The Future is Illuminating, NCTS, Hsinchu, Taiwan, online, 28th, June, 2022

Testing the early matter-dominated epoch in the early Universe in terms of BBN, CMB, PBH, DM and GWs

Kaz Kohri <sub>
尹氏</sub> 新客

SOKENDA: (PR) (SOKEK 理論センター KAVLI THEORY CENTER PMU

Abstract

We can confirm the existence of the early matter-dominated epoch by observing

- 1. Effective number of neutrino species ($N_v < 3$)
- 2. Spin of the primordial black holes (a $_*$ ~ 1)

3. Characteristic signals of stochastic gravitational wave (the Poltergeist mechanism)

Cosmic history of energy density

a=a(t): scale factor



1) MeV-scale reheating temperature

• Effective number of neutrino species ($N_v < 3$)

Freeze out of weak interaction between n and p at T ~ O(1) MeV

Weak interaction between p and n

$$n \leftrightarrow p + e^- + \bar{\nu}_e \,,$$
$$e^+ + n \leftrightarrow p + \bar{\nu}_e \,,$$
$$\nu_e + n \leftrightarrow p + e^- \,,$$

He4 mass fraction Y



Helium 4 mass fraction Y

• Primordial value of mass fraction Y

$$y_{p} \sim \frac{1}{4} + 0.01 \Delta N_{v} + 0.01 \ln(\eta_{10} / 6) + 0.1 \left(\frac{\tau_{n} - 880 \text{sec}}{880 \text{sec}} \right)$$

小玉、井岡、郡著「宇宙物理学」 (2014)
 $\Delta N_{v} = N_{v} - 3$

$$\eta = \frac{n_{\rm B}}{n_{\rm g}} = \eta_{10} \times 10^{-10} : \text{ baryon to photon ratio}$$
$$\tau_{n}: \text{ neutron life time}$$



Time evolution of neutrinos

Kawasaki, Kohri, and Sugiyama, 1998; 2000

Only photons can be heated by e+e- annihilation at T = 0.511 MeV



Imperfect thermalization of neutrinos by MeV-scale reheating

Kawasaki, Kohri, and Sugiyama, 1998; 2000



Neutrino IMPERFECT thermalisation and Big Bang Nucleosynthesis

 Modifications on interaction rates due to MeV reheating or oscillations among
 v ↔ v and/or v

$$\Gamma_{n\nu_e \to pe^-} = K \int_0^\infty dp_{\nu_e} \left[\sqrt{(p_{\nu_e} + Q)^2 - m_e^2} (p_{\nu_e} + Q) \frac{p_{\nu_e}^2}{1 + e^{-(p_{\nu_e} + Q)/T_{\gamma}}} f_{\nu_e}(p_{\nu_e}) \right]$$
$$\Delta \Gamma_{n \leftrightarrow p} < 0$$
$$\Delta Y \simeq +0.19 \left(-\Delta \Gamma_{n \leftrightarrow p} / \Gamma_{n \leftrightarrow p} \right)$$

Modifications on energy density

$$N_{\nu}^{\text{eff}} \equiv \frac{\rho_{\nu_e} + \rho_{\nu_{\mu}} + \rho_{\nu_{\tau}}}{\rho_{\text{STD}}} < 3 \quad \rightarrow \Delta \rho_{\text{tot}} < 0$$
$$\Delta Y \simeq -0.10 \left(-\Delta \rho_{\text{tot}} / \rho_{\text{tot}} \right)$$

Neutrino oscillations

Vacuum oscillation

$$P(v_i \to v_j) = \sin^2 2\theta_{ij} \sin^2 \left[\frac{L \delta m_{ij}^2}{4E} \right]$$

 $\delta m_{ij}^{2} = m_{j}^{2} - m_{i}^{2}$ L: distance E: energy $\theta_{ij}: \text{ mixing angle}$

MSW (matter effect)

$$P(v_{i} \rightarrow v_{j}) = 1 - \exp \left[-\pi \frac{\sin^{2} 2\theta_{ij}}{\cos 2\theta_{ij}} \frac{\delta m_{ij}^{2}}{4E} \frac{dt}{d \log n_{e}} \right]$$

$$n_{e}: \text{ electron \#density}$$

$$dt: \text{ time derivative}$$

Neutrino oscillation in the early Universe

- Quantum Kinetic Equation

$$\frac{d\varrho_p}{dt} = \frac{\partial \varrho_p}{\partial t} - Hp \frac{\partial \varrho_p}{\partial p} = \boxed{-i \left[\mathcal{H}_p, \varrho_p\right]} + C\left(\varrho_p\right)$$

$$\nu \text{ oscillation } \nu \text{ production/collision}$$

density matrix for ν (2 flavor) $\nu_e - \nu_x$ $e^- + e^+ \rightarrow \nu_\alpha + \nu_\alpha$

$$\varrho_p = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{ex}^* & \rho_{xx} \end{pmatrix} \qquad 0$$

diagonal: ν distribution off-diagonal: flavor coherence

$$\mathcal{H}_{p} = \underbrace{\frac{M^{2}}{2p}}_{Q} \left[-\frac{8\sqrt{2}G_{F}p}{3} \left[\frac{E_{l}}{m_{W}^{2}} + \frac{E_{\nu}}{m_{Z}^{2}} \right] \right] \text{matter effect}$$

$$M^{2} = U\mathcal{M}^{2}U^{\dagger} \quad \mathcal{M}^{2} = \begin{pmatrix} m_{1}^{2} & 0\\ 0 & m_{2}^{2} \end{pmatrix} \quad U = \begin{pmatrix} \cos\theta & \sin\theta\\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$E_{l} \sim \operatorname{diag}(\rho_{e}, 0) \quad E_{\nu} \sim \operatorname{diag}(\rho_{\nu_{e}}, \rho_{\nu_{x}})$$

MSW-like effective mass difference in the early Universe



MSW-type resonance (oscillation) in the early Universe



Thermalization of three active neutrinos

T. Hasegawa, Hiroshima, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012



Observational Helium 4 abundance Yp

 $Y_p = 0.2449 \pm 0.0040 \ (68\% \text{ C.L.})$

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

Radiative decay

Hadronic decay



Lower bound on $T_{\rm RH}$ for radiative decay

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012



Lower bounds on T_{RH} for hadronic decay

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012



Lower bounds on Reheating temperature

T.Hasegawa Hiroshima,, Kohri, Hansen, Tram, Hannestad, JCAP 12 (2019) 012

• Radiative decay

$$T_{R} > 1.8 \text{ MeV} (95\% \text{ C.L.})$$

ANnu <~ -2

Hadronic decay (B_H = 1)

$$T_{R} > 4 - 5 \text{ MeV} (95\% \text{ C.L.})$$

 $\Delta \text{Nnu} < ~ -0.3$

From CMB with He4 only for radiative decay

P.F. de Salas, M. Lattanzi, G. Mangano, G. Miele, S. Pastor, O. Pisanti, arXiv:1511.00672

$T_R > 4.7 MeV(95\% C.L.)$

Future constraints on neutrino species and mass by 21cm, CMB, and BAO



Upper bounds on reheating temperature not to produce so many dangerous gravitinos in supersymmetry/supergravity from BBN

Kawasaki, Kohri, and Moroi (2004)

e



1) Nv < 3 \rightarrow T_R \sim O(1) MeV

2) Formation of PBHs in the early Matter Dominated epoch

Extremally-spinning primordial black holes (a_{*} ~ 1)

Why PBHs?

 We can probe high-energy physics, the early Universe, and gravity with PBHs through recent and future gravitational wave observations

LIGO and Virgo have detected gravitational wave signals from Binary Black Holes

https://www.youtube.com/watch?v=1agm33iEAuo

-0.76s



GW150914 and its merger rates for 30 $$M_{\rm solar}$$ masses BBH

M. Sasaki, T. Suyama, T. Tanaka and S. Yokoyama (2016).



DECIGO discriminates BPBHs from the normal BBHs

Takashi Nakamura et al, arXiv:1607.00897 [astro-ph.HE]





Formation

Conditions for a PBH formation in Radiation dominated (RD) Universe

Zel'dovich and Novikov (1967), Hawking (1971), Carr (1975)

Harada, Yoo and KK (2013)
 Gravity could be stronger than pressure



P_{ζ} (k) and PBH abundance β (M)

 Fraction of PBH to the total with Press Schechter formalism
 For Peak Statistics,

e.g., see Yoo, Harada, KK et al (2018)(2020)

$$\beta(M) \equiv \frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}} = \int_{\delta_{\rm th}}^{\infty} d\delta \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) = \operatorname{erfc}\left(\frac{\delta_{\rm th}}{\sqrt{2\sigma}}\right)$$

= $\operatorname{erfc}\left(\frac{\delta_{\rm th}}{\sqrt{2\sigma}}\right)$

For analytical derivations, see Harada, Yoo, KK (2013) 0.43

• Relation between β and fluctuation σ (or β and $\Omega)$

$$B(\mathcal{M}) \sim \operatorname{erfc}\left(\frac{\delta_{\mathrm{th}}}{\sqrt{2}\sigma}\right) \simeq \sqrt{\frac{2}{\pi}} \frac{\sigma}{\delta_{\mathrm{th}}} \exp\left(-\frac{\delta_{\mathrm{th}}^2}{2\sigma^2}\right)$$
$$= 1.5 \times 10^{-18} \left(\frac{m_{\mathsf{PBH}}}{10^{15} \,\mathrm{g}}\right)^{1/2} \left(\frac{\Omega_{\mathsf{PBH}}h^2}{0.1}\right) \sim \mathcal{P}_{\zeta}$$

Typical quantities of PBHs in RD

• Mass (horizon mass = $\rho(t_{form}) H(t_{form})^{-3}$)

 $M_{\rm PBH} \sim \rho \left(H_{\rm form}^{-1}\right)^3 \sim M_{pl}^2 t_{\rm from} \sim \frac{M_{pl}^3}{T_{\rm form}^2} \sim 10^{15} g \left(\frac{T_{\rm form}}{3 \times 10^8 {\rm GeV}}\right)^{-2} \sim 5 \times 10^4 M_{\odot} \left(\frac{T_{\rm form}}{\rm MeV}\right)^{-2}$

- Lifetime $au_{\text{PBH}} \sim \frac{M_{\text{PBH}}^3}{M_{pl}^4} \sim 4 \times 10^{17} \sec\left(\frac{M_{\text{PBH}}}{10^{15} \text{ g}}\right)^3 \sim 1 \sec\left(\frac{M_{\text{PBH}}}{10^9 \text{ g}}\right)^3$
- Hawking Temperature

$$T_{\rm PBH} \sim \frac{M_{\rm pl}^2}{M_{\rm PBH}} \sim 10 \,{\rm MeV} \left(\frac{M_{\rm PBH}}{10^{15}{\rm g}}\right)^{-1} \sim 2 \times 10^{-9} {\rm K} \left(\frac{M_{\rm PBH}}{30 M_{\odot}}\right)^{-1}$$

- Wave number of horizon length $k = aH \sim 10^{5} \text{Mpc}^{-1} \left(\frac{M_{\text{PBH}}}{5 \times 10^{4} M_{\odot}}\right)^{-1/2} \sim 10^{5} \text{Mpc}^{-1} \left(\frac{T_{\text{form}}}{\text{MeV}}\right)^{+1}$
- Fraction to CDM $f_{\text{fraction}} \equiv \frac{\Omega_{\text{PBH}}}{\Omega_{\text{CDM}}} \sim \left(\frac{\beta}{10^{-18}}\right) \left(\frac{M_{\text{PBH}}}{10^{15} \text{g}}\right)^{-1/2} \sim \left(\frac{\beta}{10^{-8}}\right) \left(\frac{M_{\text{PBH}}}{30 \text{M}_{\odot}}\right)^{-1/2} \sim 10^8 \left(\frac{M_{\text{PBH}}}{30 \text{M}_{\odot}}\right)^{-1/2} \sqrt{P_{\delta}} \exp\left[-\frac{1}{18P_{\delta}}\right]$

Features of PBH formations in RD

• Spherical due to radiation pressure



 $(w \equiv p / \rho \sim 1 / 3)$

- Negligible evolutions of density perturbations
- Quite a small angular momentum

See, T.Chiba and S.Yokoyama, 2017 De Luca et al, 2019 Minxi He and Suyama, 2019 Harada, Yoo, Kohri, Koga and Monobe, 2020

(dimensionless Kerr parameter)

$$\sqrt{\langle a_*^2 \rangle} \simeq 6.5 \times 10^{-4} \left(\frac{M}{M_H}\right)^{-1/3}$$

Effective inspiral spin parameter of the $m_1\chi_1 \cos \theta_1 + m_2\chi_2 \cos \theta_2$ observed BHs



Credible region contours for all candidate events in the plane of chirp mass \mathcal{M} and effective inspiral spin \chieff. Each contour represents the 90\% credible region for a different event. We highlighted the previously published candidate events (cf.\ Fig.~\ref{fig:mtotqpost}), as well as \protect\NAME{GW190517A}\ and \protect\NAME{GW190514A}, which have the highest probabilities of having the largest and smallest \chieff respectively.

R. Abbott, et al, LSC P&P Committee, arXiv:2010.14527 [gr-qc]

PBH formation at the (early) matter dominated (MD) Universe

Polnarev and Khlopov (1982)

Harada, Yoo, KK, Nakao, Jhingan (2016)

Pressure is negligible, which could induce an immediate collapse and producing more PBHs?

2. Density perturbations can evolve, which produces non-spherical objects and cannot be enclosed by the Horizon. That means less PBHs can be produced?

Matter Domination

• Three radius in Lagrangian coordinate q_i

	$r_1 = (a - \alpha b)q_1$	Zel'dovich Approximation
	$r_2 = (a - \beta b)q_2$	
	$r_3=(a-\gamma b)q_3$	
Eccentricity	$e^2 = 1 - \left(\frac{r_2(t_c)}{r_3(t_c)}\right)^2 = 1$	$-\left(\frac{\alpha-\beta}{\alpha-\gamma}\right)^2$
Hoop with 2 nd Elliptic funciton E(x)		
$\mathcal{C} = 16\left(1 - rac{\gamma}{lpha} ight) E\left(\sqrt{1 - \left(rac{lpha - eta}{lpha - \gamma} ight)^2} ight)r_f,$		
Hoop conjecture for PBH production		

$$\mathcal{C} \lesssim 2\pi r_g$$
.

Abundance of PBHs formed in MD

 Probability distribution by peak statistics (BBKS) Doroshkevich (1970)

$$w(\alpha,\beta,\gamma)d\alpha d\beta d\gamma$$

$$=-\frac{27}{8\sqrt{5}\pi\sigma_3^6}\exp\left[-\frac{1}{10\sigma_3^2}(\alpha+\beta+\gamma)^2 - \frac{1}{4\sigma_3^2}\{(\alpha-\beta)^2 + (\beta-\gamma)^2 + (\gamma-\alpha)^2\}\right]$$

$$\cdot(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)d\alpha d\beta d\gamma.$$

$$\sigma_H = \sqrt{5}\sigma_3$$

• Probability

$$\beta_0 = \int_0^\infty d\alpha \int_{-\infty}^\alpha d\beta \int_{-\infty}^\beta d\gamma \ \theta(1 - h(\alpha, \beta, \gamma)) w(\alpha, \beta, \gamma)$$
$$h(\alpha, \beta, \gamma) = \frac{2}{2} \frac{\alpha - \gamma}{2} E\left(\sqrt{1 - \left(\frac{\alpha - \beta}{2}\right)^2}\right)$$

$$(\beta, \gamma) = \frac{2}{\pi} \frac{\alpha}{\alpha^2} E \left(\sqrt{1 - \left(\frac{\alpha}{\alpha - \gamma}\right)} \right)$$

 $h(\alpha, \beta, \gamma) := C/(2\pi r_g)$

Angular momentum produced by perturbations

Harada, Yoo, KK, nad Nakao (2017)

- Angular momentum 1st order effects for nonspherical V 2nd order effects $\mathbf{L}_{c} = \int_{a^{3}V} \rho \mathbf{r} \times \mathbf{v} d^{3}\mathbf{r} = \rho_{0} a^{4} \left(\int_{V} \mathbf{x} \times \mathbf{u} d^{3}\mathbf{x} + \int_{V} \mathbf{x} \delta \times \mathbf{u} d^{3}\mathbf{x} \right)$
- Density perturbation $\boldsymbol{\delta}$
- (Peculiar) Velocity perturbation $\mathbf{u} := aD\mathbf{x}/Dt$

$$\mathbf{u}_1 = -rac{t}{a}
abla\psi_1$$

• Potential perturbation

 $\psi := \Psi - \Psi_0$

Effects by finite angular momentum

Harada, Yoo, KK, Nakao (2017)

• Probability distribution

$$a_* := L/(GM^2/c)$$

$$f_{\rm BH(2)}(a_*)da_* \propto \frac{1}{a_*^{5/3}} \exp\left(-\frac{1}{2\sigma_H^{2/3}} \left(\frac{2}{5}\mathcal{I}\right)^{4/3} \frac{1}{a_*^{4/3}}\right) da_*$$

• Probability

 $\beta_0 \simeq \int_0^\infty d\alpha \int_{-\infty}^\alpha d\beta \int_{-\infty}^\beta d\gamma \theta [\delta_H(\alpha,\beta,\gamma) - \delta_{\rm th}] \theta [1 - h(\alpha,\beta,\gamma)] w(\alpha,\beta,\gamma)$

$$\delta_{H}(\alpha,\beta,\gamma) = \alpha + \beta + \gamma \qquad \delta_{\rm th} \quad := \left(\frac{2}{5}\mathcal{I}\sigma_{H}\right)^{2/3}$$

Spin distribution

More highly-spinning halos cannot collapse into PBHs, which means that the PBHs produced tend to have high spins in MD Harada, Yoo, KK, Nakao (2017)



Beta in matter-domination

Harada, Yoo, KK, Nakao (2017)



2) a_{*}~1→ PBHs produced with σ<<1 in the eMD

3) Stochastic gravitational wave produced in the early Matter Dominated epoch

Characteristic signals of stochastic gravitational wave (the Poltergeist mechanism)

The break point of Ω_{GW} marks the reheating temperature after inflation

Naoki Seto, Jun'ichi Yokoyamam, arXiv:gr-qc/0305096 Kazunori Nakayama, Shun Saito, Yudai Suwa, Jun'ichi Yokoyama, arXiv:0804.1827 [astro-ph]



The 2nd order GWs with gradual transition from MD to RD

Inomata, Kohri, Nakama, Terada, JCAP10(2019)071, arXiv:1904.12878 [astro-ph.CO]

$$\Omega_{\rm GW}(\eta, k) = \frac{\rho_{\rm GW}(\eta, k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24} \left(\frac{k}{\mathcal{H}(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta, k)}$$

$$\overline{\mathcal{P}_h(\eta, k)} = 4 \int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2 - (1+v^2 - u^2)^2}{4vu}\right)^2 \overline{I^2(u, v, k, \eta, \eta_{\rm R})} \mathcal{P}_{\zeta}(uk) \mathcal{P}_{\zeta}(vk).$$

$$\overline{\mathcal{P}_h(\eta, k)} \sim \int \int f^2(\mathbf{u}, v, \mathbf{x}, \mathbf{x}_{\rm R})$$

$$f(u, v, \bar{x}, x_{\rm R}) = \frac{3 \left(2(5+3w) \Phi(u\bar{x}) \Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x}) \Phi(v\bar{x}) + \Phi(u\bar{x}) \Phi'(v\bar{x})) + 4\mathcal{H}^{-2} \Phi'(u\bar{x}) \Phi'(v\bar{x})\right)}{25(1+w)}$$



mechanism for gravitational wave production

Logo by ©Takahiro Terada

The 2nd order GWs enhanced at a sudden transition from MD to RD

Inomata, Kohri, Nakama, Terada, Phys. Rev. D 100, 043532 (2019), arXiv:1904.12879

 $\overline{\mathcal{P}_{h}(\eta, k)} \sim \int \int f^{2}(u, v, x, x_{\mathsf{R}}) f^{2}(u, v, x, x_{\mathsf{R}})$ $f(u, v, \bar{x}, x_{\mathsf{R}}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x})+4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x})+\Phi(u\bar{x})\Phi'(v\bar{x}))+4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}$ This is big!

Gravitational potential

 $\Phi(x, x_{\mathrm{R}}) = \begin{cases} 1 & (\text{for } x \leq x_{\mathrm{R}}), \quad \mathsf{eMD} \\ A(x_{\mathrm{R}})\mathcal{J}(x) + B(x_{\mathrm{R}})\mathcal{Y}(x) & (\text{for } x \geq x_{\mathrm{R}}), \quad \mathsf{RD} \end{cases}$

Enhancement at T_R

 $\mathcal{H}^{-2}\Phi'\Phi'\sim\,(k\eta_{\rm R})^2\Phi^2\,\gg\,\Phi^2$

Amplitude should be less than unity The transition occurs in a finite time



Sudden decay from $\phi \rightarrow 2\chi$ only when M > 2 m_x~ $\chi\lambda/2 \tau$

• Lagrangian

$$\begin{split} \mathcal{L} &= -\frac{1}{2} \partial^{\mu} \phi \partial_{\mu} \phi - \frac{1}{2} \partial^{\mu} \chi \partial_{\mu} \chi - \frac{1}{2} \partial^{\mu} \tau \partial_{\mu} \tau - V, \\ V &= & \frac{1}{2} M^2 \phi^2 + \frac{1}{2} m^2 \tau^2 + \frac{\lambda}{4} \tau^2 \chi^2 + \frac{c}{2} M \phi \chi^2, \end{split}$$

• Decay rate

$$\Gamma = \frac{c^2 M}{32\pi} \sqrt{1 - \frac{m_{\chi,\text{eff}}^2}{(M/2)^2}} \Theta \left(M^2 - 4m_{\chi,\text{eff}}^2 \right)$$

• Effective mass of χ

$$m_{\chi,\text{eff}}^2 = \langle \lambda \tau^2 / 2 \rangle$$

Applications of this mechanism (The poltergeist mechanism)

• Evaporating PBHs with their domination

Keisuke Inomata, Masahiro Kawasaki, Kyohei Mukaida, Takahiro Terada, Tsutomu T. Yanagida, arXiv:2003.10455 [astro-ph.CO]

 Poisson fluctuation of evaporating PBHs themselves with their domination

Guillem Domènech, Chunshan Lin, Misao Sasaki, arXiv:2012.08151 [gr-qc] Guillem Domènech, Volodymyr Takhistov, Misao Sasaki, arXiv:2105.06816 [astroph.CO]

Extension from Theodoros Papanikolaou, Vincent Vennin, David Langlois,

arXiv:2010.11573 [astro-ph.CO]

• Sudden decays of Q-balls

Graham White, Lauren Pearce, Daniel Vagie, Alex Kusenko, arXiv:2105.11655 [hep-ph]

Summary

We can confirm the existence of the early matter-dominated epoch by observing

- Effective number of neutrino species ($N_v < 3$)
- Spin of the primordial black holes (a_{*} ~ 1)
- Characteristic signals of stochastic GW produced from inflation at the level of $\Omega_{GW} \sim 10^{-16}$ or secondary GW nonlienary-produced by large curvature perturbation at small scales (The Poltergeist mechanism)