Starobinsky R^2 Inflation with Weyl Symmetry and Dark Matter

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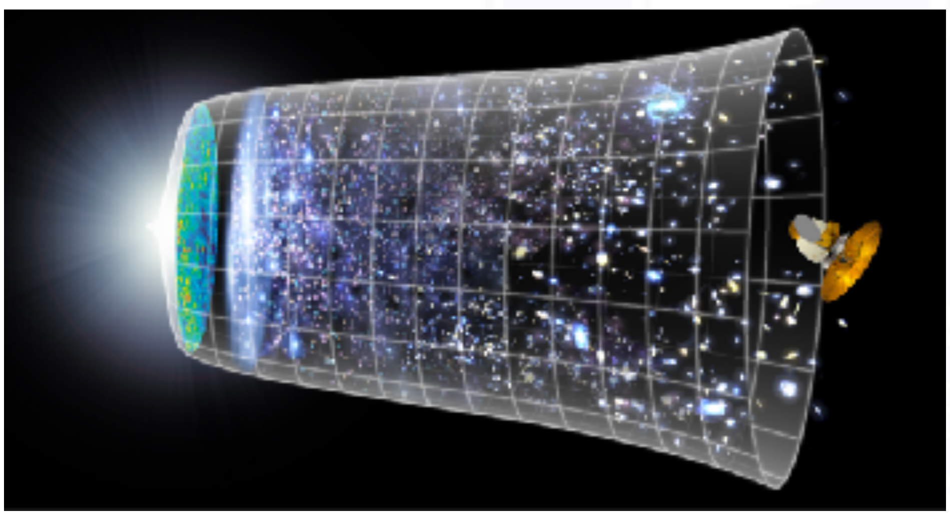
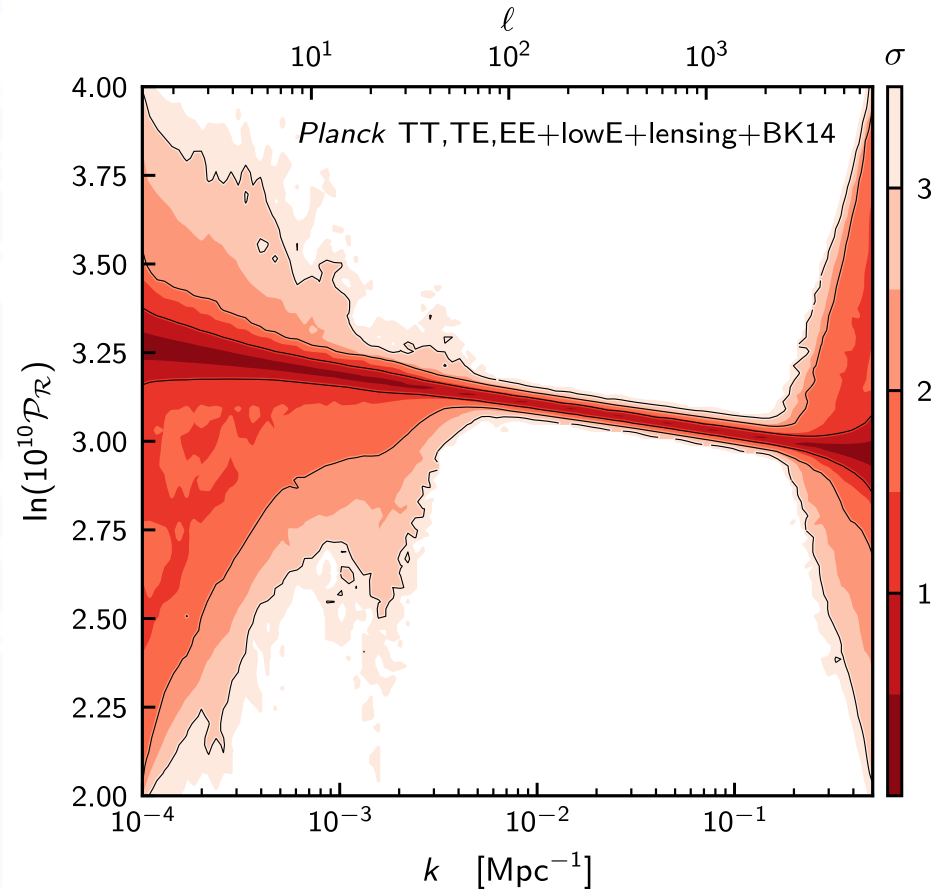
Inflation

- Inflation is a very attractive solution to many cosmic problems
  - flatness, horizon, initial cond
  - conformal/Weyl symmetry
  - breaking uncertain yet



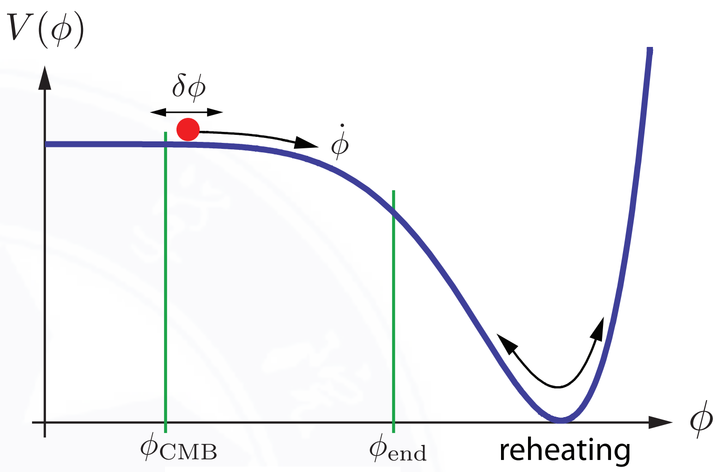


$$\Delta^2_\mathcal{R} \equiv \mathcal{P}_\mathcal{R} = A_\mathcal{R}\left(\frac{k}{k_*}\right)^{n_s-1}$$

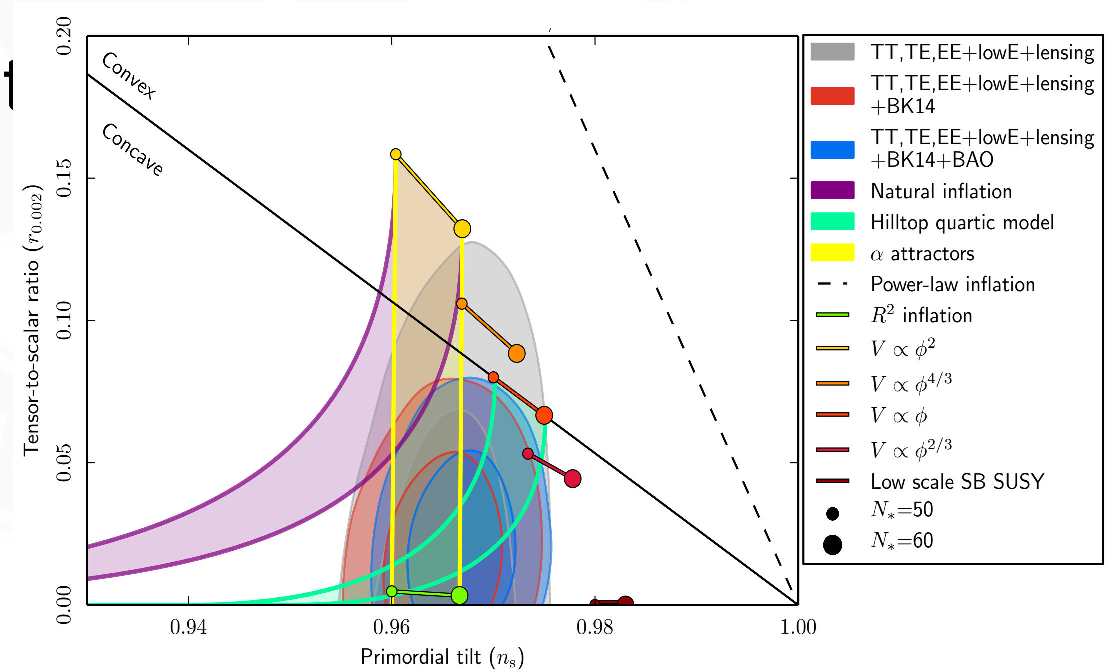
$$n_s \simeq 0.9649 \pm 0.0042$$

Inflation

- Inflation is a very attractive solution to many cosmic problems
  - flatness, horizon, initial cond
  - conformal/Weyl symmetry
  - breaking uncertain yet



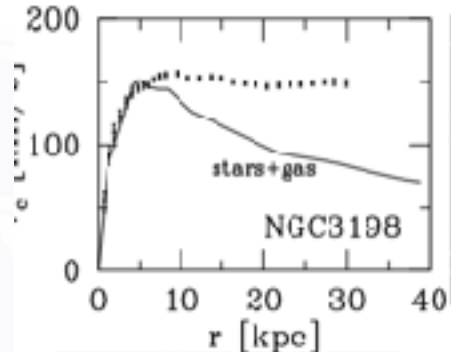
- Cosmological observat
now have entered a precision era
  - primordial GW
  - power spectrum

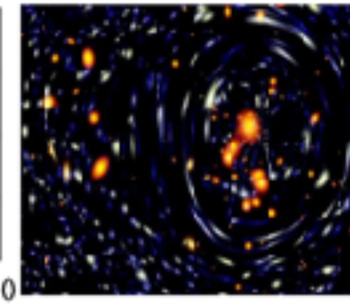
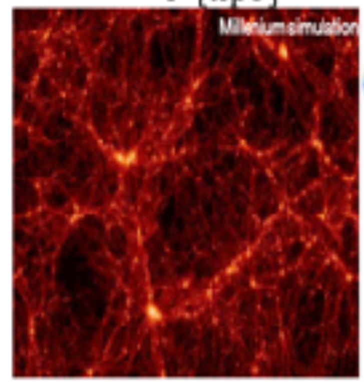


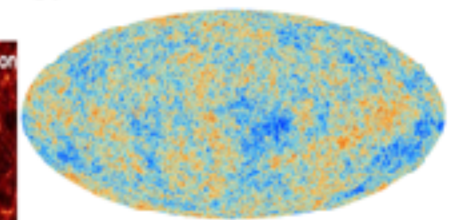
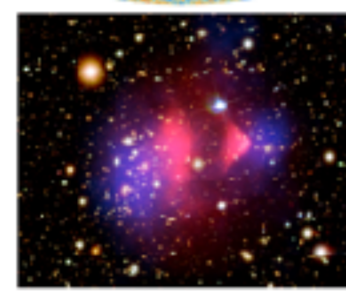
In future $r \sim 10^{-3}$

Dark Matter

- Dark Matter is challenging our standard model in particle physics
- So far only evidence for the gravitational interaction
- Extensions of Einstein's GR may give DM candidate



Symmetry determines interaction.

We propose a scenario based on Weyl symmetry,
$\Rightarrow$ Inflation and Dark Matter.

Weyl Symmetry

- Weyl symmetry was referred to

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x) = e^{2\theta(x)} g_{\mu\nu}(x)$$
$$W_\mu(x) \to W'_\mu(x) = W_\mu(x) - \partial_\mu \theta(x)$$

- First proposed by Weyl around 1919 in order to unify general relativity and electromagnetic interaction.
- Later Weyl modified it to

$$\psi(x) \to \psi'(x) = e^{i\theta(x)} \psi(x) \qquad \partial_\mu \to \partial_\mu - igW_\mu$$
$$W_\mu(x) \to W'_\mu(x) = W_\mu(x) - \partial_\mu \theta(x)$$

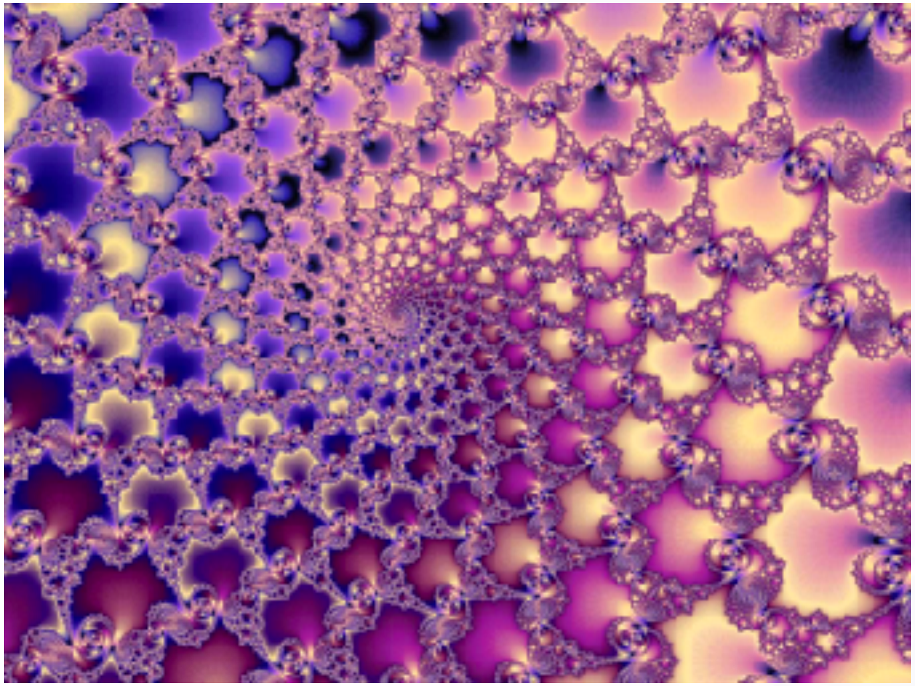
Gauge symmetry

- To describe electron after quantum mechanics.

Scale and Conformal

- Scale invariance

$$x^{\mu} \to \lambda\, x^{\mu}$$




Fractals

Scale and Conformal

- Scale invariance

$$x^\mu \to \lambda\, x^\mu$$

- Conformal invariance, in addition to above,

$$x^\mu \to \frac{x^\mu + v^\mu x^2}{1 + 2 v^\mu x_\mu + v^2 x^2}$$

They will also induce field transformations, $\phi(x) \to F[\phi(x)]$

- Weyl symmetry is different from the above ones, since coordinates do not change, only fields.

$$\phi(x) \to F[\phi(x)]$$

Most people use Weyl/Conformal interchangeably. **Ref. Y. Nakayama, 1302.0884**

Weyl Symmetry

- Terminology
  - conformal ~ Weyl ~ scale/scaling
- Weyl symmetry has wide applications in physics
  - Two-dimensional CFT—string theory
  - Conformal gravity—unitarity and instability

$$\frac{1}{2}W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} = \frac{1}{6}R^2 - R_{\mu\nu}R^{\mu\nu} + \frac{1}{2}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

  - Gauge theory of gravity
    - WU Yue-Liang, arXiv:1712.04537(EPJC)
      arXiv:1506.01807(PRD)
  - Particle Physics ……

R^2 inflation

Einstein Gravity

- Einstein-Hilbert action,

$$S=\int d^4x\mathcal{L}\ \ ,\ \mathcal{L}=\sqrt{-g}\Big[\frac{M_p^2}{2}R-\Lambda\Big]\qquad M_P^2=\frac{1}{8\pi G}\equiv 1$$

- Einstein frame

scalars do not couple to *R*

$$\frac{\mathcal{L}}{\sqrt{-g}}=\boxed{\frac{1}{2}R}+\frac{1}{2}f(\phi)\partial_\mu\phi\partial^\mu\phi-V(\phi)$$

- Jordan frame

$$\frac{\mathcal{L}}{\sqrt{-g}}=\boxed{\frac{1}{2}g(\phi)R}+\frac{1}{2}f(\phi)\partial_\mu\phi\partial^\mu\phi-V(\phi)$$

Conformal Transformation

- Jordan frame

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2} g(\phi) R + \frac{1}{2} f(\phi) \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

- After conformal transformation

$$g_{\mu\nu}(x) \Rightarrow \bar{g}_{\mu\nu}(x) = I(\phi) g_{\mu\nu}(x)$$

$$R = I(\phi) \left(\bar{R} + \frac{3}{2} \partial_\mu \ln I(\phi) \partial^\mu \ln I(\phi) \right)$$

$$I(\phi) = g(\phi)$$

- We are now in Einstein frame

$$\frac{\mathcal{L}}{\sqrt{-\bar{g}}} = \frac{1}{2} \bar{R} + \frac{1}{2} \Omega(\phi) \partial_\mu \phi \partial^\mu \phi - \bar{V}(\phi)$$

- Canonical kinetic terms?

Conformal Transformation

- With N scalar fields

YT and Y.L Wu, 2106.04726(PRD)

$$\frac{\mathcal{L}}{\sqrt{-g}} = f(\phi^i)R + \frac{1}{2}\delta_{ij}\nabla^\mu\phi^i\nabla_\mu\phi^j - V(\phi^i),$$

$$\tilde{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu},\ \Omega^2(x) = 2f(\phi^i).$$

- Einstein frame

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{1}{2}\tilde{R} - \frac{1}{2}\mathcal{G}_{ij}\tilde{\nabla}^\mu\phi^i\tilde{\nabla}_\mu\phi^j - \Omega^{-4}V(\phi^i), \mathcal{G}_{ij} = \frac{1}{2f}\left(\delta_{ij} + \frac{3}{f}f_i f_j\right).$$

- Canonical kinetic term flat field metric $\mathcal{G}_{ij}$, not possible for $N > 1$ generally. But conformally flat

$f(\phi^i) =$	constant	$\begin{cases} a\,(c + b_i\phi^i)^{\frac{1}{1-p}}, & p \neq 1 \\ a\exp(b_i\phi^i), & p = 1 \end{cases}$	$a + b_i\phi^i - \frac{1}{12}\delta_{ij}\phi^i\phi^j$
$\hat{\mathcal{G}}_{ij} =$	δ_{ij}	$\begin{cases} \delta_{ij} + \frac{3ab_ib_j}{(1-p)^2}\,(c + b_k\phi^k)^{\frac{2p-1}{1-p}}, & p \neq 1 \\ \delta_{ij} + 3ab_ib_j\exp(b_k\phi^k), & p = 1 \end{cases}$	$\delta_{ij} + \frac{3(b_i - \frac{\phi^i}{6})(b_j - \frac{\phi^j}{6})}{a + b_k\phi^k - \frac{1}{12}\delta_{kl}\phi^k\phi^l}$
$\hat{\Gamma}^i_{jk} =$	0	$\propto (p - \frac{1}{2})\,b_ib_jb_k$	$\propto \left(-\frac{1}{6}f_i\delta_{jk} - \frac{1}{2f}f_if_jf_k\right)$
$\hat{\mathcal{R}}_{ijkl} =$	0	0	$\propto \left(\delta_{i[k}\delta_{l]j} - \frac{6}{f}f_{[i}\delta_{j][k}f_{l]}\right)$

Conformal Transformation

- With N scalar fields

YT and Y.L Wu, 2106.04726(PRD)

$$\frac{\mathcal{L}}{\sqrt{-g}} = f(\phi^i)R + \frac{1}{2}\delta_{ij}\nabla^\mu\phi^i\nabla_\mu\phi^j - V(\phi^i),$$

$$\tilde{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu},\ \Omega^2(x) = 2f(\phi^i).$$

- Einstein frame

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{1}{2}\tilde{R} - \frac{1}{2}\mathcal{G}_{ij}\tilde{\nabla}^\mu\phi^i\tilde{\nabla}_\mu\phi^j - \Omega^{-4}V(\phi^i), \mathcal{G}_{ij} = \frac{1}{2f}\left(\delta_{ij} + \frac{3}{f}f_i f_j\right).$$

- Canonical kinetic term flat field metric $\mathcal{G}_{ij}$, not possible for $N > 1$ generally. But conformally flat
- Conformal flatness is always possible in the case of *f(R)* gravity with multiple scalars, due to the additional scalar degree of freedom.

R^2 Gravity

- Starobinsky Inflation

Starobinsky, Phys. Lett. B91, 99 (1980).

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R + \frac{\alpha}{12}R^2$$

- Equivalent to

Stelle, Gen. Rel. Grav. 9, 353 (1978).
Whitt, Phys. Lett. 145B, 176 (1984).

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\left(1 + \frac{\alpha}{3}\chi^2\right)R - \frac{\alpha}{12}\chi^4$$

- χ an auxiliary scalar field

$$\frac{\delta \mathcal{L}}{\delta \chi} = 0 \Rightarrow \chi^2 = R$$

- Equivalence can be shown by using EoM

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$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\left(1 + \frac{\alpha}{3}\chi^2\right) R - \frac{\alpha}{12}\chi^4$$

Jordan frame

- Conformal transformation

$$g_{\mu\nu}(x) \Rightarrow \overline{g}_{\mu\nu}(x) = \left(1 + \frac{\alpha}{3}\chi^2\right) g_{\mu\nu}(x)$$

$$\frac{\mathcal{L}}{\sqrt{-\overline{g}}} = \frac{1}{2}\overline{R} + \frac{1}{2}\Omega(\chi)\partial_\mu\chi\partial^\mu\chi - \overline{V}(\chi)$$

Einstein frame

R^2 Gravity

- Starobinsky Inflation

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$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R + \frac{\alpha}{12}R^2$$

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$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\left(1 + \frac{\alpha}{3}\chi^2\right)R - \frac{\alpha}{12}\chi^4$$

$\boxed{\text{Jordan frame}}$

- After conformal transformation

$$\frac{\mathcal{L}}{\sqrt{-\overline{g}}} = \frac{1}{2}\overline{R} + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \overline{V}(\sigma), \ \overline{V}(\sigma) = \frac{3}{4\alpha}\left[1 - \exp(-\sqrt{\frac{2}{3}}\sigma)\right]^2$$

$\boxed{\text{Einstein frame}}$

$$\sigma = \sigma\left(\chi\right)$$

Starobinsky Inflation

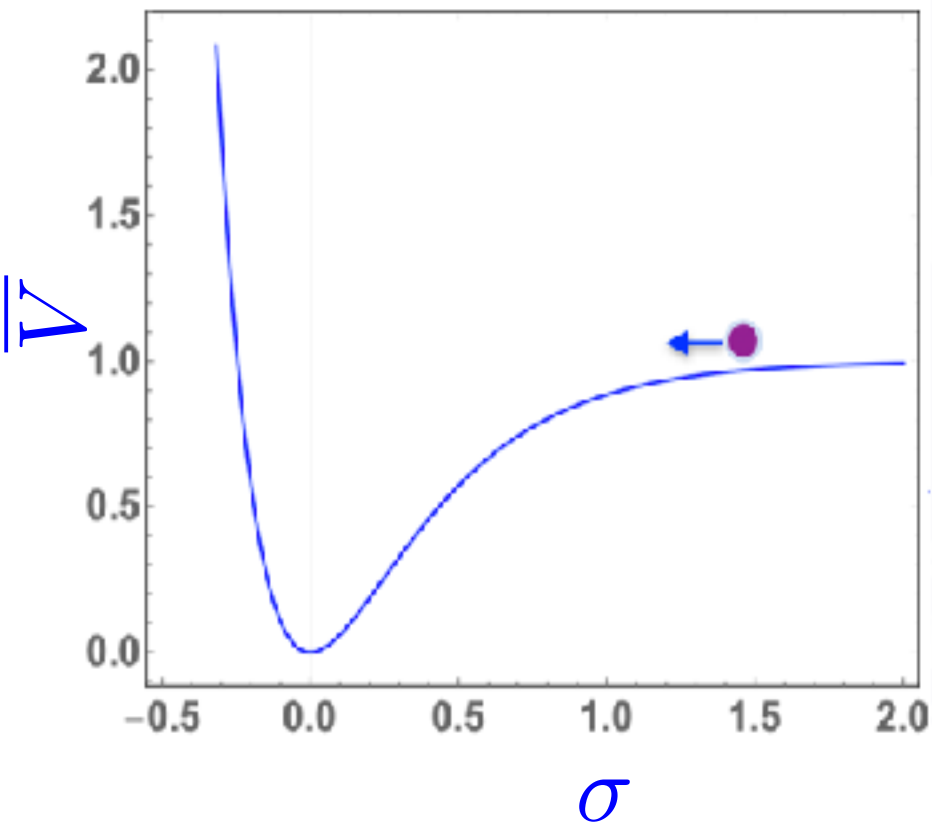
- Potential

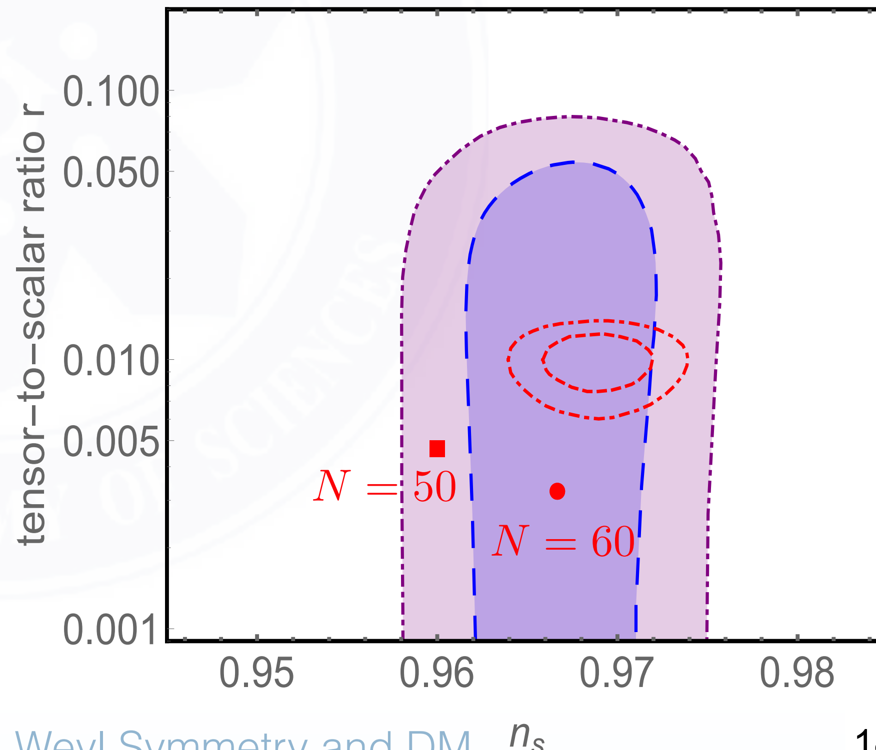
$$\overline{V}(\sigma) = \frac{3}{4\alpha}\left[1 - \exp(-\sqrt{\frac{2}{3}}\sigma)\right]^2$$

$$\epsilon = \frac{1}{2}\left(\frac{\overline{V}'(\sigma)}{\overline{V}(\sigma)}\right)^2, \; \eta = \frac{\overline{V}''(\sigma)}{\overline{V}(\sigma)}$$

$$n_s = 1 - 6\epsilon + 2\eta, r = 16\epsilon$$

$$n_s \simeq 1 - \frac{2}{N}, \; r \simeq \frac{12}{N^2}$$





R^2 Gravity

- Starobinsky Inflation

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R + \frac{\alpha}{12}R^2$$

- Global scaling symmetry if EH action is absent

$$g_{\mu\nu}(x) \to \overline{g}_{\mu\nu}(x) = f^2 g_{\mu\nu}(x) \Rightarrow \mathcal{L}[g] = \overline{\mathcal{L}}[\overline{g}]$$

- But not local symmetry

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x) = e^{2\theta(x)} g_{\mu\nu}(x)$$

$$W_\mu(x) \to W'_\mu(x) = W_\mu(x) - \partial_\mu\theta(x)$$

Weyl $\hat{R}^2$ Inflation

- Covariant derivative

YT, Y.L.Wu, arXiv:2006.02811

$$\partial_\mu g_{\rho\sigma} \to (\partial_\mu + 2W_\mu)\, g_{\rho\sigma}$$

$$D_\mu = \partial_\mu + qW_\mu$$

$$q = \begin{cases} +2 \text{ metric} \\ -1 \text{ scalar} \\ -\frac{3}{2} \text{ fermion} \\ 0 \quad \text{vector} \end{cases}$$

- The modified connection

$$\hat{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + \left[W_\mu \delta^\rho_\nu + W_\nu \delta^\rho_\mu - W^\rho g_{\mu\nu}\right],$$
$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right)$$

- And the modified Ricci tensor and scalar $\hat{R}$

$$\hat{R}^\rho_{\sigma\mu\nu} = \partial_\mu \hat{\Gamma}^\rho_{\sigma\nu} - \partial_\nu \hat{\Gamma}^\rho_{\sigma\mu} + \hat{\Gamma}^\rho_{\mu\tau}\hat{\Gamma}^\tau_{\sigma\nu} - \hat{\Gamma}^\rho_{\nu\tau}\hat{\Gamma}^\tau_{\sigma\mu},$$
$$\hat{R}_{\sigma\nu} = \hat{R}^\rho_{\sigma\rho\nu}, \hat{R} = g^{\sigma\nu}\hat{R}_{\sigma\nu}$$

Warm-up

- First consider the Weyl $\hat{R}$ gravity

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\phi^2\hat{R} + \frac{1}{2}\zeta D^\mu\phi D_\mu\phi - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} - \lambda\phi^4\right]$$

- It is invariant under scaling transformation

$$g_{\mu\nu} \to g'_{\mu\nu} = f^2 g_{\mu\nu}$$

$$W_\mu \to W'_\mu = W_\mu - \partial_\mu \ln f \quad \hat{\Gamma}'^{\rho}_{\mu\nu} = \hat{\Gamma}^{\rho}_{\mu\nu}, \hat{R}'^{\rho}_{\sigma\mu\nu} = \hat{R}^{\rho}_{\sigma\mu\nu}, D'_\mu\phi' = f D_\mu\phi$$

$$\phi \to \phi' = f^{-1}\phi$$

- One can write in a familiar form by using

$$\hat{R} = R + 6W_\mu W^\mu + 6\nabla_\mu W^\mu, \nabla_\mu W^\mu = \frac{6}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}W^\mu\right)$$

$$\phi^2\hat{R} = \phi^2 R - 6\partial_\mu\phi\partial^\mu\phi + 6D_\mu\phi D^\mu\phi$$

- When W_μ is absent $\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(\phi^2 R - 6\partial_\mu\phi\partial^\mu\phi\right)$

Warm-up

- First consider the Weyl $\hat{R}$ gravity

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\phi^2\hat{R} + \frac{1}{2}\zeta D^\mu\phi D_\mu\phi - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} - \lambda\phi^4\right]$$

- Einstein frame is set by $\phi^2 = 1$ due to Weyl symmetry

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}R - \lambda - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\zeta + 6)\,W_\mu W^\mu\right]$$

- Massive W_μ with discrete symmetry, might be a dark matter candidate.
- However, inflation is not implemented and other extension is needed.

Weyl $\hat{R}^2$ Inflation

YT, Y.L.Wu, arXiv:2006.02811
Ghilencea, arXiv:1906.11572
Ferreira et al,arXiv:1906.03415

- Now let us consider

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\phi^2\hat{R} + \frac{\alpha}{12}\hat{R}^2 + \frac{1}{2}\zeta D^\mu\phi D_\mu\phi - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} - \lambda\phi^4\right]$$

- Similarly introduce an auxiliary field χ

$$\sqrt{-g}\left[\frac{1}{2}\left(\phi^2 + \frac{\alpha}{3}\chi^2\right)\hat{R} - \frac{\alpha}{12}\chi^4 + \frac{1}{2}\zeta D^\mu\phi D_\mu\phi - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} - \lambda\phi^4\right]$$

- The equivalence can be shown with $\frac{\delta\mathcal{L}}{\delta\chi} = 0 \Rightarrow \chi^2 = \hat{R}$
- Fix to Einstein frame $\phi^2 + \frac{\alpha}{3}\chi^2 = 1$

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R + \frac{1}{2}\zeta D^\mu\phi D_\mu\phi - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} + 3W^\mu W_\mu - \frac{3}{4\alpha}\left(1-\phi^2\right)^2 - \lambda\phi^4$$

Continued

- We can combine the Weyl boson term

$$\frac{1}{2}\zeta D^\mu\phi D_\mu\phi + 3W^\mu W_\mu = \frac{1}{2}\left[\left(6+\zeta\phi^2\right)W^\mu W_\mu - \zeta W^\mu \partial_\mu\phi^2\right] + \frac{1}{2}\zeta\partial^\mu\phi\partial_\mu\phi$$

$$= \frac{1}{2}\left(6+\zeta\phi^2\right)\left[W_\mu - \frac{1}{2}\partial_\mu \ln\left(6+\zeta\phi^2\right)\right]^2 + \frac{1}{2}\frac{6\zeta}{6+\zeta\phi^2}\partial^\mu\phi\partial_\mu\phi$$

- With gauge transformation $\boxed{\overline{W}_\mu = W_\mu - \frac{1}{2}\partial_\mu \ln\left(6+\zeta\phi^2\right)}$

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R - \frac{1}{4g_W^2}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \frac{1}{2}\left(6+\zeta\phi^2\right)\overline{W}^\mu\overline{W}_\mu + \frac{3\zeta}{6+\zeta\phi^2}\partial^\mu\phi\partial_\mu\phi - \frac{3}{4\alpha}\left(1-\phi^2\right)^2 - \lambda\phi^4$$

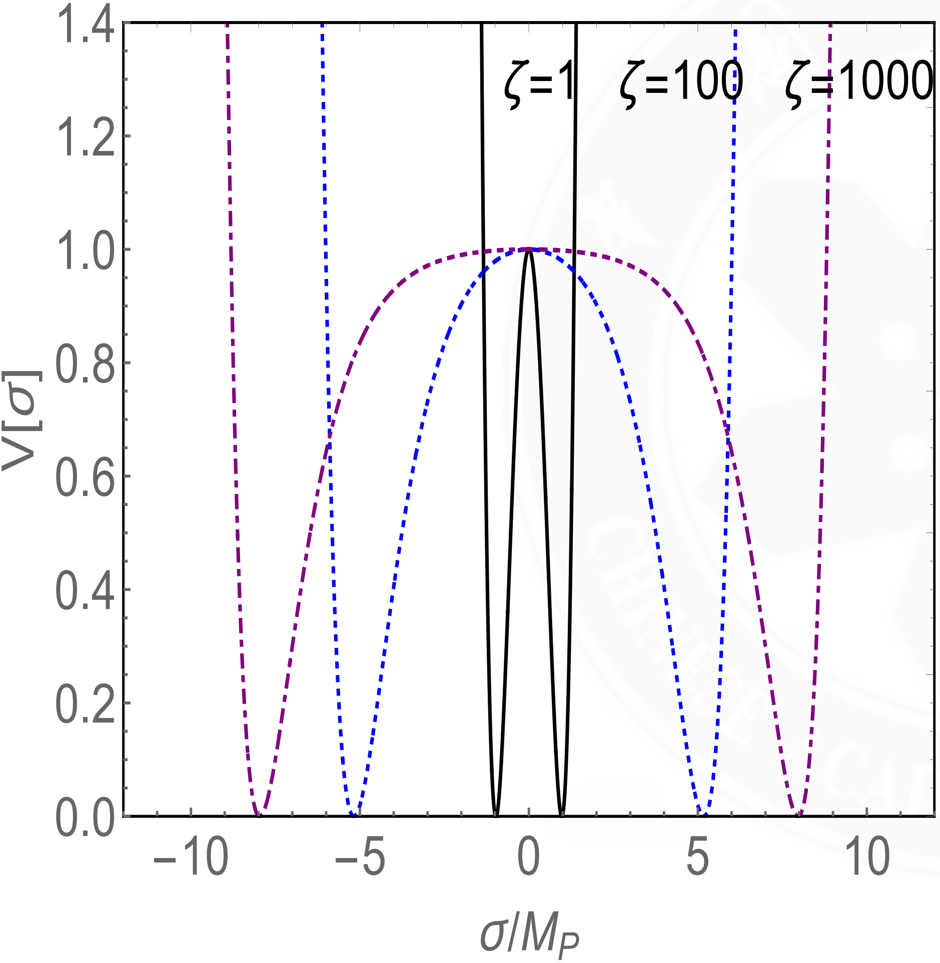
$$\boxed{\frac{d\sigma}{d\phi} = \pm\sqrt{\frac{6\zeta}{6+\zeta\phi^2}} \Rightarrow \begin{array}{ll} \phi = \sqrt{\frac{6}{+\zeta}}\sinh\frac{\pm\sigma}{\sqrt{6}}, & \text{for } \zeta > 0 \\ \phi = \sqrt{\frac{6}{-\zeta}}\cosh\frac{\pm\sigma}{\sqrt{6}}, & \text{for } \zeta < 0 \end{array}}$$

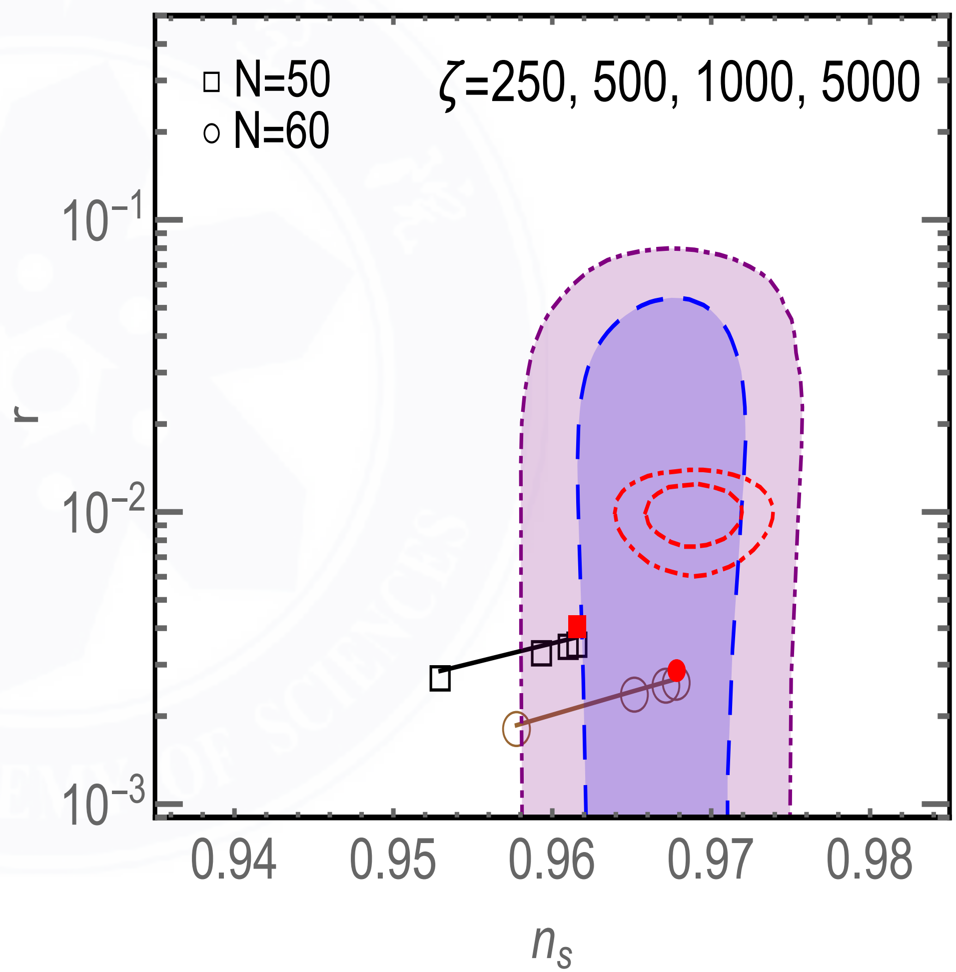
- finally

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R - \frac{1}{4g_W^2}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \frac{1}{2}\left(6+\zeta\phi^2\right)\overline{W}^\mu\overline{W}_\mu + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{3}{4\alpha}\left(1-\phi^2\right)^2 - \lambda\phi^4$$

Inflation

$$V(\sigma) \simeq \frac{3}{4\alpha}\left[1 - \frac{6}{\zeta}\sinh^2\left(\frac{\sigma}{\sqrt{6}}\right)\right]^2 \xrightarrow{\zeta\to\infty} V(\bar{\sigma}) = \frac{3}{4\alpha'}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\bar{\sigma}\right)\right]^2$$





Analytic Limit

- We have seen

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R - \frac{1}{4g_W^2}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \frac{1}{2}\left(6+\zeta\phi^2\right)\overline{W}^{\mu}\overline{W}_{\mu} + \frac{3\zeta}{6+\zeta\phi^2}\partial^{\mu}\phi\partial_{\mu}\phi - \frac{3}{4\alpha}\left(1-\phi^2\right)^2 - \lambda\phi^4$$

- and

$$\frac{d\sigma}{d\phi} = \pm\sqrt{\frac{6\zeta}{6+\zeta\phi^2}} \xrightarrow{\zeta\to\infty} \pm\frac{\sqrt{6}}{\phi}$$

$$\phi = \exp\left(\pm\frac{\sigma}{\sqrt{6}}\right)$$

- So the potential is approximated as

$$\overline{V}(\sigma) = \frac{3}{4\alpha}\left[1-\exp(-\sqrt{\frac{2}{3}}\sigma)\right]^2$$

- same as the one in Starobinsky model

General Case

- If the Lagrangian has the following form

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}F(\hat{R},\phi) + \frac{1}{2}\zeta D^{\mu}\phi D_{\mu}\phi - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} - \lambda\phi^4.$$

- One can similarly get

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\left[F(\chi^2,\phi) + F_{\hat{R}}(\chi^2,\phi)\left(\hat{R}-\chi^2\right)\right] + \frac{1}{2}\zeta D^{\mu}\phi D_{\mu}\phi - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} - \lambda\phi^4$$

where $F_{\hat{R}}$ denotes the derivative of $F(\hat{R},\phi)$ over $\hat{R}$ and $F(\chi^2,\phi) \equiv F(\hat{R}\to\chi^2,\phi)$. One may check that the equation of motion for χ still gives $\chi^2 = \hat{R}$.

- We can set to Einstein frame by $F_{\hat{R}}(\chi^2,\phi) = 1$
- The potential now is

$$\mathbf{V}(\phi) = -\left[F(\chi^2,\phi) - \chi^2 F_{\hat{R}}(\chi^2,\phi)\right]/2 + \lambda\phi^4$$

General Case

- If the Lagrangian has the following form

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}F(\hat{R},\phi) + \frac{1}{2}\zeta D^\mu\phi D_\mu\phi - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} - \lambda\phi^4.$$

- One can similarly get

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\left[F(\chi^2,\phi) + F_{\hat{R}}\left(\chi^2,\phi\right)\left(\hat{R}-\chi^2\right)\right] + \frac{1}{2}\zeta D^\mu\phi D_\mu\phi - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} - \lambda\phi^4$$

- For example

$$F(\hat{R},\phi) = \phi^2\hat{R} + \frac{\alpha}{6}\hat{R}^2 + \frac{\beta}{6\phi^2}\hat{R}^3.$$

- The potential is

$$F_{\hat{R}}\left(\chi^2,\phi\right) = \phi^2 + \frac{\alpha}{3}\chi^2 + \frac{\beta}{2\phi^2}\chi^4,\ F\left(\chi^2,\phi\right) = \phi^2\chi^2 + \frac{\alpha}{6}\chi^4 + \frac{\beta}{6\phi^2}\chi^6.$$

$$\mathbf{V}(\phi) = \frac{\alpha}{6}\chi^4 + \frac{\beta}{3\phi^2}\chi^6 + \lambda\phi^4. \quad \chi^2 = \frac{\alpha/3 \pm \sqrt{\alpha^2/9 + 2\beta(1-\phi^2)/\phi^2}}{\beta}\phi^2.$$

Dark Matter

Weyl gauge boson as Dark Matter

- Lagrangian

$$\frac{\mathcal{L}}{\sqrt{-g}} \supset \sum_{i=1}^{N} \frac{\alpha_i}{2} \left(\phi_i^2 R - 6\partial_\mu \phi_i \partial^\mu \phi_i\right) + \frac{1}{2}\sum_{i=1}^{N} \zeta_i D_\mu \phi_i D^\mu \phi_i - V\left(\phi_i\right) - \frac{1}{4g_W^2} F_{\mu\nu}F^{\mu\nu}$$

- At first sight, we have linear term of W_μ

$$2\mathcal{L}_k = \sum_{i=1}^{N}\left[\zeta_i \partial_\mu \phi_i \partial^\mu \phi_i - W_\mu \partial^\mu \left(\zeta_i \phi_i^2\right) + \zeta_i \phi_i^2 W_\mu W^\mu\right]$$

$$= \Xi \left(W_\mu W^\mu - W_\mu \frac{\partial^\mu \Xi}{\Xi}\right) + \sum_{i=1}^{N} \zeta_i \partial_\mu \phi_i \partial^\mu \phi_i$$

$$= \Xi \left(W_\mu - \frac{1}{2}\partial^\mu \ln \Xi\right)^2 - \frac{1}{4}\Xi \times \left(\partial^\mu \ln \Xi\right)^2 + \sum_{i=1}^{N} \zeta_i \partial_\mu \phi_i \partial^\mu \phi_i$$

$$= \Xi \overline{W}_\mu \overline{W}^\mu - \frac{\Xi}{4}\left[\partial_\mu \ln \Xi\right]^2 + \sum_{i=1}^{N} \zeta_i \partial_\mu \phi_i \partial^\mu \phi_i$$

$$\Xi \equiv \sum_{i=1}^{N} \zeta_i \phi_i^2,$$

$$\overline{W}_\mu = W_\mu - \frac{1}{2}\partial_\mu \ln \Xi$$

- Z_2 symmetry, $\overline{W}_\mu \to -\overline{W}_\mu$ **Stable !**

Weyl gauge boson as Dark Matter

- Lagrangian with $\hat{R}^2$

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\hat{R}\sum_I \beta_I \phi_I^2 + \frac{\alpha}{12}\hat{R}^2 + \frac{1}{2}\sum_I \zeta_I D^\mu \phi_I D_\mu \phi_I - \frac{1}{4g_W^2}F_{\mu\nu}F^{\mu\nu} - V(\phi_I)$$

- We can show

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}R - \frac{1}{4g_W^2}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \frac{\left(6+\sum_I \zeta_I \phi_I^2\right)}{2}\overline{W}_\mu \overline{W}^\mu$$
$$+\frac{1}{2}\frac{1}{6+\sum_I \zeta_I \phi_I^2}\left[6\sum_I \zeta_I \partial_\mu \phi_I \partial^\mu \phi_I - \sum_{I\neq J}\zeta_I \zeta_J \phi_I \phi_J \partial_\mu \phi_I \partial^\mu \phi_J\right] - V(\phi_I)$$

- The gauge transformation $\overline{W}_\mu = W_\mu - \frac{1}{2}\partial_\mu \ln\left(6 + \sum_I \zeta_I \phi_I^2\right)$
- Again, we have

$$Z_2 \text{ symmetry, } \overline{W}_\mu \to -\overline{W}_\mu$$

Stable !

Production

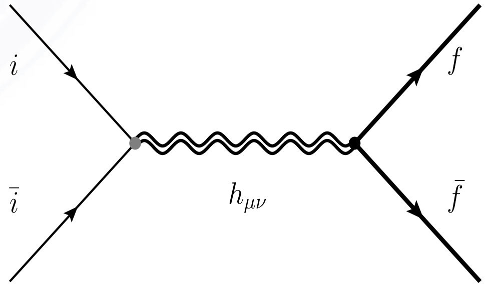
- W_μ couples gravitationally and to all scalars ϕ_i with gauge coupling .
- The simplest way is through gravitational production
  - Vacuum fluctuation (expanding universe)

$$\Omega_W \simeq \Omega_{\rm DM} \times \sqrt{\frac{m_W}{6\times 10^{-11}\mathrm{GeV}}} \times \left(\frac{\mathcal{H}}{10^{13}\mathrm{GeV}}\right)^2$$

Graham, Mardon, Rajendran,1504.02102(PRD)
Ema, Nakayama, **YT**, 1903.10973(JHEP)
Ema, Nakayama, **YT**, 1804.07471(JHEP)…

  - Gravitational Scattering

YT&Y.-L. Wu, 1604.04701(PLB), 1708.05138(PLB)
Garny, Sandora, Sloth, 1511.03278(PRL)
Garny,Palessandro, Sandora, Sloth, 1709.09688(JCAP)



EFT in Einstein's Gravity

- Einstein-Hilbert action

$$S = \int \mathcal{L}\, d^4x \,, \quad \mathcal{L} = \sqrt{-g}\left[\frac{1}{16\pi G}R + L_m\right]$$

- EFT for *$E<<M_p$* ← Justified after inflation

$$\mathcal{L}_{\rm int} = \frac{\kappa}{2} h_{\mu\nu}T^{\mu\nu}, \qquad \kappa \propto \frac{1}{M_p}$$

Energy-Momentum Tensor

$$T_S^{\mu\nu} = -\eta^{\mu\nu}\partial^\alpha S^\dagger \partial_\alpha S + \eta^{\mu\nu} m_S^2 S^\dagger S + \partial^\mu S^\dagger \partial^\nu S + \partial^\nu S^\dagger \partial^\mu S,$$

$$T_F^{\mu\nu} = -\eta^{\mu\nu}\left(\overline{F} i\partial\!\!\!/ F - m_F \overline{F} F\right) + \frac{1}{2}\overline{F} i\gamma^\mu \partial^\nu F + \frac{1}{2}\overline{F} i\gamma^\nu \partial^\mu F$$
$$+ \frac{1}{2}\eta^{\mu\nu}\partial^\alpha\left(\overline{F} i\gamma_\alpha F\right) - \frac{1}{4}\partial^\mu\left(\overline{F} i\gamma^\nu F\right) - \frac{1}{4}\partial^\nu\left(\overline{F} i\gamma^\mu F\right),$$

$$T_V^{\mu\nu} = \eta^{\mu\nu}\left(\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} - \frac{1}{2}m_V^2 V^\alpha V_\alpha\right) - \left(F^{\mu\alpha}F^\nu{}_\alpha - m_V^2 V^\mu V^\nu\right),$$

$$T_\gamma^{\mu\nu} = \frac{1}{4}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} - F^{\mu\alpha}F^\nu{}_\alpha .$$

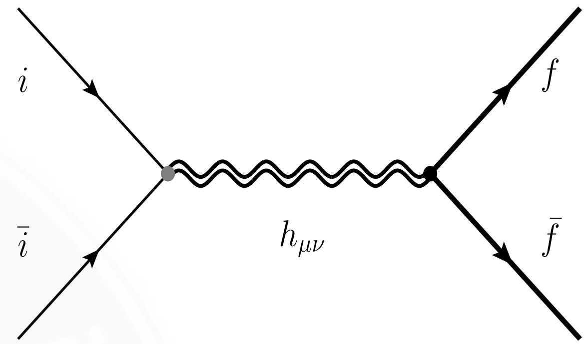
Non-minimal coupling

$\zeta S^\dagger S R \to 2\zeta(\partial^\mu\partial^\nu - \eta^{\mu\nu}\partial_\alpha\partial^\alpha)S^\dagger S$

Annihilation Processes

- Boltzmann Equation

$$\frac{d\left(n_X a^3\right)}{a^3 dt} = \frac{dn_X}{dt} + 3\mathcal{H}n_X = \langle\sigma v\rangle\left[n_X^2 - \left(n_{\rm eq}\right)^2\right]$$



- Neglect n_X, reduced to

$$\frac{d\left(n_X a^3\right)}{a^3 dt} = \frac{g^2 T}{32\pi^4}\int ds\,\sigma\,\sqrt{s}(s-4m^2)K_1\left(\frac{\sqrt{s}}{T}\right),$$

yield $Y \equiv n_X/s$

$$\frac{dY}{d\ln T} \simeq \frac{n_{\rm eq}\langle\sigma v\rangle}{H}$$

- The core

$$\sigma = \frac{\kappa^4}{32\pi s\left(S g_i^2\right)}\frac{|\vec{p}_f|}{|\vec{p}_i|}\mathcal{A}$$

$$|\vec{p}_i| = \sqrt{s^2/4 - m^2},\ |\vec{p}_f| = \sqrt{s^2/4 - M^2}$$

Massless limit
$\sigma \propto \kappa^4 s$
Tang & Wu, **1604.04701**

Various Contributions

YT&Y.-L. Wu, 1604.04701(PLB), 1708.05138(PLB)

- Scalar

$$
\begin{aligned}
\mathcal{A}(S\to S) =& \frac{7m^4M^4}{30s^2} - \frac{m^2M^2}{30s}\left(m^2+M^2\right), \\
&+\frac{1}{40}\left(m^4+4m^2M^2+M^4\right)+\frac{s}{120}\left(m^2+M^2\right)+\frac{s^2}{240}, \\
\mathcal{A}(F\to S) =& -\frac{7m^4M^4}{15s^2} - \frac{m^2M^2}{60s}(M^2-4m^2) \\
&+\frac{1}{60}\left(2M^4+3m^2M^2-3m^4\right)-\frac{s}{240}(4M^2-m^2)+\frac{s^2}{480}, \\
\mathcal{A}(V\to S) =& \frac{101m^4M^4}{30s^2} - \frac{m^2M^2}{10s}\left(11M^2+m^2\right) \\
&+\frac{1}{120}\left(19M^4+76m^2M^2+49m^4\right)-\frac{7s}{120}\left(m^2+M^2\right)+\frac{s^2}{80}, \\
\mathcal{A}(\gamma\to S) =& \frac{1}{120}\left(s-4M^2\right)^2,
\end{aligned}
$$

Various Contributions

- Fermion

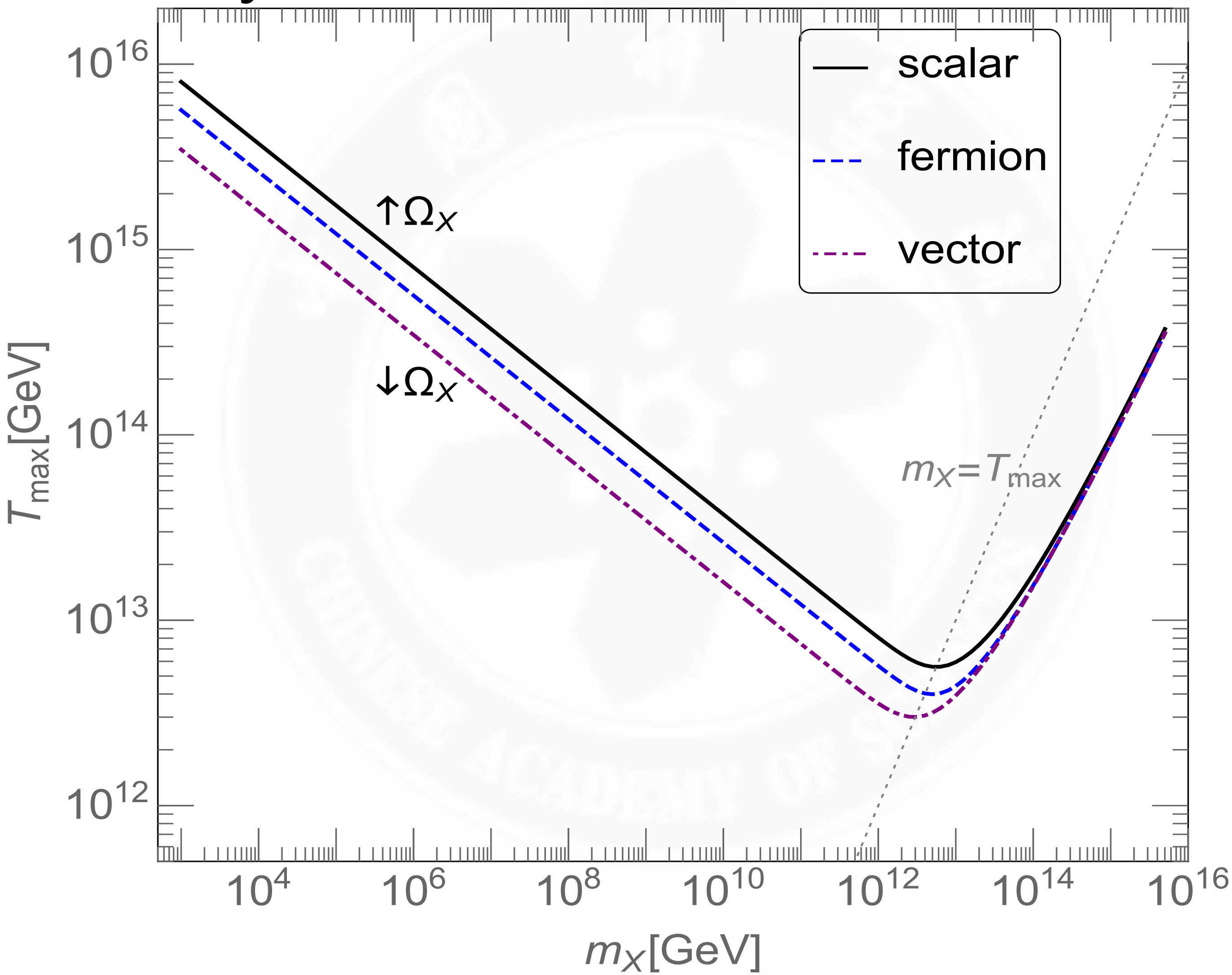
$$\mathcal{A}(F \to F) = \frac{14m^4M^4}{15s^2} + \frac{m^2M^2}{30s}\left(m^2+M^2\right),$$
$$-\frac{1}{120}\left(8m^4 - 3m^2M^2 + 8M^4\right) - \frac{s}{120}\left(m^2+M^2\right) + \frac{s^2}{160},$$
$$\mathcal{A}(V \to F) = -\frac{101m^4M^4}{15s^2} + \frac{m^2M^2}{20s}\left(44M^2 - m^2\right)$$
$$-\frac{1}{60}\left(19M^4 - 19m^2M^2 - 26m^4\right) - \frac{s}{240}\left(7M^2 + 52m^2\right) + \frac{13s^2}{480},$$
$$\mathcal{A}(\gamma \to F) = \frac{1}{120}\left(s - 4M^2\right)\left(3s + 8M^2\right),$$

- Vector

$$\mathcal{A}(V \to V) = \frac{2983m^4M^4}{30s^2} - \frac{293m^2M^2}{10s}\left(m^2+M^2\right),$$
$$+\frac{1}{120}\left(257m^4 + 1188m^2M^2 + 257M^4\right) - \frac{37s}{40}\left(m^2+M^2\right) + \frac{29s^2}{240},$$
$$\mathcal{A}(\gamma \to V) = \frac{13}{120}\left(s - 4M^2\right)^2,$$
$$\mathcal{A}(\gamma \to \gamma) = \frac{s^2}{10}.$$

Gravitational Scattering

- Relic density



Scalar Annihilations

- Special contribution

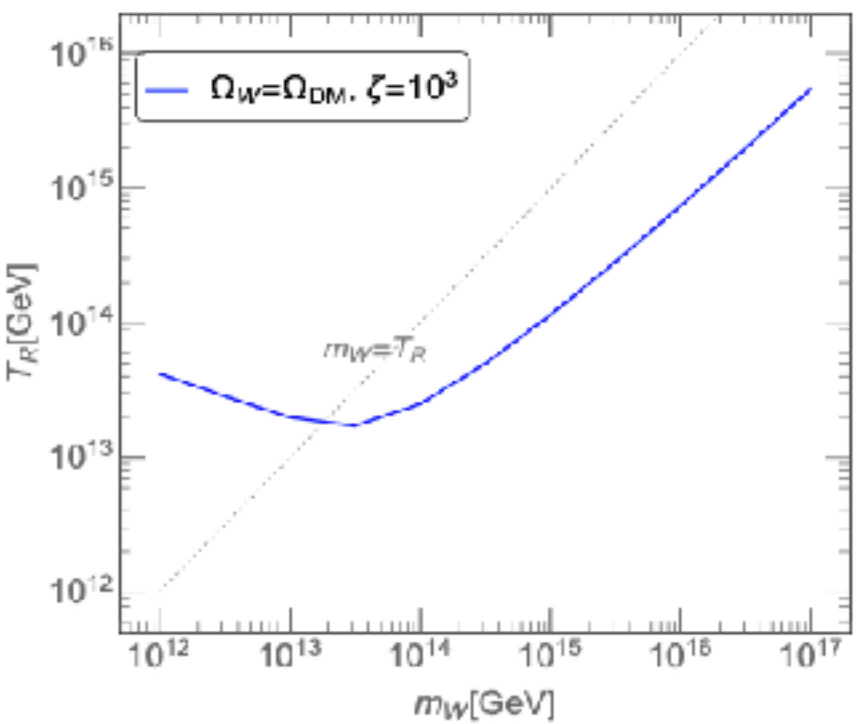
$$\Phi + \Phi \to \overline{W}_\mu + \overline{W}_\mu$$

- Dominated by the longitudinal model

$$\sigma \sim \frac{g_W^4}{E^2}\frac{E^4}{m_W^4} = \frac{E^2}{(6+\zeta)^2 M_p^4},$$

- Similar behavior as the gravitational scattering
- Valid up to Planck scale
- Maximal mass

$$m_W \sim 5 \times 10^{16}\mathrm{GeV}$$



Summary

- The universe is almost scaling invariant during inflation, however how this symmetry is broken is still uncertain.
- In Starobinsky model, global scaling symmetry is broken by the Einstein-Hilbert action.
- With gauge scaling symmetry, Weyl $\hat{R}^2$ is different from the Starobinsky model, as a deformation.
- Weyl gauge boson W_μ has a Z_2 symmetry, and is a dark matter candidate through gravitational production.

Thanks for your attention.