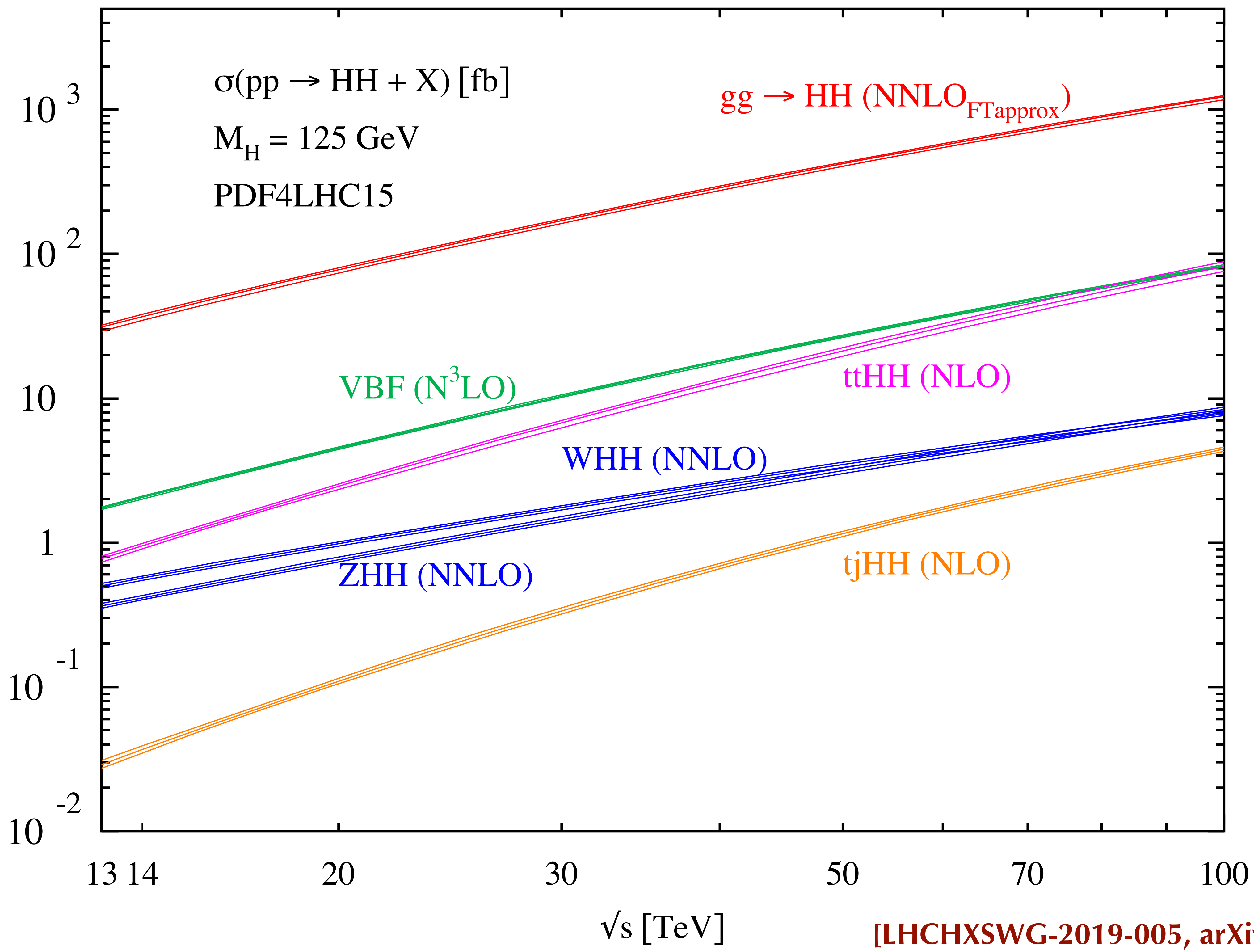
Possible partonic channels



Possible partonic channels



Gluon fusion is the dominant channel



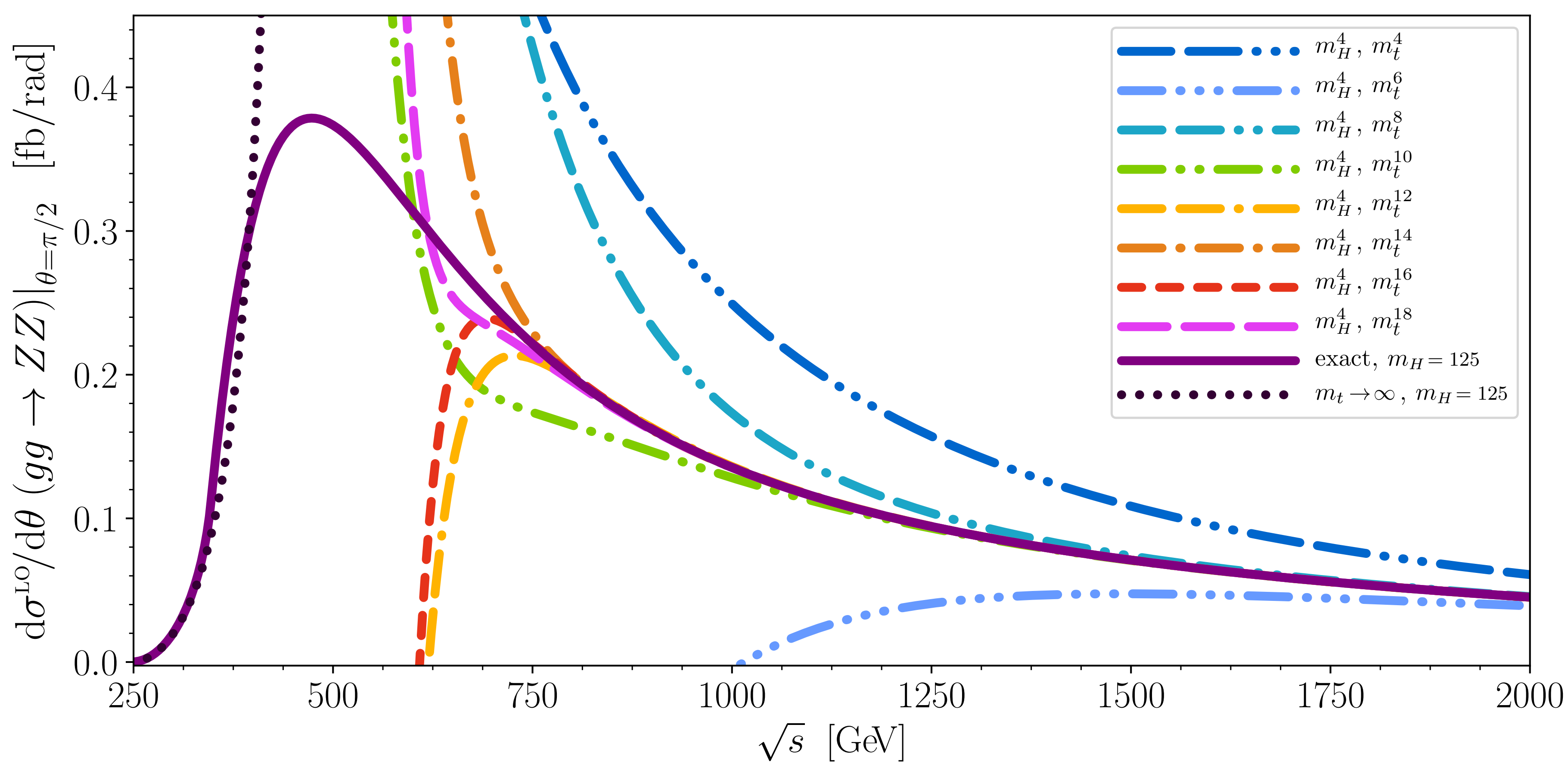
[LHCHXSWG-2019-005, arXiv:1901.00012]

What was done so far

LO: known analytically since 1988 (Glover&van der Bij, Plehn&Spira&Zerwas)

NLO: exact numerical results known only recently 2016-
many many many approximations are investigated

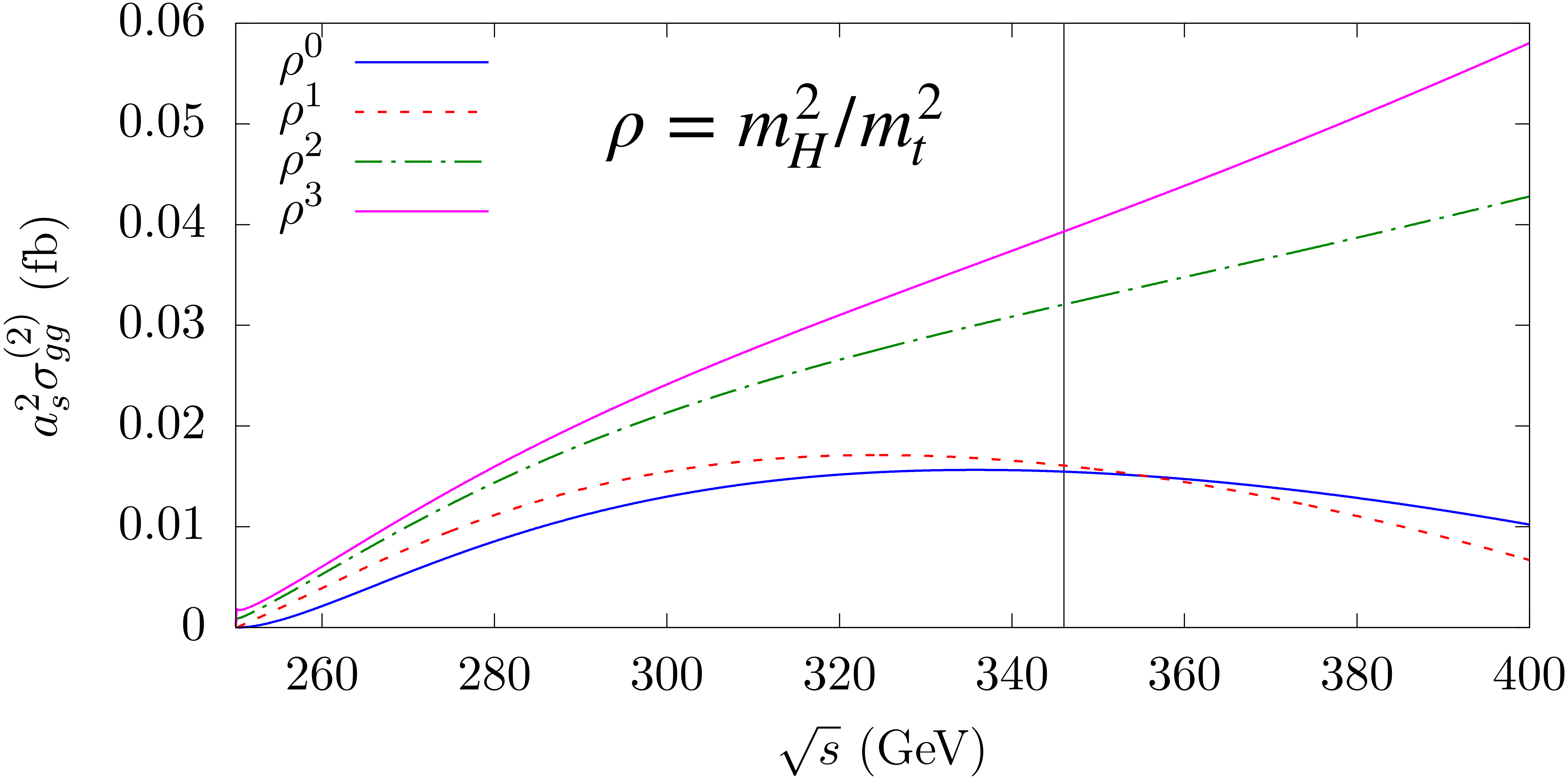
NNLO: known in Higgs Effective Field Theory (HEFT: mt -> infinity), numerically



NLO: large-mt approx.
and high-energy approx.

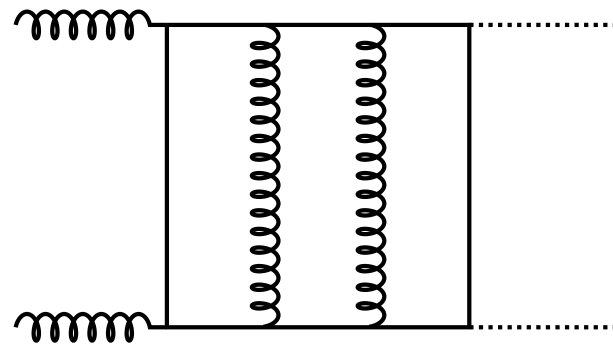
[D. Wellmann, Ph. D. thesis]

Our achievement: analytic, 1/mt corrections

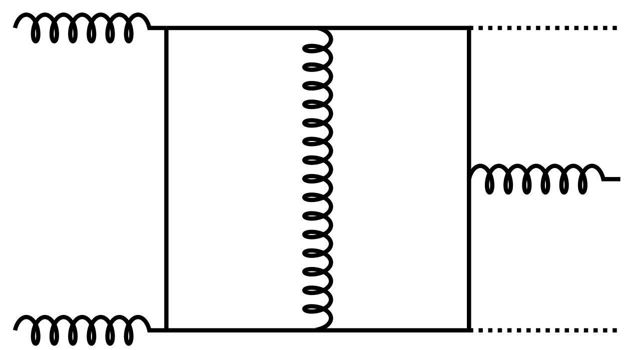


Ingredients of NNLO cross section

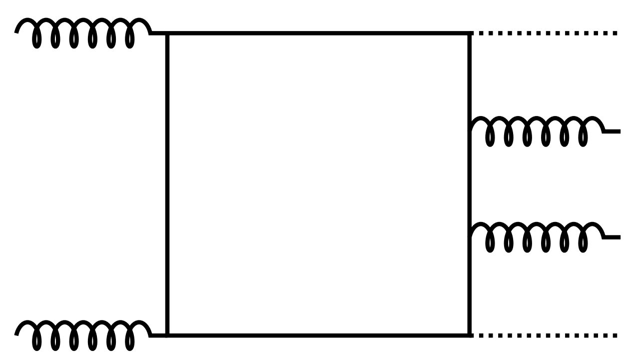
3-loop 2 -> 2
(virtual-virtual)



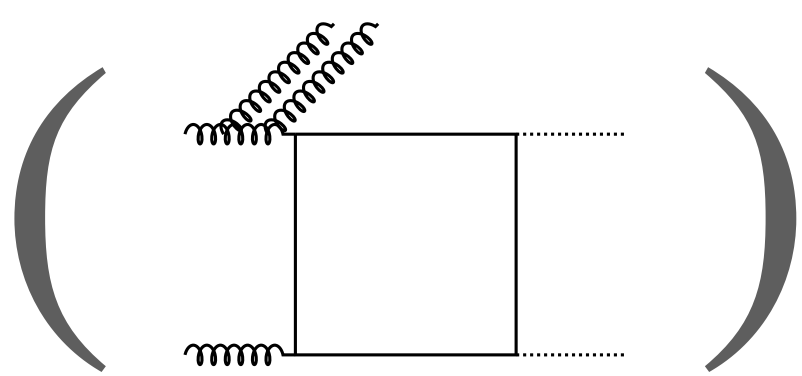
2-loop 2 -> 3
(real-virtual)



1-loop 2 -> 4
(real-real)



collinear
counter term



Individual pieces
are divergent!

We obtain
finite quantity
only after
adding them all.

collinear counter term can be explained as "renormalization of PDFs"

$$\sigma_{pp\to HHX}(s) = \sum_{i,j=g,q,\bar{q}} \int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2) \sigma_{ij\to HH}(x_1 x_2 s)$$

PDFs should be determined experimentally by using the same renormalization scheme.

Common used convention is Altarelli-Parisi splitting functions.

$$\sigma^{(2)}_{gg,\mathrm{coll}} \;=\; -\frac{1}{\epsilon}\left(\frac{\mu^2}{\mu_f^2}\right)^{\epsilon} \int_{1-\delta}^{1} \mathrm{d}z\, P^{(0)}_{gg}(z)\sigma^{(1)}_{gg}(x/z) + \ldots\,,$$

NLO splitting functions

$$P_{\rm ps}^{(1)}(x) \;=\; 4C_F n_f\Big(\frac{20}{9}\frac{1}{x} - 2 + 6x - 4{\rm H}_0 + x^2\Big[\frac{8}{3}{\rm H}_0 - \frac{56}{9}\Big] + (1+x)\Big[5{\rm H}_0 - 2{\rm H}_{0,0}\Big]\Big) \qquad (4.7)$$

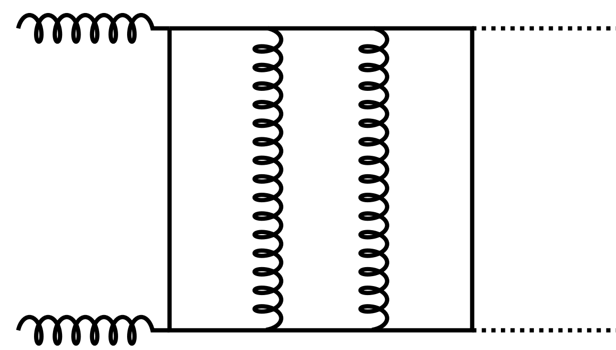
$$\begin{aligned} P_{\rm qg}^{(1)}(x) \;=\;& 4C_A n_f\Big(\frac{20}{9}\frac{1}{x} - 2 + 25x - 2p_{\rm qg}(-x){\rm H}_{-1,0} - 2p_{\rm qg}(x){\rm H}_{1,1} + x^2\Big[\frac{44}{3}{\rm H}_0 - \frac{218}{9}\Big] \\ &+4(1-x)\Big[{\rm H}_{0,0} - 2{\rm H}_0 + x{\rm H}_1\Big] - 4\zeta_2 x - 6{\rm H}_{0,0} + 9{\rm H}_0\Big) + 4C_F n_f\Big(2p_{\rm qg}(x)\Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 \\ &-\zeta_2\Big] + 4x^2\Big[{\rm H}_0 + {\rm H}_{0,0} + \frac{5}{2}\Big] + 2(1-x)\Big[{\rm H}_0 + {\rm H}_{0,0} - 2x{\rm H}_1 + \frac{29}{4}\Big] - \frac{15}{2} - {\rm H}_{0,0} - \frac{1}{2}{\rm H}_0\Big) \qquad (4.8)\end{aligned}$$

$$\begin{aligned} P_{\rm gq}^{(1)}(x) \;=\;& 4C_A C_F\Big(\frac{1}{x} + 2p_{\rm gq}(x)\Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \frac{11}{6}{\rm H}_1\Big] - x^2\Big[\frac{8}{3}{\rm H}_0 - \frac{44}{9}\Big] + 4\zeta_2 - 2 \\ &-7{\rm H}_0 + 2{\rm H}_{0,0} - 2{\rm H}_1 x + (1+x)\Big[2{\rm H}_{0,0} - 5{\rm H}_0 + \frac{37}{9}\Big] - 2p_{\rm gq}(-x){\rm H}_{-1,0}\Big) - 4C_F n_f\Big(\frac{2}{3}x \\ &-p_{\rm gq}(x)\Big[\frac{2}{3}{\rm H}_1 - \frac{10}{9}\Big]\Big) + 4C_F^{\,2}\Big(p_{\rm gq}(x)\Big[3{\rm H}_1 - 2{\rm H}_{1,1}\Big] + (1+x)\Big[{\rm H}_{0,0} - \frac{7}{2} + \frac{7}{2}{\rm H}_0\Big] - 3{\rm H}_{0,0} \\ &+1 - \frac{3}{2}{\rm H}_0 + 2{\rm H}_1 x\Big) \qquad (4.9)\end{aligned}$$

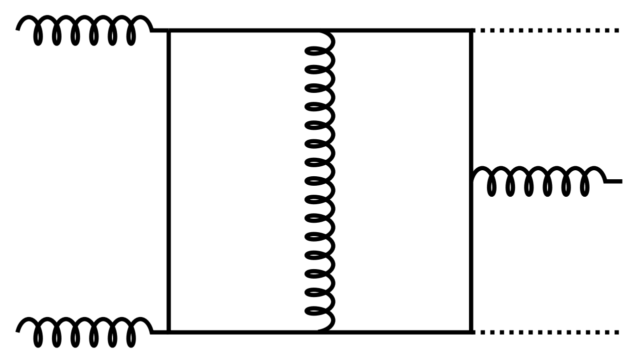
$$\begin{aligned} P_{\rm gg}^{(1)}(x) \;=\;& 4C_A n_f\Big(1 - x - \frac{10}{9}p_{\rm gg}(x) - \frac{13}{9}\Big(\frac{1}{x} - x^2\Big) - \frac{2}{3}(1+x){\rm H}_0 - \frac{2}{3}\delta(1-x)\Big) + 4C_A^{\,2}\Big(27 \\ &+(1+x)\Big[\frac{11}{3}{\rm H}_0 + 8{\rm H}_{0,0} - \frac{27}{2}\Big] + 2p_{\rm gg}(-x)\Big[{\rm H}_{0,0} - 2{\rm H}_{-1,0} - \zeta_2\Big] - \frac{67}{9}\Big(\frac{1}{x} - x^2\Big) - 12{\rm H}_0 \\ &-\frac{44}{3}x^2{\rm H}_0 + 2p_{\rm gg}(x)\Big[\frac{67}{18} - \zeta_2 + {\rm H}_{0,0} + 2{\rm H}_{1,0} + 2{\rm H}_2\Big] + \delta(1-x)\Big[\frac{8}{3} + 3\zeta_3\Big]\Big) + 4C_F n_f\Big(2{\rm H}_0 \\ &+\frac{2}{3}\frac{1}{x} + \frac{10}{3}x^2 - 12 + (1+x)\Big[4 - 5{\rm H}_0 - 2{\rm H}_{0,0}\Big] - \frac{1}{2}\delta(1-x)\Big)\;. \qquad (4.10)\end{aligned}$$

Ingredients of NNLO cross section

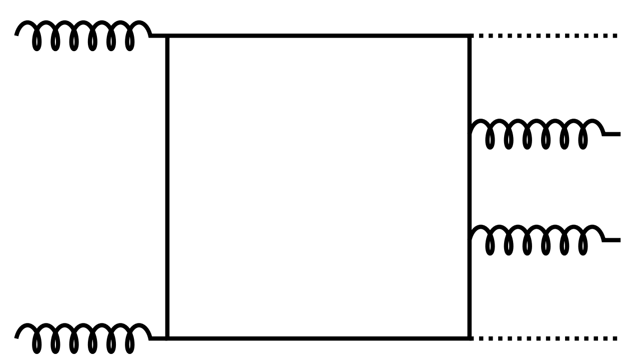
3-loop 2 -> 2
(virtual-virtual)



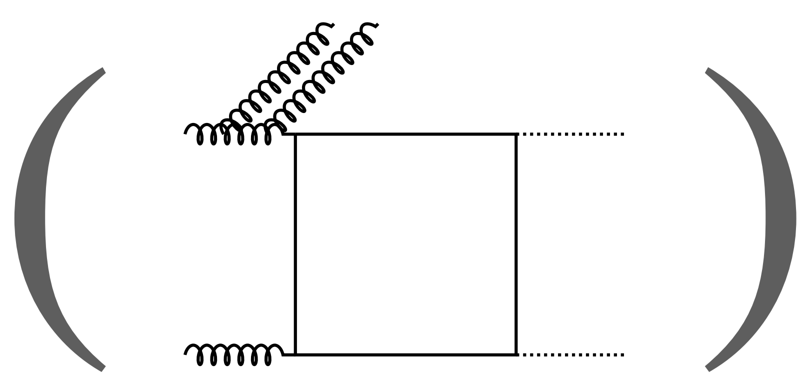
2-loop 2 -> 3
(real-virtual)



1-loop 2 -> 4
(real-real)



collinear
counter term



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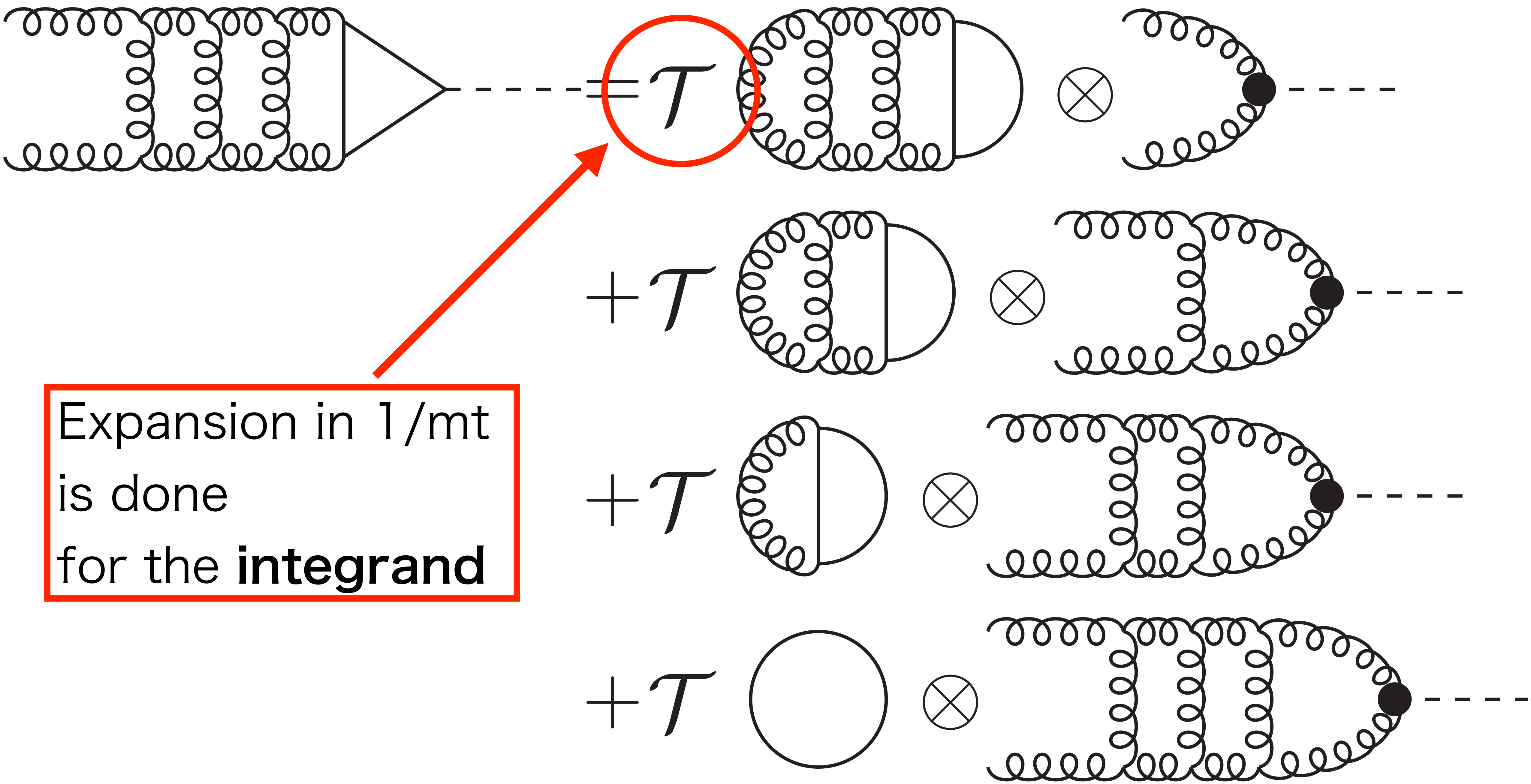
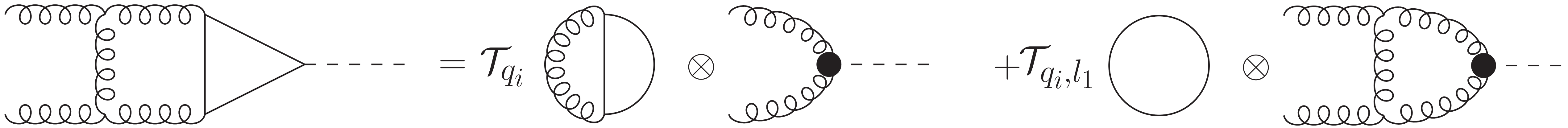
Expansion in 1/mt during the integration



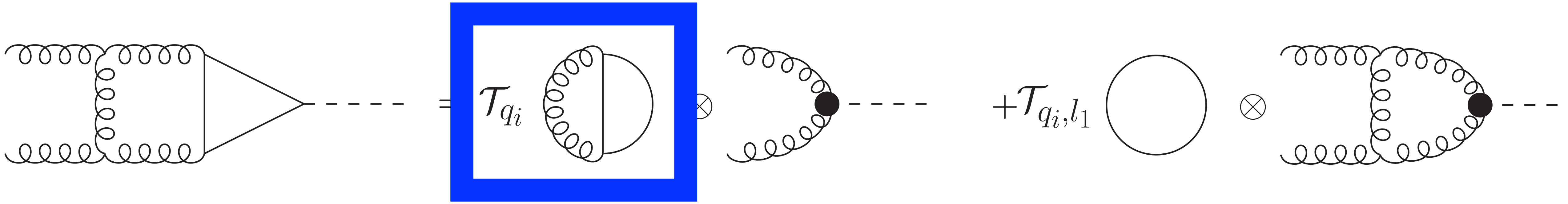
Expansion in 1/mt
is done
for the **integrand**



$$\int d^d\ell_1 d^d\ell_2 \frac{1}{(\ell_2-\ell_1)^2-m_t^2}\frac{1}{(\ell_2+q_1)^2-m_t^2}\frac{1}{(\ell_2-q_2)^2-m_t^2}\frac{1}{\ell_1^2}\frac{1}{(\ell_1+q_1)^2}\frac{1}{(\ell_1-q_2)^2}$$

$$=\int d^d\ell_1 d^d\ell_2 \frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_2^2-m_t^2}\frac{1}{(\ell_2-\ell_1)^2-m_t^2}\frac{1}{\ell_1^2}\frac{1}{\ell_1^2}\frac{1}{\ell_1^2}+\mathcal{O}(q_i)$$

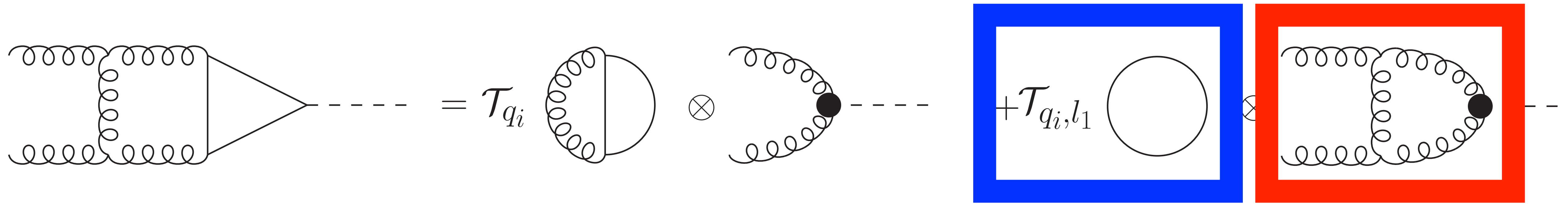
$$+\int d^d\ell_1 d^d\ell_2 \frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_1^2}\frac{1}{(\ell_1+q_1)^2}\frac{1}{(\ell_1-q_2)^2}+\mathcal{O}(q_i)$$



$$\int d^d\ell_1 d^d\ell_2 \frac{1}{(\ell_2-\ell_1)^2-m_t^2}\frac{1}{(\ell_2+q_1)^2-m_t^2}\frac{1}{(\ell_2-q_2)^2-m_t^2}\frac{1}{\ell_1^2}\frac{1}{(\ell_1+q_1)^2}\frac{1}{(\ell_1-q_2)^2}$$

$$= \int d^d\ell_1 d^d\ell \boxed{\frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_2^2-m_t^2}\frac{1}{(\ell_2-\ell_1)^2-m_t^2}\frac{1}{\ell_1^2}\frac{1}{\ell_1^2}\frac{1}{\ell_1^2}} + \mathcal{O}(q_i)$$

$$+ \int d^d\ell_1 d^d\ell_2 \frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_1^2}\frac{1}{(\ell_1+q_1)^2}\frac{1}{(\ell_1-q_2)^2} + \mathcal{O}(q_i)$$

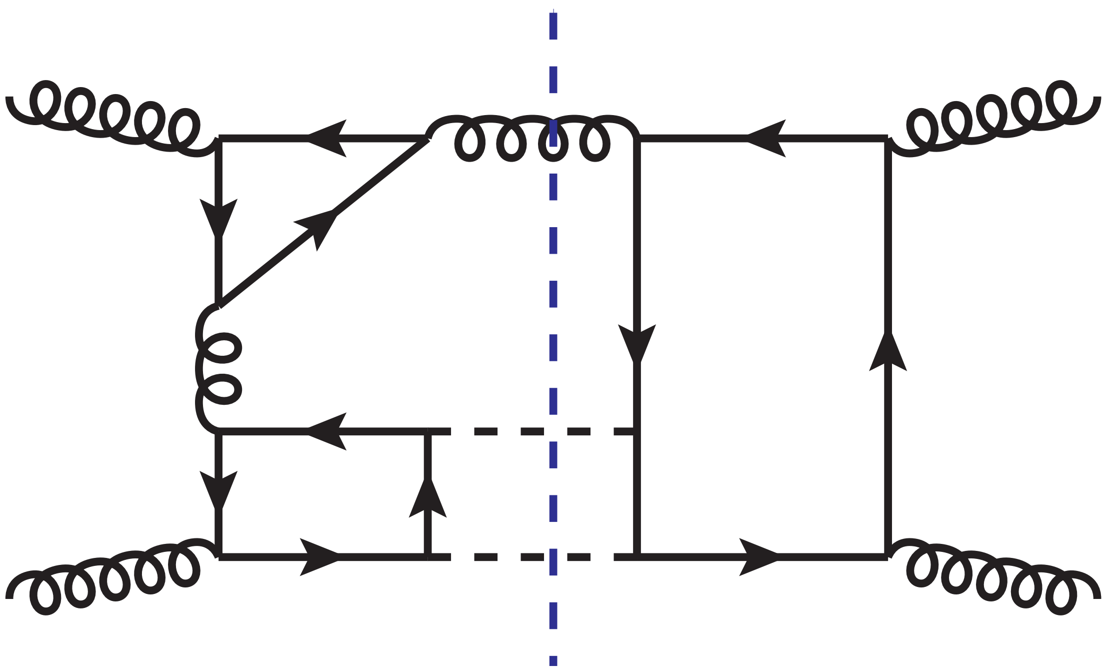


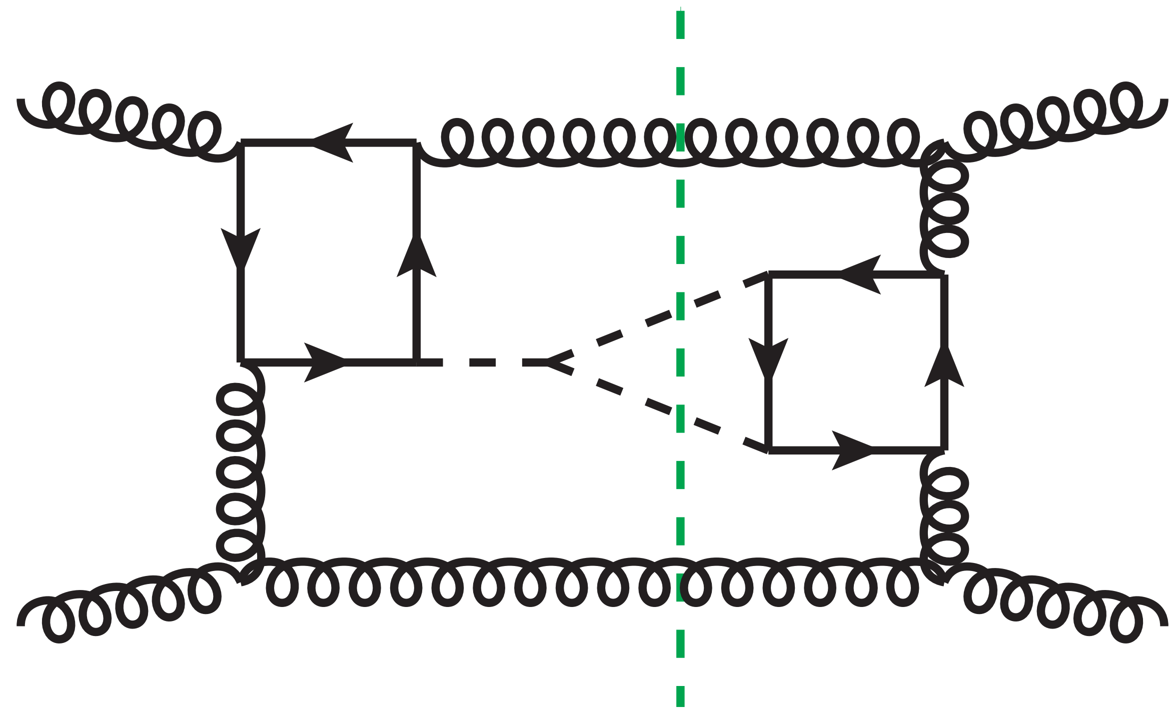
$$\int d^d\ell_1 d^d\ell_2 \frac{1}{(\ell_2-\ell_1)^2-m_t^2}\frac{1}{(\ell_2+q_1)^2-m_t^2}\frac{1}{(\ell_2-q_2)^2-m_t^2}\frac{1}{\ell_1^2}\frac{1}{(\ell_1+q_1)^2}\frac{1}{(\ell_1-q_2)^2}$$

$$= \int d^d\ell_1 d^d\ell_2 \frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_2^2-m_t^2}\frac{1}{(\ell_2-\ell_1)^2-m_t^2}\frac{1}{\ell_1^2}\frac{1}{\ell_1^2}\frac{1}{\ell_1^2} + \mathcal{O}(q_i)$$

$$+ \int d^d\ell_1 d^d\ell_2 \frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_2^2-m_t^2}\frac{1}{\ell_1^2}\frac{1}{(\ell_1+q_1)^2}\frac{1}{(\ell_1-q_2)^2} + \mathcal{O}(q_i)$$

For real radiations, we use the Optical Theorem





2-loop 2 -> 3
x
1-loop 2 -> 3

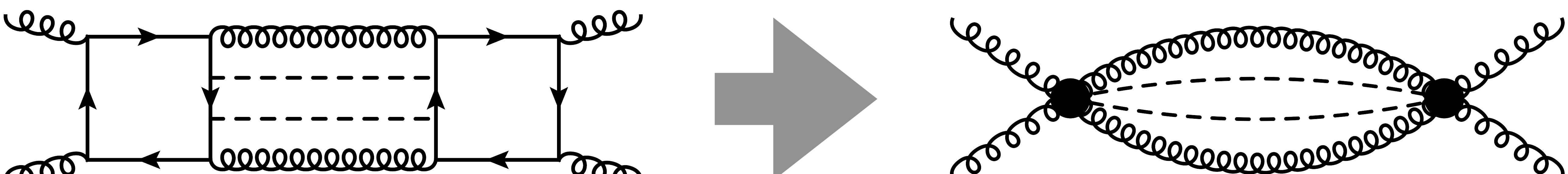
1-loop 2 -> 4
x
1-loop 2 -> 4

The expansion method explained before is also used.

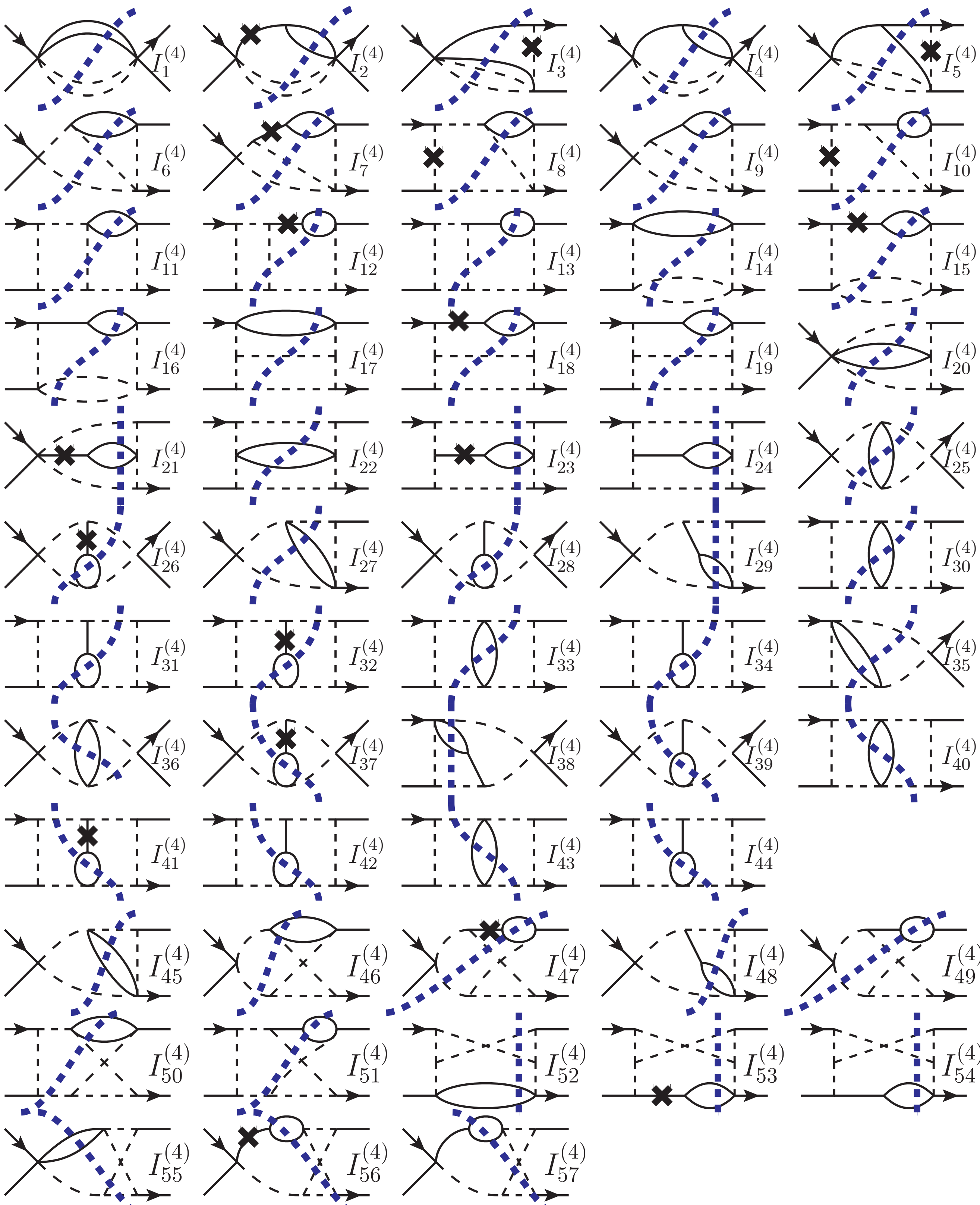
There are many diagrams

Channel	`qgraf` diagrams	`gen`-filtered diagrams	building block diagrams
gg	16,631,778	160,154	4,612
gc	1,671,006	5,426	336
$c\bar{c}$	406,662	3,879	243
cc	(not considered)	(not considered)	8
gq	1,671,006	5,426	336
cq	(not considered)	(not considered)	8
$q\bar{q}$	406,662	3,879	243
qq	(not considered)	(not considered)	8
qq'	203,331	34	4

using "building block"



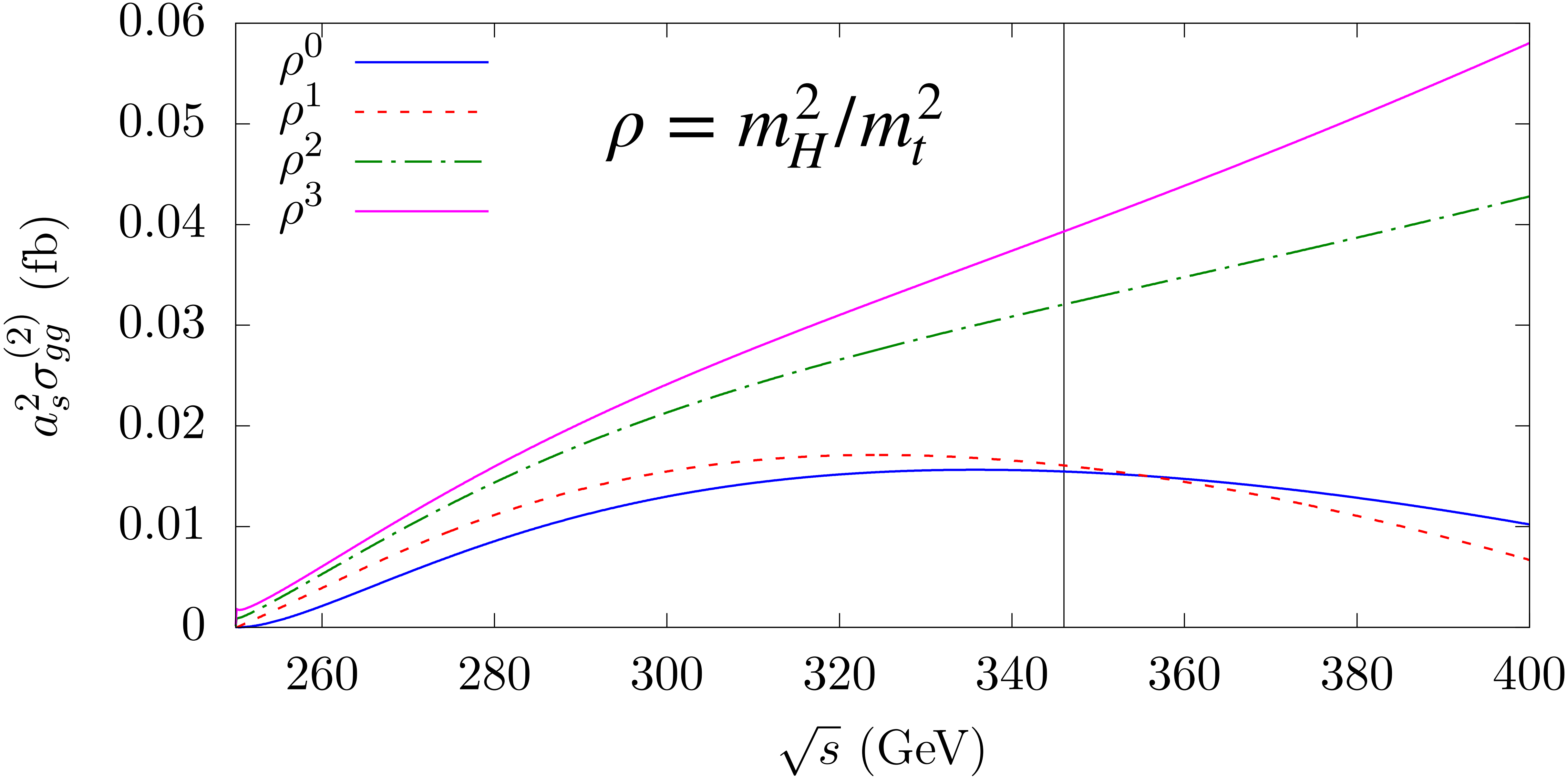
After an algebraic reduction (Integration-by-Parts reduction), there are only 17 1-loop 3-particle phase-space integral and 57 tree-level 4-particle phase-space integral



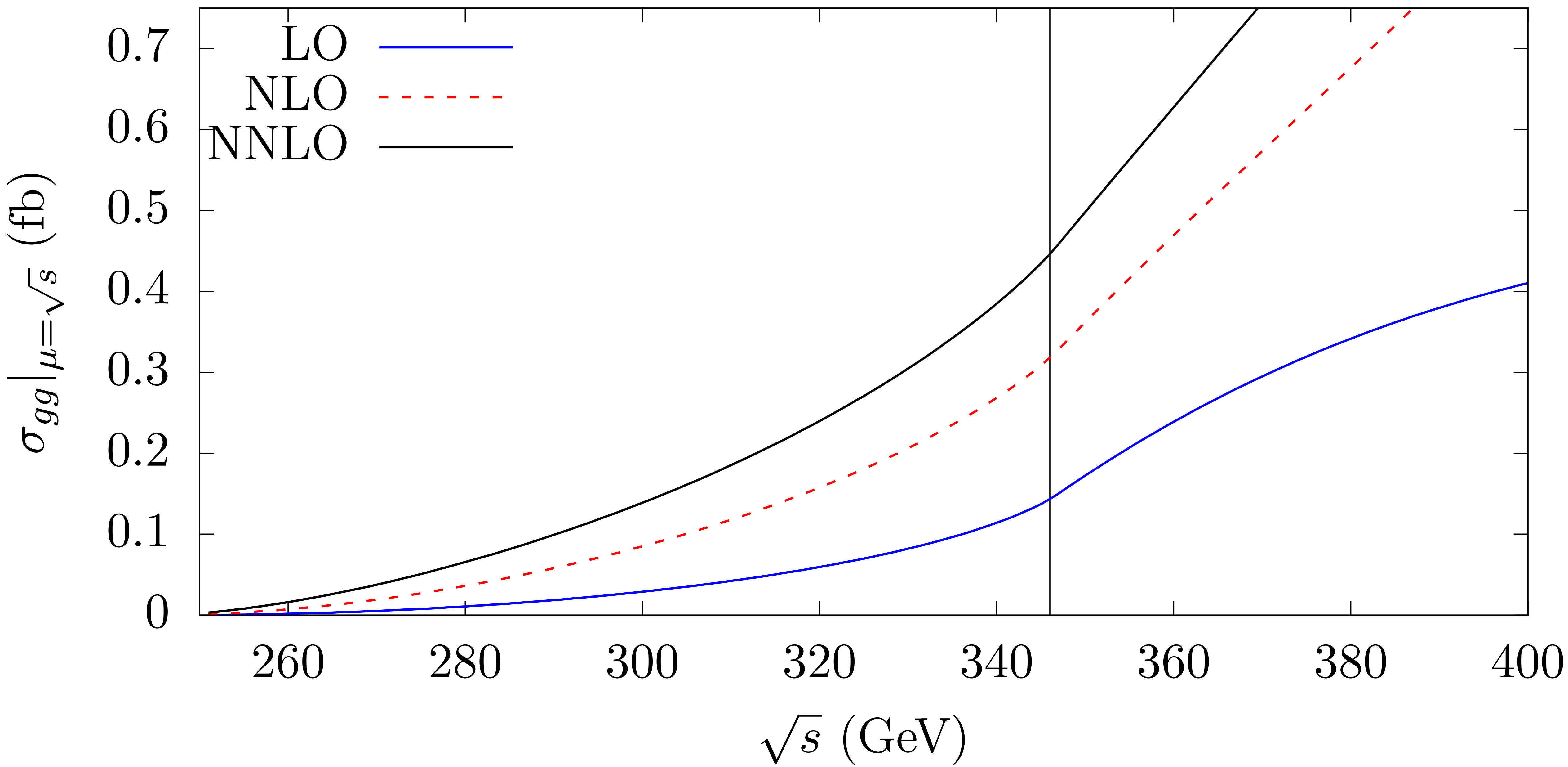
Due to the Optical Theorem, tree-level 4-particle phase-space integrals look like three-loop integrals.

We solve them analytically for the first time.

Result of partonic cross section



Result of partonic XS (preliminary)



Summary

- Higgs pair production is the simplest channel to examine the triple Higgs coupling.
- We improved the existing NNLO cross section by implementing the higher order 1/mt corrections analytically.