Generation and detection of discrete-variable multipartite entanglement

with multi-rail encoding in linear optics networks

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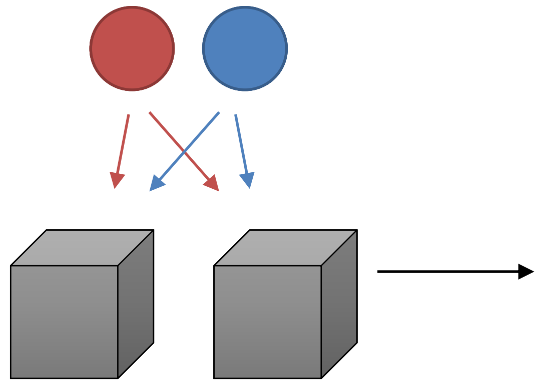
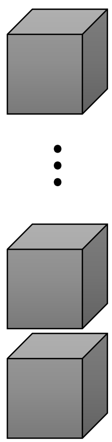
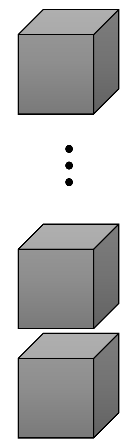
淡江大學 Tamkang University

MOST 科技部 Ministry of Science and Technology

Entanglement







Probability	Red (Bob)	Blue (Bob)
Red (Alice)	0	½
Blue (Alice)	½	0

Q: Is this correlation caused by entanglement?

A separable state

$$\rho = \frac{1}{2}|H\,V\rangle\langle H\,V| + \frac{1}{2}|V\,H\rangle\langle V\,H|$$

Bipartite entanglement

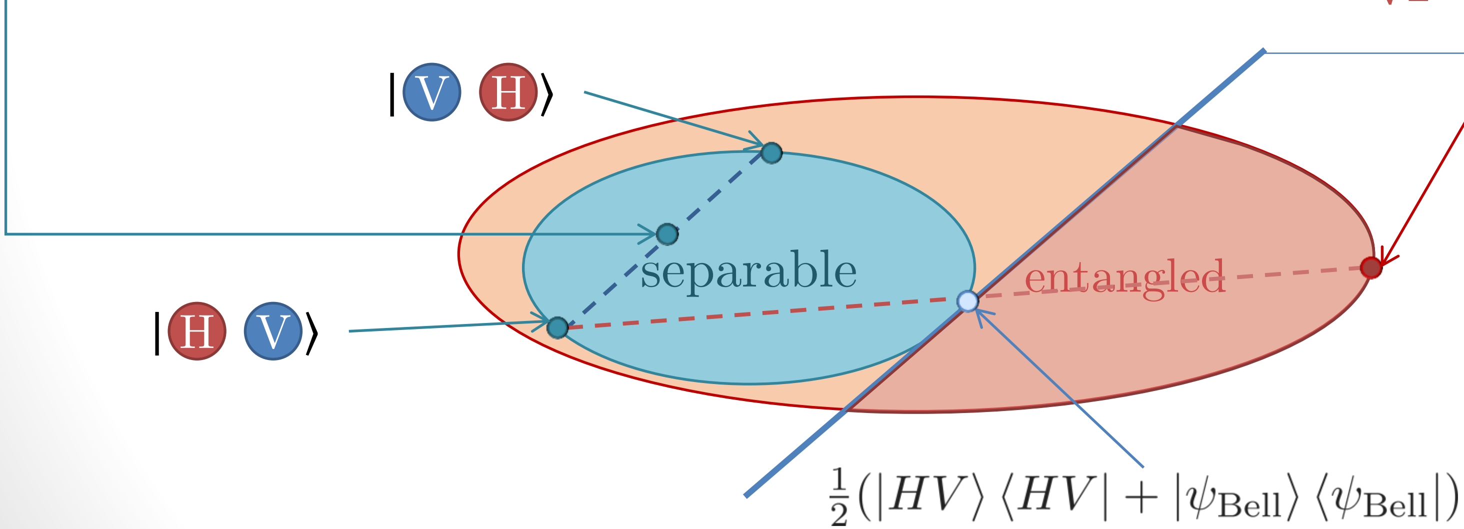
Definition (Entanglement). *[1] A state ρ_{AB} in the Hilbert space $\mathbb{H}_A \otimes \mathbb{H}_B$ is* separable, *if and only if it can be expressed as a convex combination of separable states*

$$\rho_{AB} = \sum p_i \rho_i^{(A)} \otimes \rho_i^{(B)}.$$

A state which is not separable is called entangled.

$$\rho = \frac{1}{2}|HV\rangle\langle HV| + \frac{1}{2}|VH\rangle\langle VH|$$

$$|\psi_{Bell}\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle)$$

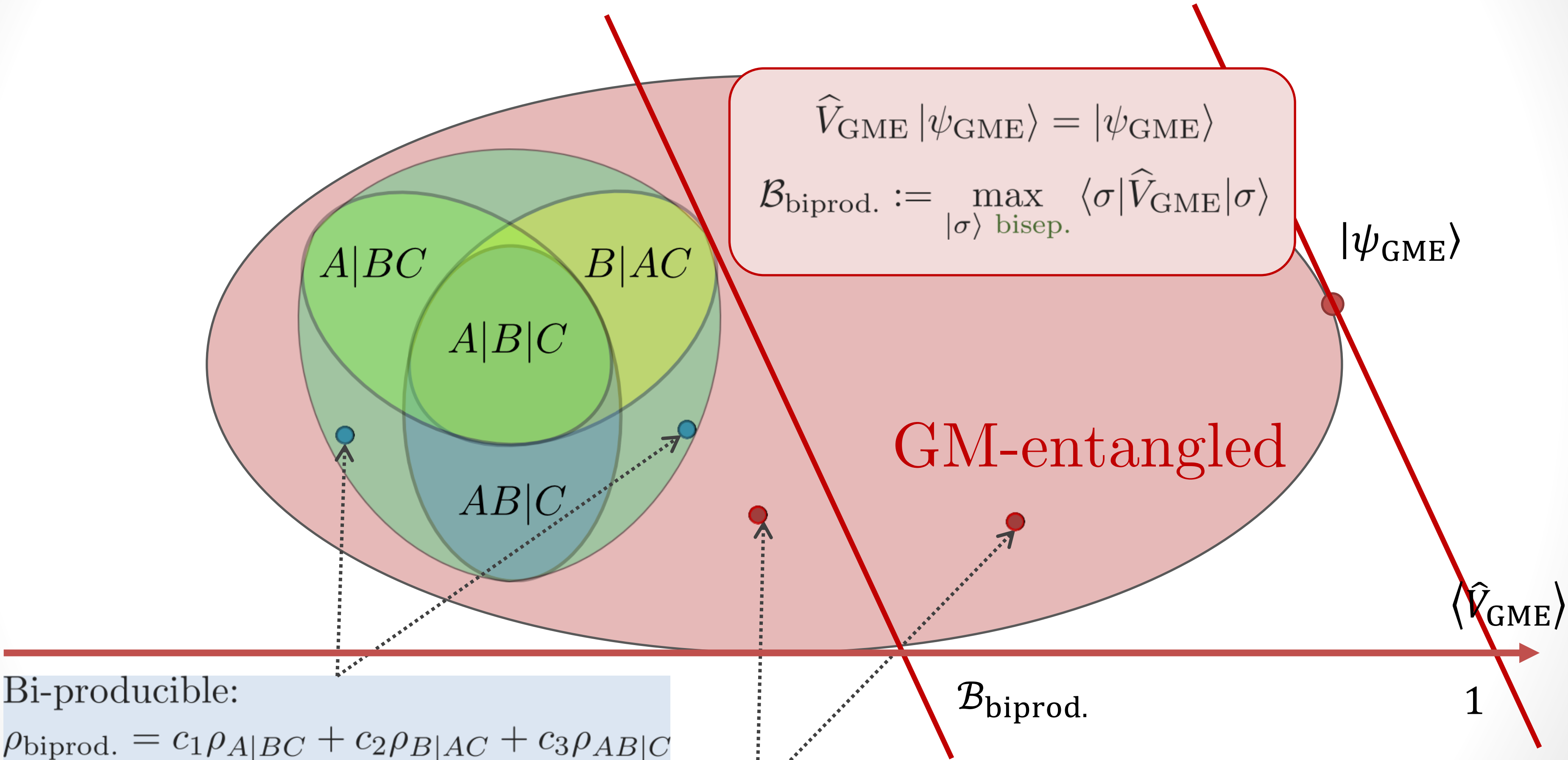


[1] R. F. Werner, Phys. Rev. A, 40, 4277-4281, 1989

Bipartite systems

Multipartite systems

Genuine multipartite entanglement (GME)[1]



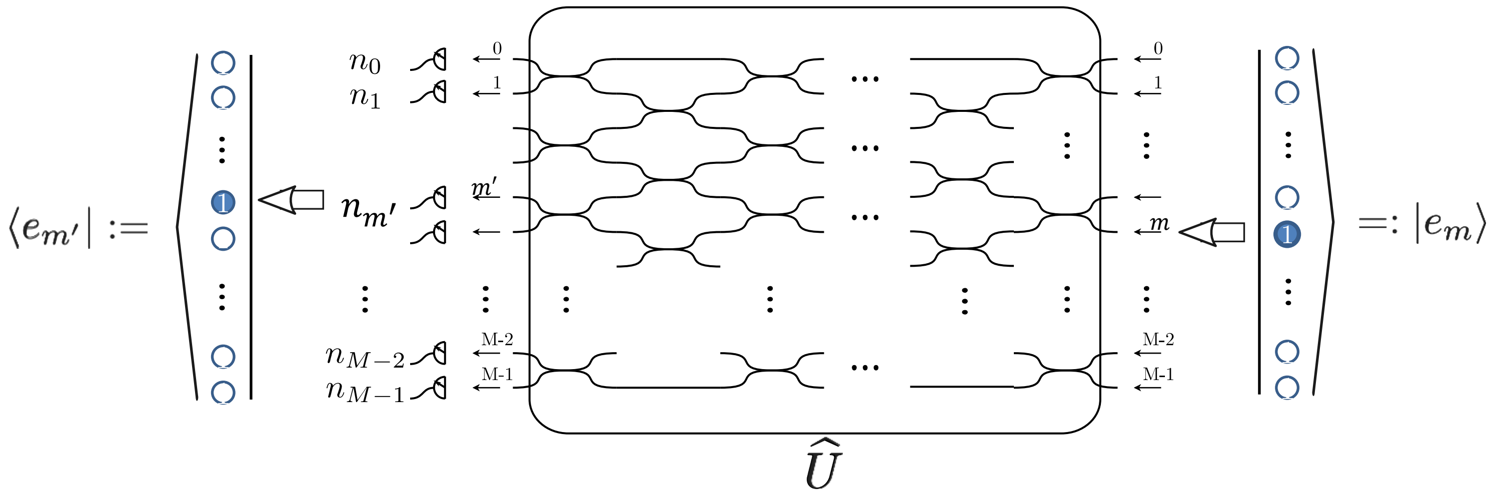
Bi-producible:
$\rho_{\mathrm{biprod.}} = c_1\rho_{A|BC} + c_2\rho_{B|AC} + c_3\rho_{AB|C}$

Genuinely multipartite (GM) entangled: $\rho \neq c_1\rho_{A|BC} + c_2\rho_{B|AC} + c_3\rho_{AB|C}$

[1] Acin, et al., PRL, 87:040401, 2001.; Bourennane, et al. PRL, 92:087902, 2004.

Multi-rail encoding in single-photon linear optics networks (LONs)

All M-mode linear optics networks can be constructed with beam splitters[1,2]

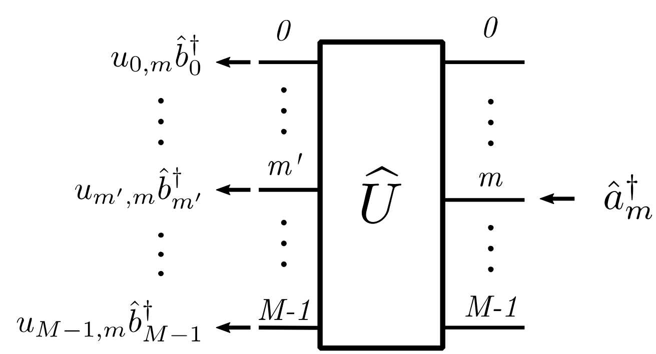


A linear optics transform $\widehat{U}$ of modes $\widehat{a}_m^\dagger \to \widehat{b}_{m'}^\dagger$

$$\widehat{U}\widehat{a}_m^\dagger\widehat{U}^\dagger = \sum_{m'} u_{m',m}\widehat{b}_{m'}^\dagger$$

is an $M \times M$ unitary in a single-photon system,

$$\left\{\left\langle e_{m'} \middle| \widehat{U} \middle| e_m \right\rangle\right\}_{m',m} = \begin{pmatrix} u_{00} & \cdots & u_{M-1,0} \\ \vdots & u_{m'm} & \vdots \\ u_{0,M-1} & \cdots & u_{M-1,M-1} \end{pmatrix}$$

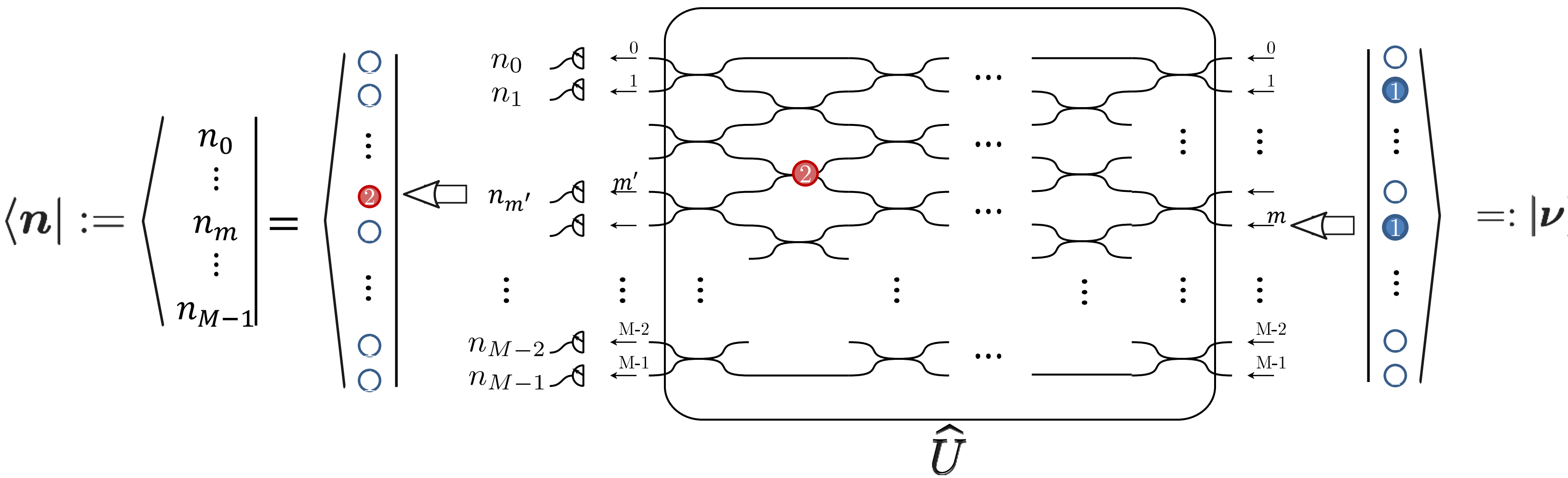


[1] M. Reck et al. PRL, 73, 1, 58–618 (1994); [2] Clements, et al. Optica , 3, 12, 1460-1465 (2016)

Distinguishable systems   Indistinguishable systems

?????...

Complexity in multiphoton LONs



Boson sampling is a #P-hard[1] problem: permanent of a matrix $\mathcal{U}_{\boldsymbol{n},\boldsymbol{\nu}}$,

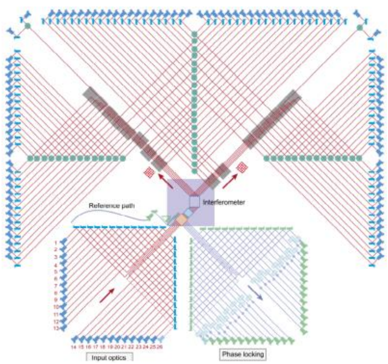
$$\left\langle \boldsymbol{n} \middle| \hat{U} \middle| \boldsymbol{\nu} \right\rangle = \mathrm{Perm}(\mathcal{U}_{\boldsymbol{n},\boldsymbol{\nu}}) =?$$

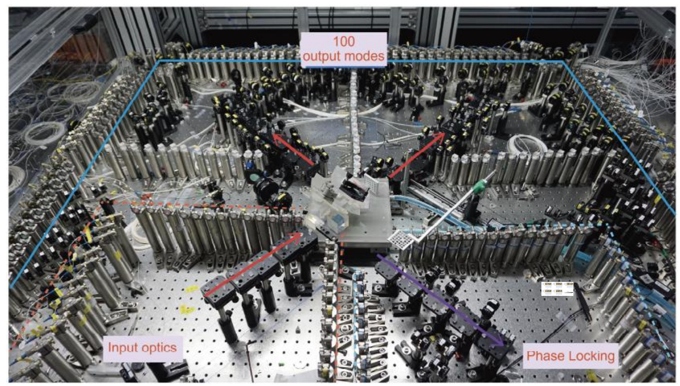
where $\mathcal{U}_{\boldsymbol{n},\boldsymbol{\nu}}$ is an $N \times N$-matrix.

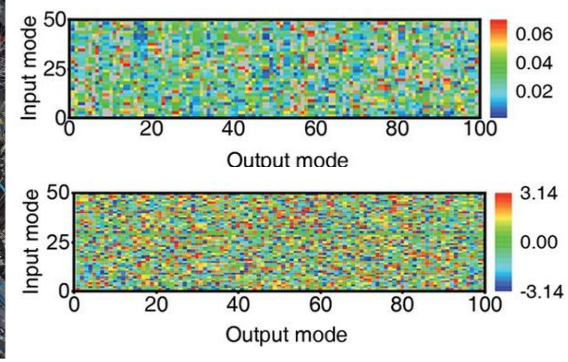
[1] S. Aaronson. Proc. of the Roy. Soc. of London A: Math., Phys. and Engr. Sciences (2011), 467, 2136, 3393–3405;
S. Aaronson and A. Arkhipov. Proc. of the 43th Ann. ACM Symp. on Theory of Computing(2011), 333–342;

Quantum computation for boson sampling

Jiuzhang 九章[a]

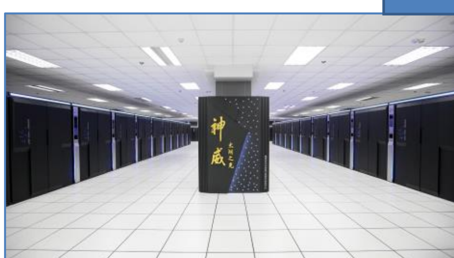




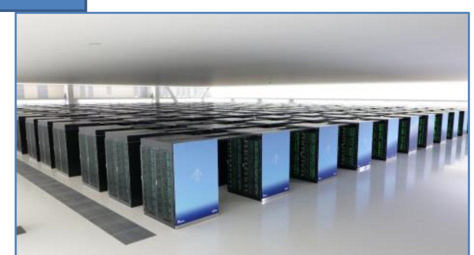


Quantum computation: ~200 s

Classical computation

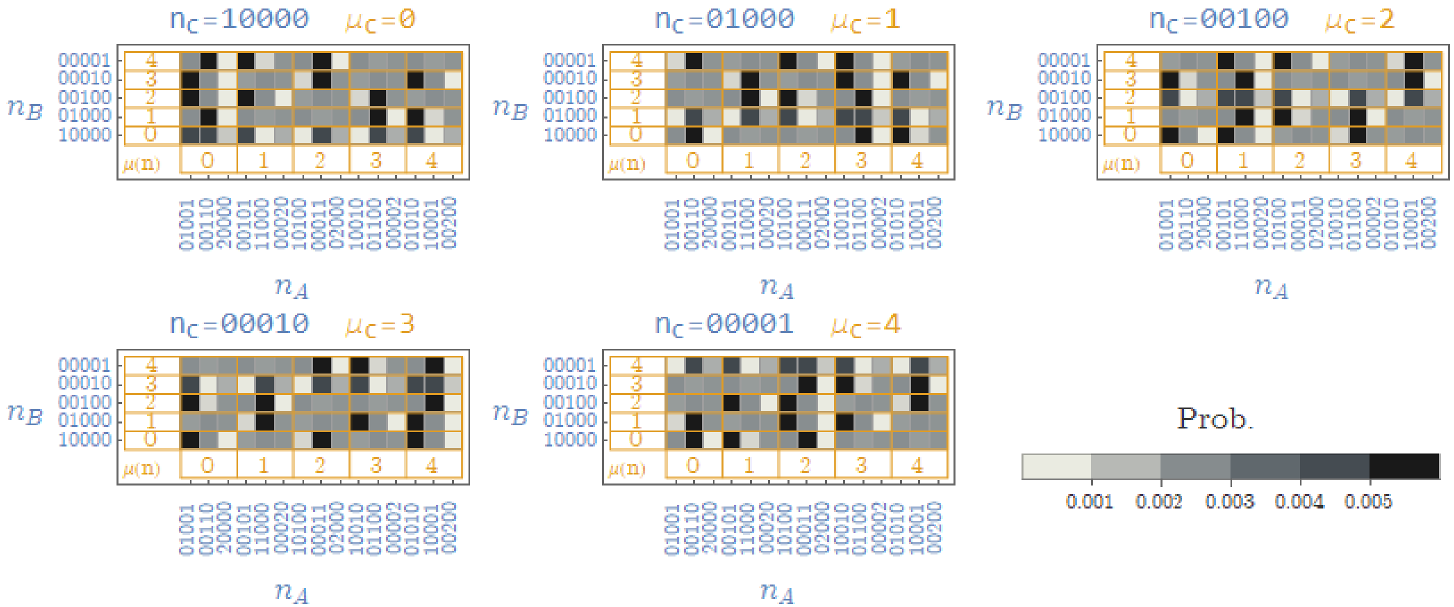


TaihuLight 太湖之光
2.5 billion years



Fugaku 富岳
0.6 billion years

[a] H.-S. Zhong, et. al., C.-Y. Lu, J.-W. Pan, Science 10.1126/science.abe8770 (2020).



Generation and detection

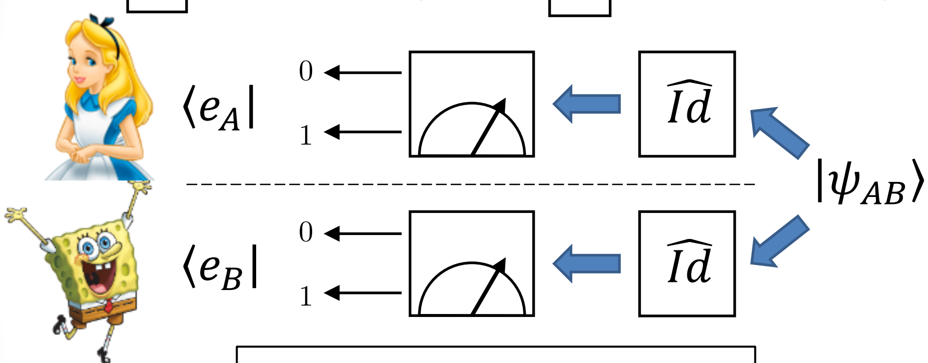
of discrete-variable multipartite entanglement with multi-rail encoding in linear optics networks

- Complementary measurement in multiphoton LONs
- GME detection in single-photon LONs

Entanglement verifier in complementary local measurements[*]

Target state: $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|H_A V_B\rangle + |V_A H_B\rangle) = \frac{1}{\sqrt{2}}(|P_A P_B\rangle - |M_A M_B\rangle)$

$\widehat{Id}$ = Identity gate $\hat{H}$ = Hadamard gate

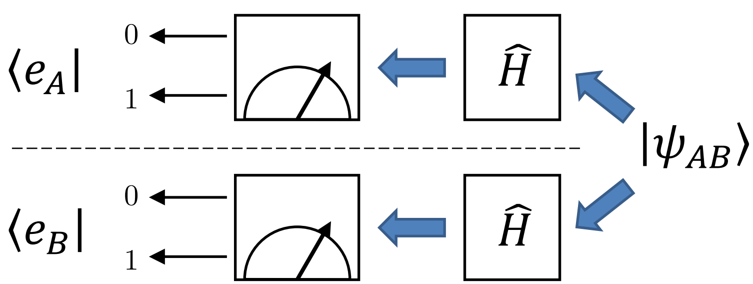


$|\mathbb{E}_0(Id)\rangle = |H\rangle$ $|\mathbb{E}_1(Id)\rangle = |V\rangle$

Measurement 1	$\left\|\mathbb{E}_0^{(A)}(Id)\right\rangle$	$\left\|\mathbb{E}_1^{(A)}(Id)\right\rangle$
$\left\|\mathbb{E}_0^{(B)}(Id)\right\rangle$	0	½
$\left\|\mathbb{E}_1^{(B)}(Id)\right\rangle$	½	0

$\hat{V}_{id} = |H_A V_B\rangle\langle H_A V_B| + |V_A H_B\rangle\langle V_A H_B|$

$|\langle\mathbb{E}_e(H)|\mathbb{E}_e(id)\rangle| = 1/\sqrt{2}$



$|\mathbb{E}_0(H)\rangle = |P\rangle$ $|\mathbb{E}_1(H)\rangle = |M\rangle$

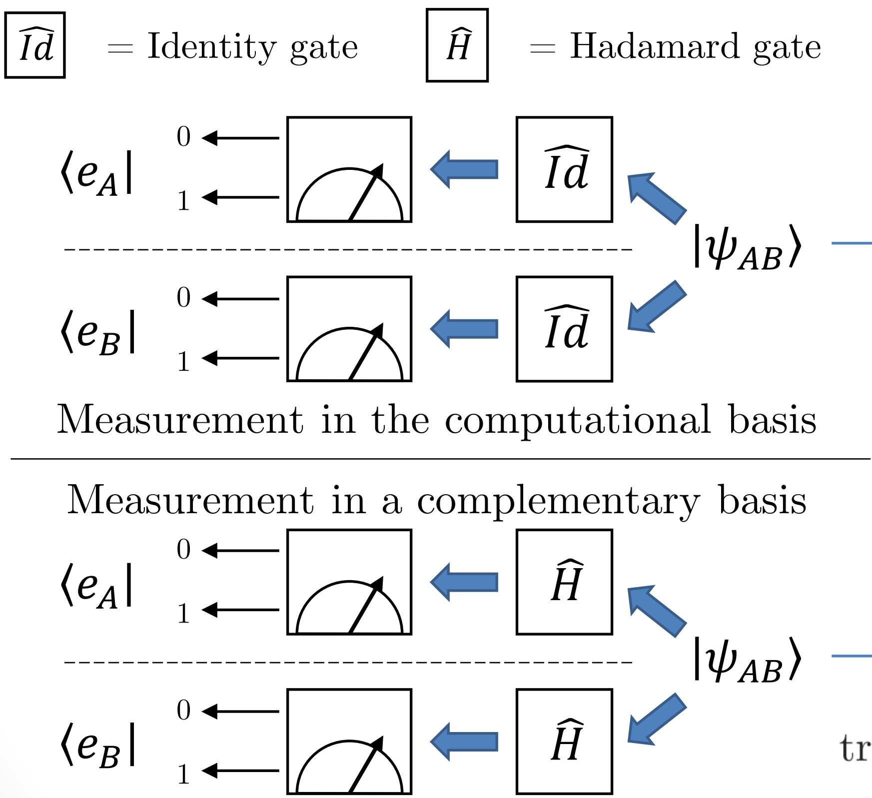
Measurement 2	$\left\|\mathbb{E}_0^{(A)}(H)\right\rangle$	$\left\|\mathbb{E}_1^{(A)}(H)\right\rangle$
$\left\|\mathbb{E}_0^{(B)}(H)\right\rangle$	½	0
$\left\|\mathbb{E}_1^{(B)}(H)\right\rangle$	0	½

$\hat{V}_H = |P_A P_B\rangle\langle P_A P_B| + |M_A M_B\rangle\langle M_A M_B|$

[*] G. Tóth and O. Gühne, PRA 72.022340, 94.060501 (2005); L. Maccone, D. Bruß, C. Macchiavello, PRL 114.130401 (2015); C. Spengler, M. Huber, S. Brierley, A. Stephen, H. Theodor, B.C. Hiesmayr, PRA, 86. 022311 (2012);

Entanglement verifier in complementary local measurements[*]

Target state: $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|H_A V_B\rangle + |V_A H_B\rangle) = \frac{1}{\sqrt{2}}(|P_A P_B\rangle - |M_A M_B\rangle)$



$$\widehat{V}_{id} = |H_A V_B\rangle\langle H_A V_B| + |V_A H_B\rangle\langle V_A H_B|$$

$$\widehat{V}_{H} = |P_A P_B\rangle\langle P_A P_B| + |M_A M_B\rangle\langle M_A M_B|$$

$$\widehat{V}_{\text{ent.}} = \frac{1}{2}\widehat{V}_{id} + \frac{1}{2}\widehat{V}_{H} \qquad \widehat{V}_{\text{ent.}}|\psi_{AB}\rangle = |\psi_{AB}\rangle$$

$$\mathcal{B}_{\text{sep.}} = \max_{|\sigma\rangle\ \text{sep.}} |\langle\sigma|\widehat{V}_{\text{ent.}}|\sigma\rangle| = \frac{3}{4}$$

$$\text{tr}\left(\widehat{V}_{\text{ent.}}\widehat{\rho}_{\text{test}}\right) > \mathcal{B}_{\text{sep.}} = \frac{3}{4} \quad \Rightarrow \quad \widehat{\rho}_{\text{test}} \text{ is entangled.}$$

[*] G. Tóth and O. Gühne, PRA 72.022340, 94.060501 (2005); L. Maccone, D. Bruß, C. Macchiavello, PRL 114.130401 (2015); C. Spengler, M. Huber, S. Brierley, A. Stephen, H. Theodor, B.C. Hiesmayr, PRA, 86. 022311 (2012);

Heisenberg-Weyl operators for complementary measurements

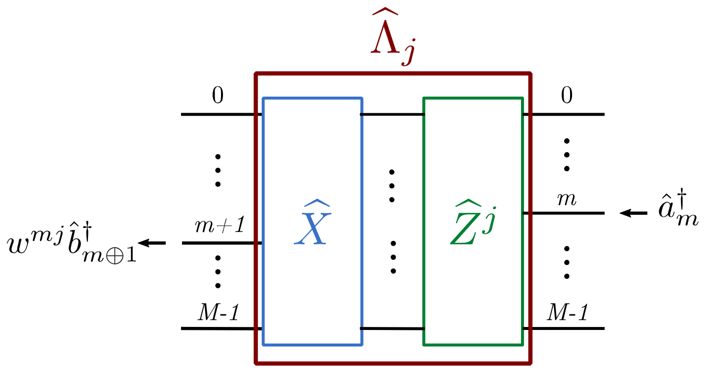
Heisenberg-Weyl operators:

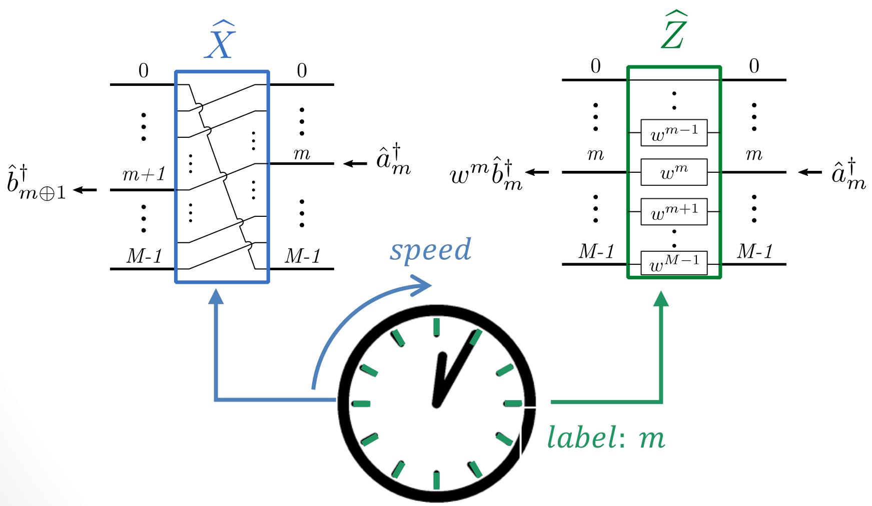
$$\widehat{\Lambda}_j := \widehat{X}^i \widehat{Z}^j,$$

where $\widehat{X}$ and $\widehat{Z}$ are the mode shifting operator and phase shifting operator, respectively,

$$\widehat{X}\hat{a}^\dagger_m \widehat{X}^\dagger = \hat{b}^\dagger_{m\oplus 1} \quad \text{and} \quad \widehat{Z}\hat{a}^\dagger_m \widehat{Z}^\dagger = w^m \hat{b}^\dagger_m,$$

with $\omega = \exp(\mathrm{i}\, 2\pi/M)$.





The Heisenberg-Weyl operators specify mutually unbiased bases $\{|\mathbb{E}_m(\Lambda_j)\rangle\}_m$

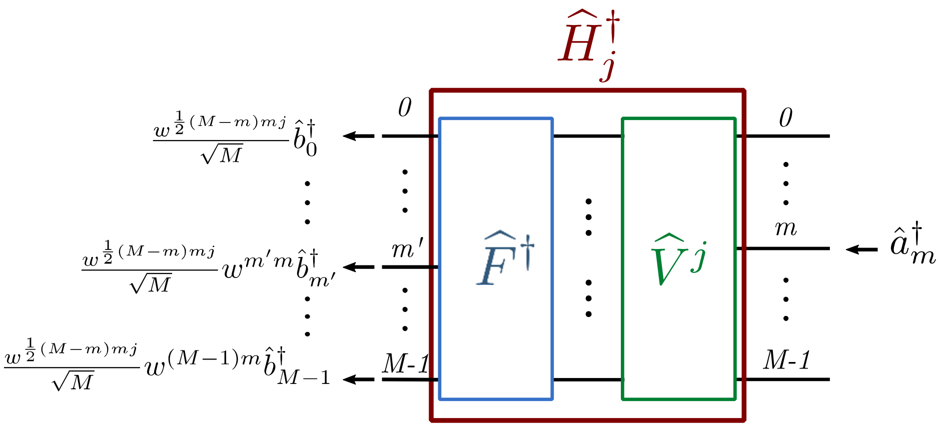
$$|\mathbb{E}_m(\Lambda_j)\rangle := \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} w^{-(m+\frac{1}{2}jd)k+\frac{1}{2}jk^2} |e_k\rangle .$$

[a] T. Durt et al. Int. J. of Quan. Inf. (2010), 08, 04, 535–640;

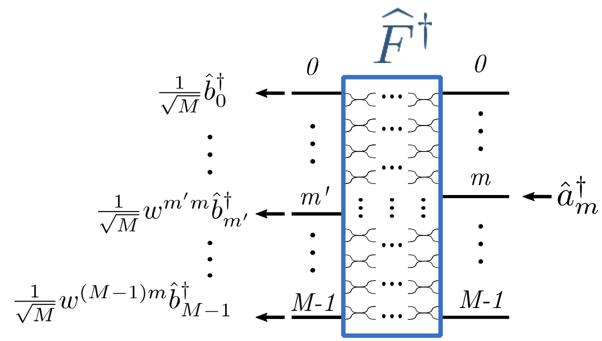
Pauli measurements in single-photon LONs

A generalized Hadamard transform $\widehat{H}_j$ maps the computational basis to the $\widehat{\Lambda}_j$ eigenbasis,

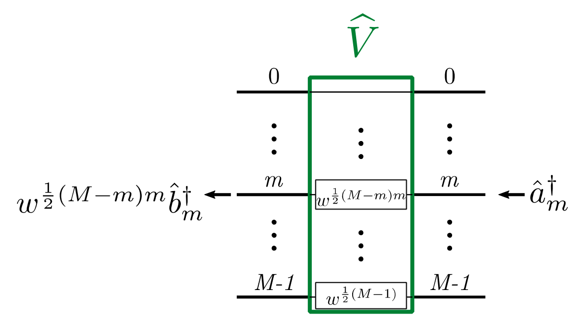
$$\widehat{H}_j \left| e_m \right\rangle = \left| \mathbb{E}_m(\Lambda_j) \right\rangle \quad \text{with} \quad \widehat{H}_j^\dagger \hat{a}_m^\dagger \widehat{H}_j = w^{\frac{1}{2}(M-m)mj} \sum_{m'} \frac{w^{m'm}}{\sqrt{M}} \hat{b}_{m'}^\dagger .$$



$$\widehat{H}_j^\dagger = \widehat{F}^\dagger \widehat{V}^j$$



Discrete Fourier transform

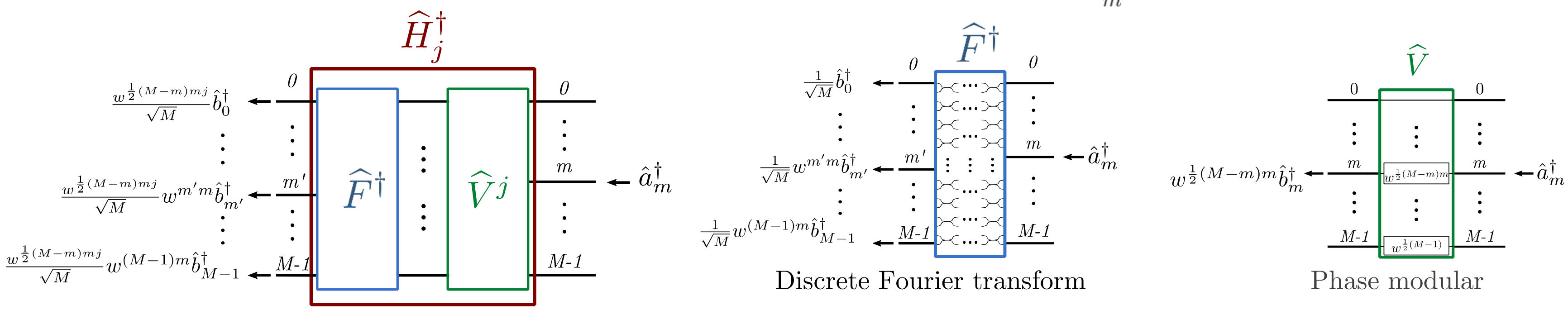


Phase modular

Pauli measurements in single-photon LONs

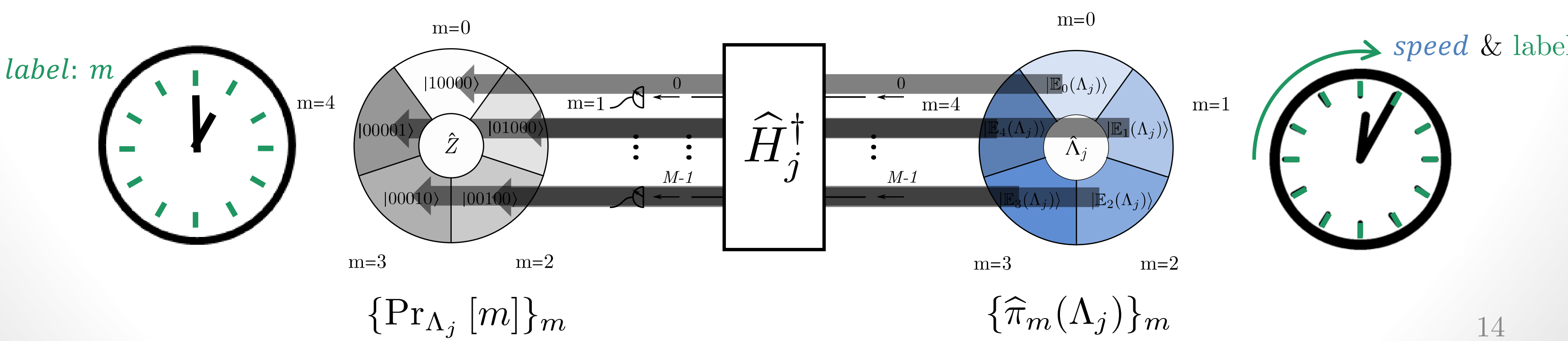
- A generalized Hadamard transform $\widehat{H}_j$ transforms the computational basis to the $\widehat{\Lambda}_j$ eigenbasis,

$$\widehat{H}_j \left|e_m\right\rangle = \left|\mathbb{E}_m(\Lambda_j)\right\rangle \quad \text{with} \quad \widehat{H}_j^\dagger \hat{a}_m^\dagger \widehat{H}_j = w^{\frac{1}{2}(M-m)mj} \sum_{m'} \frac{w^{m'm}}{\sqrt{M}} \hat{b}_{m'}^\dagger .$$



- A Λ_j-Pauli measurement $\{\widehat{\pi}_m(\Lambda_j) = \left|\mathbb{E}_m(\Lambda_j)\right\rangle \left\langle\mathbb{E}_m(\Lambda_j)\right|\}_m$ can be implemented with the inverse Hadamard

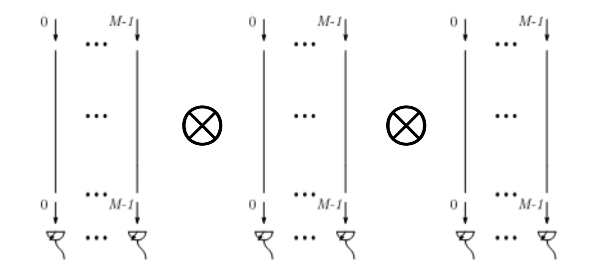
$$\Pr_{\Lambda_j}[m] = \left\langle e_m\right| \widehat{H}_j^\dagger \widehat{\rho} \widehat{H}_j \left|e_m\right\rangle = \mathrm{tr}\left[\widehat{\pi}_m(\Lambda_j)\,\widehat{\rho}\right].$$



Entanglement verifier in single-photon linear optics networks

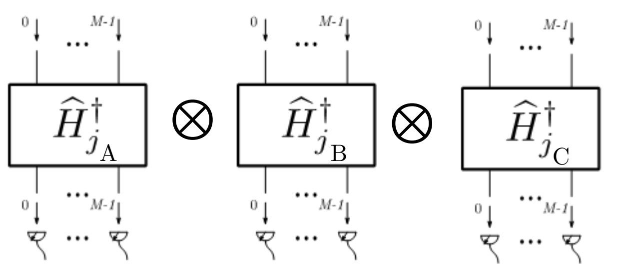
$\hat{\rho}_{ABC}$

$\{\hat{\pi}_m(Z)\}_m \quad \{\hat{\pi}_m(Z)\}_m \quad \{\hat{\pi}_m(Z)\}_m$



…

$\{\hat{\pi}_m(\Lambda_{j_A})\}_m \quad \{\hat{\pi}_m(\Lambda_{j_B})\}_m \quad \{\hat{\pi}_m(\Lambda_{j_C})\}_m$



…

$$\hat{V}_{Z,Z,Z}(\psi_{ABC}) = \sum_{\substack{(m_A,m_B,m_C)\in \\ \mathbb{S}_{Z\otimes Z\otimes Z}(\psi_{ABC})}} |e_{m_A}\rangle\langle e_{m_A}| \otimes \cdots \otimes |e_{m_C}\rangle\langle e_{m_C}|$$

$$\hat{V}_{j_A,j_B,j_C}(\psi_{ABC}) = \sum_{\substack{(m_A,m_B,m_C)\in \\ \mathbb{S}_{j_A,j_B,j_C}(\psi_{ABC})}} |\mathbb{E}_{m_A}(\Lambda_{j_A})\rangle\langle \mathbb{E}_{m_A}(\Lambda_{j_A})| \otimes \cdots \otimes |\mathbb{E}_{m_C}(\Lambda_{j_C})\rangle\langle \mathbb{E}_{m_C}(\Lambda_{j_C})|$$

$$\hat{V}_{Z,Z,Z}|\psi_{ABC}\rangle = |\psi_{ABC}\rangle$$

$$\hat{V}_{j_A,j_B,j_C}|\psi_{ABC}\rangle = |\psi_{ABC}\rangle$$

$$\hat{V}_{\mathrm{ent.}}(\psi_{ABC}) = u_Z \hat{V}_{Z,Z,Z} + \sum_{j_A,j_B,j_C} u_{j_A,j_B,j_C} \hat{V}_{j_A,j_B,j_C}(\psi_{ABC}) \quad \text{with} \quad u_Z + \sum_{j_A,j_B,j_C} u_{j_A,j_B,j_C} = 1$$

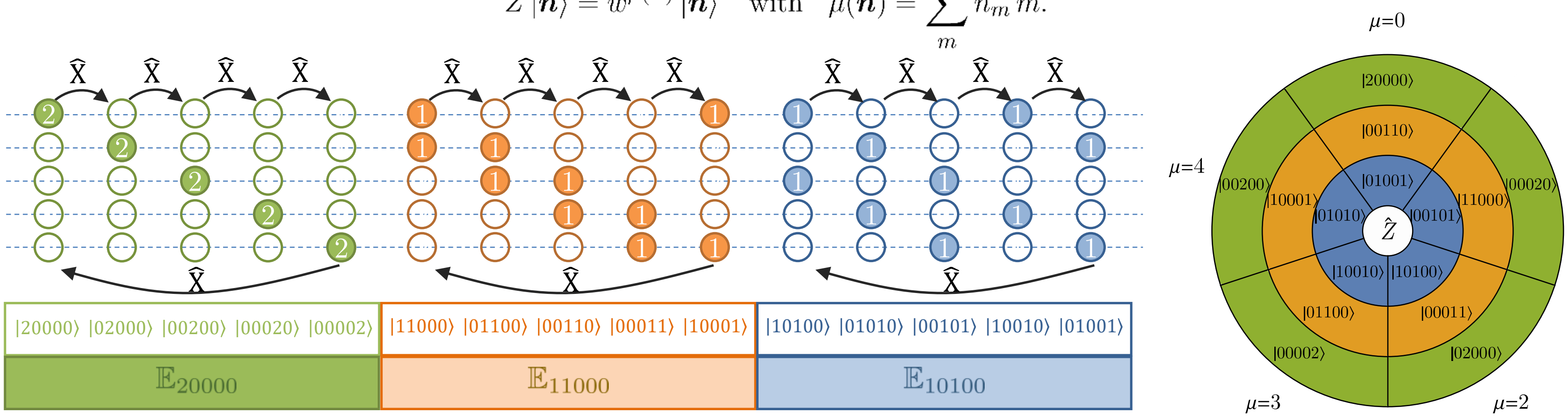
Eigensystems of Heisenberg-Weyl operators

The effect of $\widehat{\Lambda}_j$ performed on a Fock state is a combination of the mode shift and phase shift,

$$\widehat{\Lambda}_j \left|\boldsymbol{n}\right\rangle = \widehat{X}\widehat{Z}^j \left|\boldsymbol{n}\right\rangle = w^{j\mu(\boldsymbol{n})}\widehat{X}\left|\boldsymbol{n}\right\rangle = w^{j\mu(\boldsymbol{n})}\left|n_{M-1}, n_0, ..., n_{M-2}\right\rangle,$$

where $\mu(\boldsymbol{n})$ is the total phase added by $\widehat{Z}$ called $\widehat{Z}$-clock label,

$$\widehat{Z}\left|\boldsymbol{n}\right\rangle = w^{\mu(\boldsymbol{n})}\left|\boldsymbol{n}\right\rangle \quad \text{with} \quad \mu(\boldsymbol{n}) = \sum_m n_m m.$$



Definition (Pauli classes and subspaces). *A Pauli class* $\mathbb{E}_{\boldsymbol{n}}$ *is a set of Fock states, whose elements are generated by the mode-shift operator performed,*

$$\mathbb{E}_{\boldsymbol{n}} := \left\{\widehat{X}^k \left|\boldsymbol{n}\right\rangle : k = 0, ..., d_{\mathbb{E}_{\boldsymbol{n}}} - 1\right\},$$

where $d_{\mathbb{E}_{\boldsymbol{n}}}$ *is the cardinality.* $\widehat{\Lambda}_j$*-Pauli eigenstates can be constructed within the Pauli subspace* $\text{span}(\mathbb{E}_{\boldsymbol{n}})$ *as*

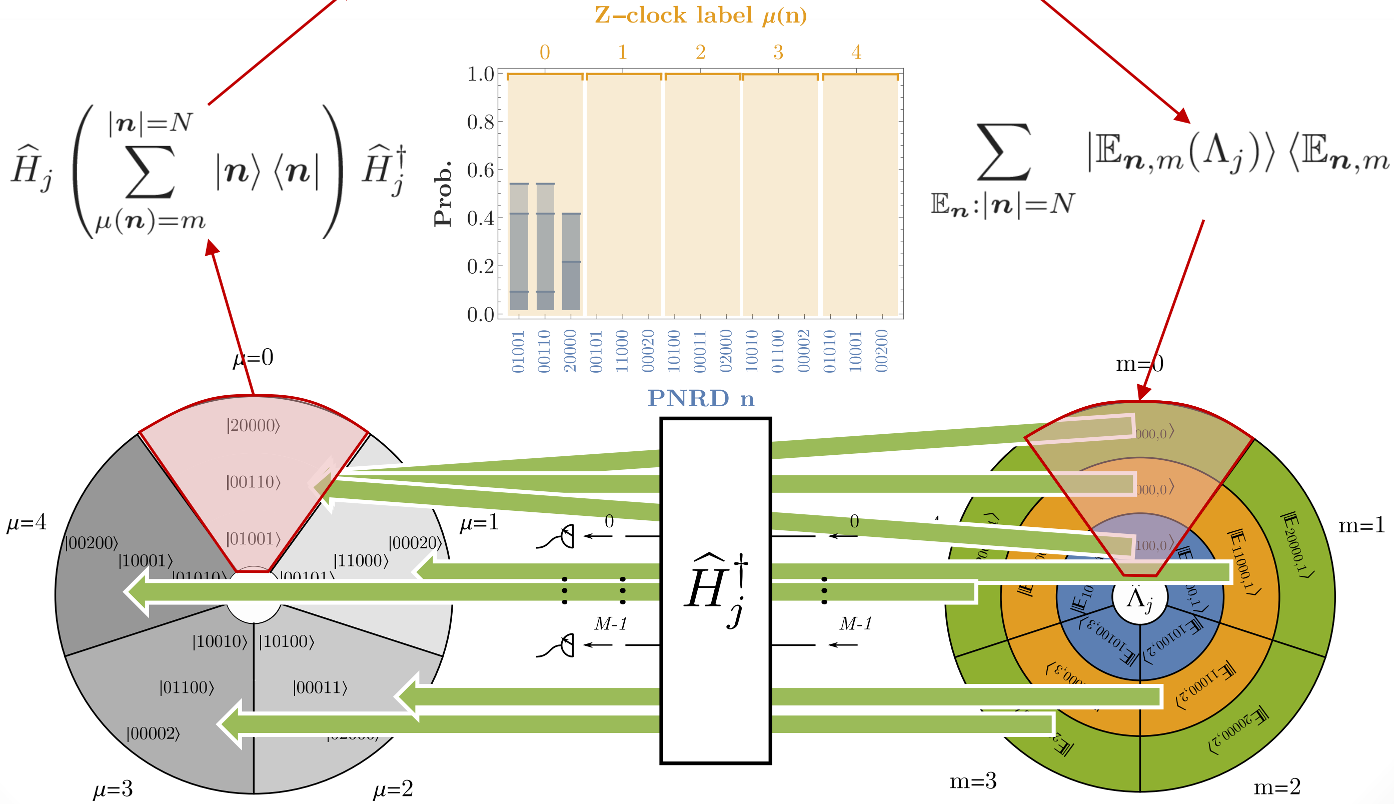
$$\left|\mathbb{E}_{\boldsymbol{n},m}(\Lambda_j)\right\rangle := \frac{1}{\sqrt{d_{\mathbb{E}_{\boldsymbol{n}}}}}\sum_{k=0}^{d_{\mathbb{E}_{\boldsymbol{n}}}-1} w^{-(\frac{1}{2}(M-1)j|\boldsymbol{n}|+m)k}\widehat{\Lambda}_j^k \left|\boldsymbol{n}\right\rangle.$$

Pauli measurements

Theorem 1 (Pauli measurement) *Given a quantum state $\widehat{\rho}$, its expectation value of a $\widehat{\Lambda}_j$-Pauli projector $\widehat{\pi}_{N,m}(\Lambda_j)$ can be evaluated by simply counting the probability of detecting photon number occupations $\boldsymbol{n}$ satisfying $\mu(\boldsymbol{n}) = m$ in the output modes of a $\widehat{H}_j^\dagger$ transform*

$$\Pr\left[(|\boldsymbol{n}| = N, \mu(\boldsymbol{n}) = m) \mid \widehat{H}_j^\dagger \widehat{\rho} \widehat{H}_j\right] = \langle \widehat{\pi}_{N,m}(\Lambda_j) \rangle .$$

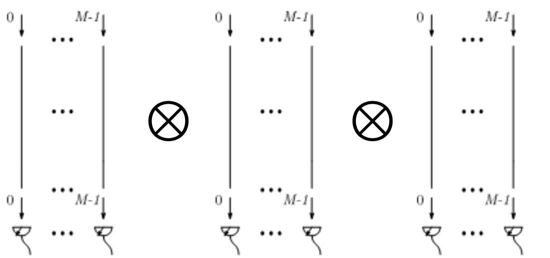
$$\widehat{H}_j \left(\sum_{\mu(\boldsymbol{n})=m}^{|\boldsymbol{n}|=N} |\boldsymbol{n}\rangle\langle\boldsymbol{n}| \right) \widehat{H}_j^\dagger \qquad \sum_{\mathbb{E}_{\boldsymbol{n}}:|\boldsymbol{n}|=N} |\mathbb{E}_{\boldsymbol{n},m}(\Lambda_j)\rangle\langle\mathbb{E}_{\boldsymbol{n},m}(\Lambda_j)|$$

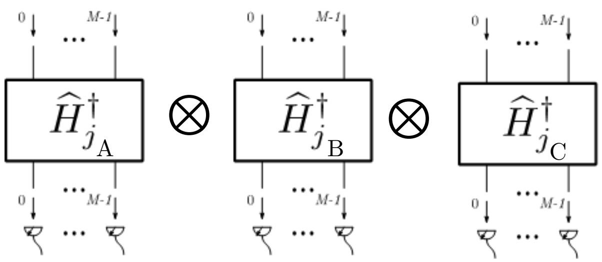


Entanglement verifier in ~~single-photon~~ multi-photon linear optics networks

$\hat{\rho}_{ABC}$

$\{\hat{\pi}_m(Z)\}_m \quad \{\hat{\pi}_m(Z)\}_m \quad \{\hat{\pi}_m(Z)\}_m$ ... $\{\hat{\pi}_m(\Lambda_{j_A})\}_m \quad \{\hat{\pi}_m(\Lambda_{j_B})\}_m \quad \{\hat{\pi}_m(\Lambda_{j_C})\}_m$ ...





$$\hat{V}_{Z,Z,Z}(\psi_{ABC}) = \sum_{\substack{(\boldsymbol{n}_A,\boldsymbol{n}_B,\boldsymbol{n}_C)\in \\ \mathbb{S}_{Z\otimes Z\otimes Z}(\psi_{ABC})}} |\boldsymbol{n}_A,\boldsymbol{n}_B,\boldsymbol{n}_C\rangle\langle\boldsymbol{n}_A,\boldsymbol{n}_B,\boldsymbol{n}_C|$$

$$\hat{V}_{j_A,j_B,j_C}(\psi_{ABC}) = \sum_{\substack{(m_A,m_B,m_C)\in \\ \mathbb{S}_{j_A,j_B,j_C}(\psi_{ABC})}} \hat{\pi}_{N_A,m_A}(\Lambda_{j_A})\otimes\hat{\pi}_{N_B,m_B}(\Lambda_{j_B})\otimes\hat{\pi}_{N_C,m_C}(\Lambda_{j_C})$$

$$\hat{V}_{Z,Z,Z}|\psi_{ABC}\rangle = |\psi_{ABC}\rangle \qquad \hat{V}_{j_A,j_B,j_C}|\psi_{ABC}\rangle = |\psi_{ABC}\rangle$$

$$\hat{V}_{\text{ent.}}(\psi_{ABC}) = u_Z\hat{V}_{Z,Z,Z} + \sum_{j_A,j_B,j_C} u_{j_A,j_B,j_C}\hat{V}_{j_A,j_B,j_C}(\psi_{ABC}) \quad \text{with} \quad u_Z + \sum_{j_A,j_B,j_C} u_{j_A,j_B,j_C} = 1$$

Generation of multipartite mode entanglement

Input state: $|\varphi\rangle = \sum_n c_\varphi(n)\,|n\rangle$

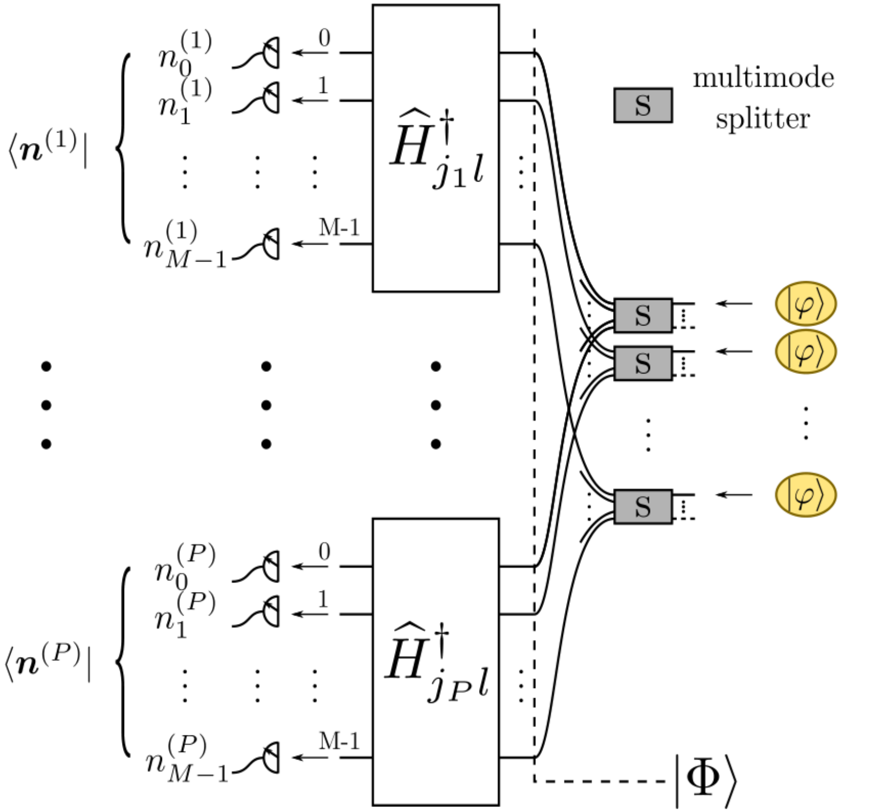
$$|\Phi(\varphi)\rangle = \sum_{\boldsymbol{n}_1,\dots,\boldsymbol{n}_P} \frac{c_\varphi(\boldsymbol{\nu})}{\sqrt{P^{|\boldsymbol{\nu}|}}}\sqrt{\frac{\boldsymbol{\nu}(\boldsymbol{n}_1,\dots,\boldsymbol{n}_P)!}{\boldsymbol{n}_1!\cdots\boldsymbol{n}_P!}}\,|\boldsymbol{n}_1,\dots,\boldsymbol{n}_P\rangle$$

where

$$\boldsymbol{\nu} := \sum_i \boldsymbol{n}_i, \quad \text{and} \quad c_\varphi(\boldsymbol{\nu}) = \prod_{m=0}^{M-1} c_\varphi(\nu_m)$$

Postselection on $(|\boldsymbol{n}_1|,\dots,|\boldsymbol{n}_P|) = (N_1,\dots,N_P)$,

$$|\Phi(\varphi)\rangle \to |\Phi_{N_1,\dots,N_P}(\varphi)\rangle$$



Tripartite (5,5,5)-mode (2,1,1)-photon entanglement

Input: x-displaced r-squeezed state
$|\varphi\rangle = |\sigma(r,x)\rangle = \widehat{D}(x)|\sigma_r\rangle = \widehat{D}(x)\hat{S}(r)|vac\rangle$

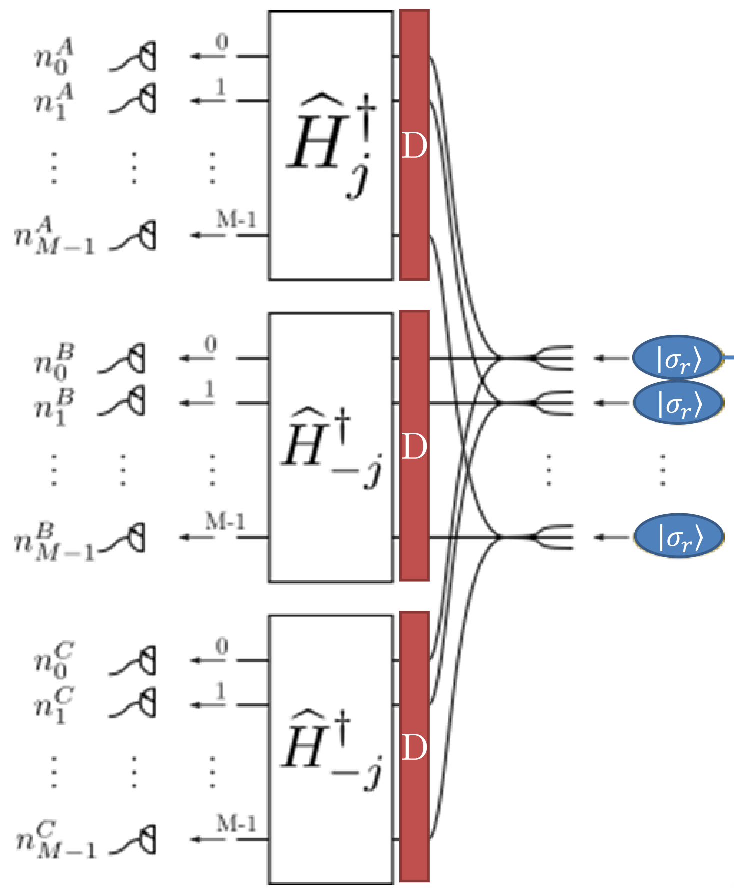
Postselection on (2,1,1) photon:

$$|\Phi_{2_A 1_B 1_C}(r,x)\rangle = \sum_{\kappa=0}^{4} c_\kappa(r,x)\,|\psi_{0,\kappa}(r,x)\rangle$$

is a superposition of GM-entangled states with Heisenberg-Weyl symmetry,

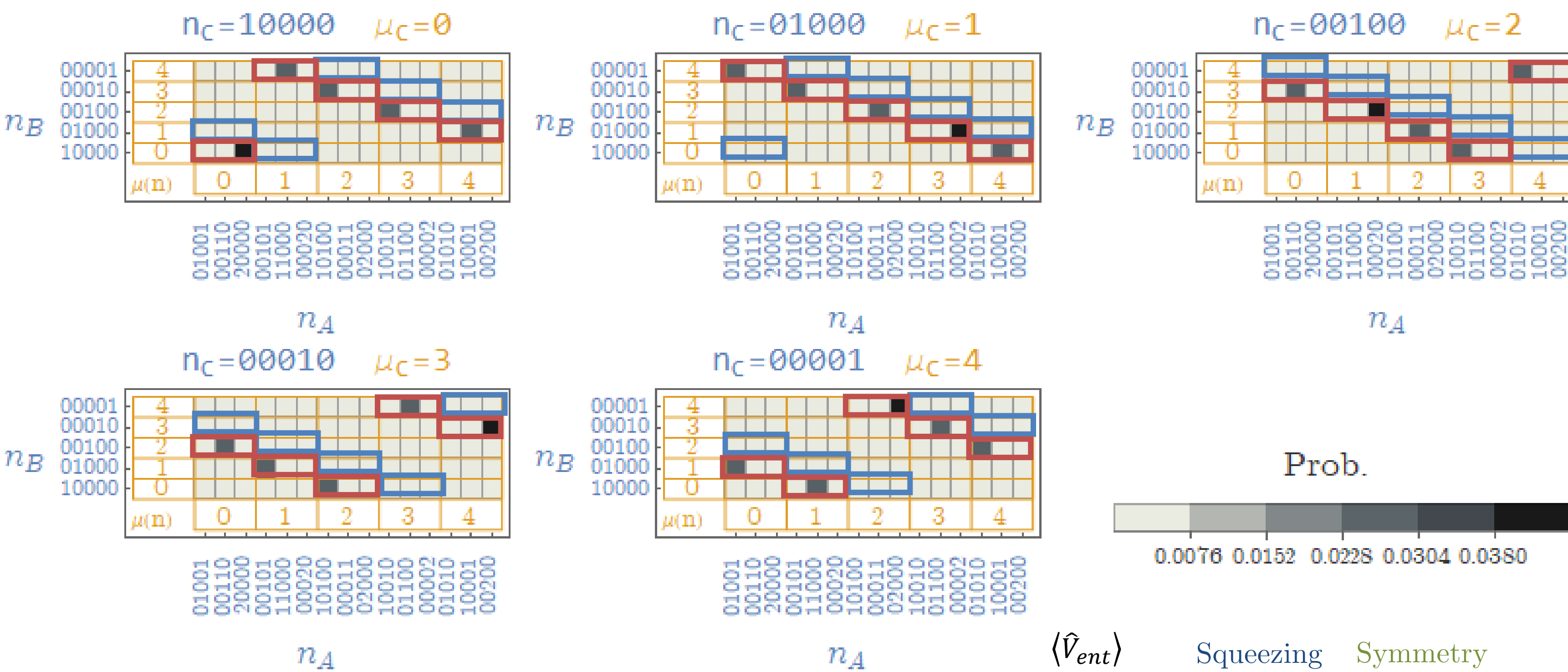
$$\widehat{\Lambda}_j \otimes \widehat{\Lambda}_{-j} \otimes \widehat{\Lambda}_{-j}\,|\psi_{k,\kappa}(r,x)\rangle = \omega^{k+\kappa j}\,|\psi_{k,\kappa}(r,x)\rangle.$$

Select a target GM-entangled state:
$|\psi_{target}\rangle = |\psi_{k=0,\kappa}\rangle$



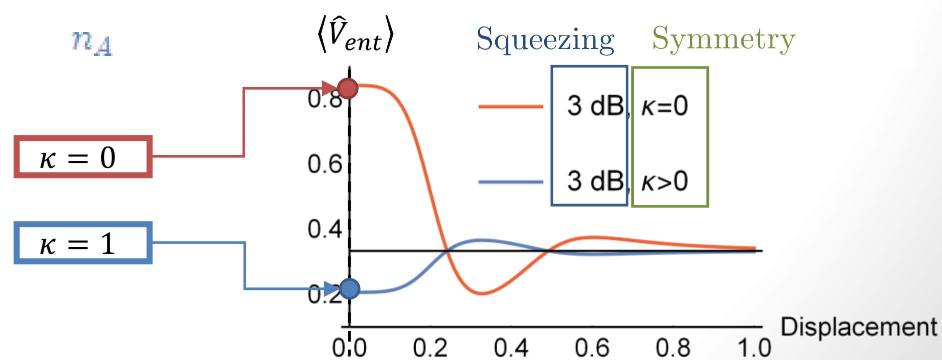
$$\widehat{V}_{ent.} = \frac{1}{6}\left(\widehat{V}_{Z\otimes Z\otimes Z} + \sum_{j=0}^{5} \widehat{V}_{\Lambda_j\otimes\Lambda_{-j}\otimes\Lambda_{-j}}\right) \qquad \mathcal{B}_{sep.} = \max_{|\sigma\rangle \text{ is bi-sep.}} \langle\sigma|\widehat{V}_{ent.}|\sigma\rangle = \frac{1}{3}$$

Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement

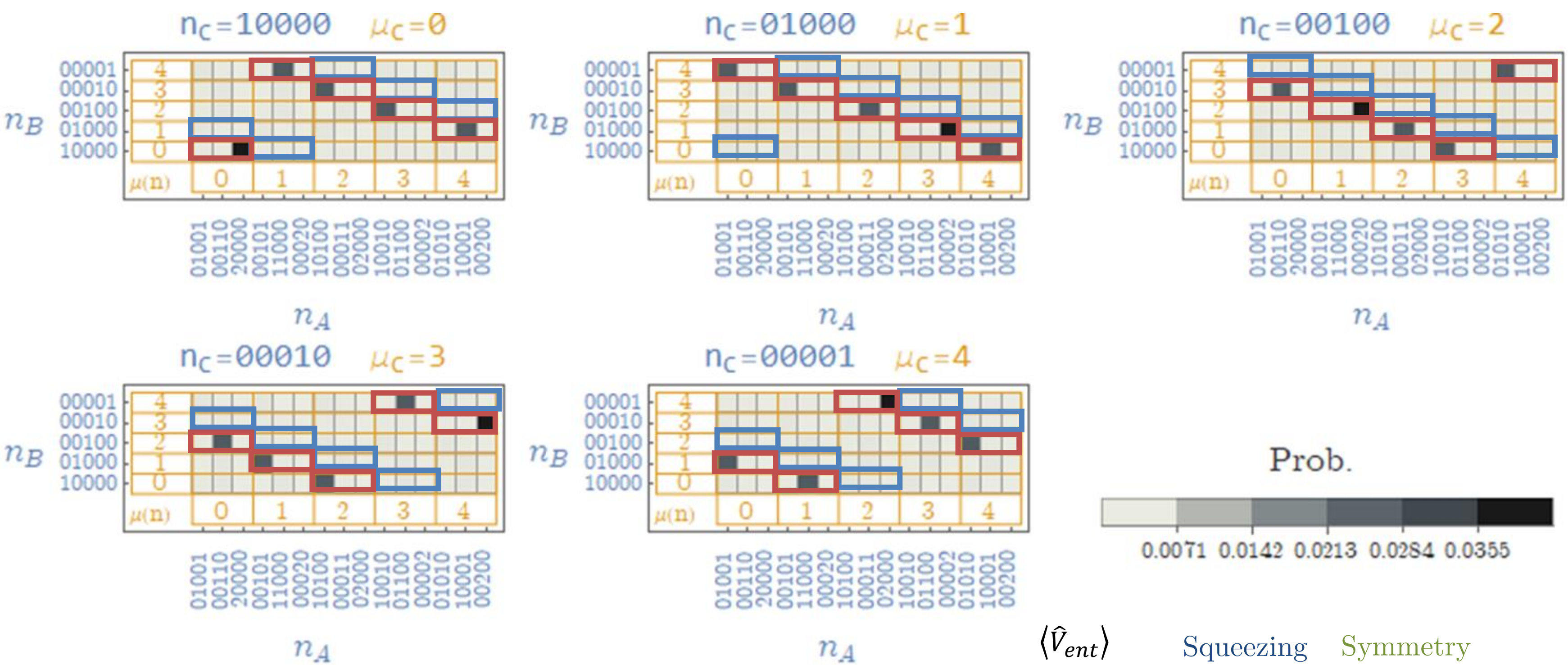


Target state: $|\psi_{k=0,\kappa}\rangle$

$$\hat{\Lambda}_j \otimes \hat{\Lambda}_{-j} \otimes \hat{\Lambda}_{-j} \, |\psi_{k=0,\kappa}\rangle = \omega^{\kappa} \, |\psi_{k=0,\kappa}\rangle$$

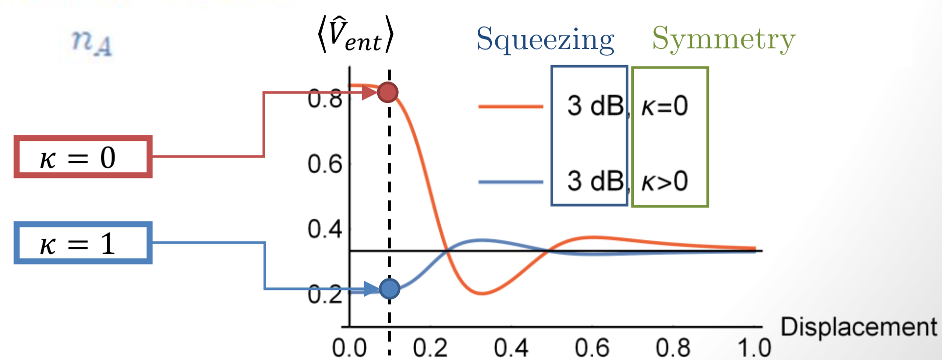


Simulated photon statistics of $\widehat{\Lambda}_1 \otimes \widehat{\Lambda}_{-1} \otimes \widehat{\Lambda}_{-1}$ measurement

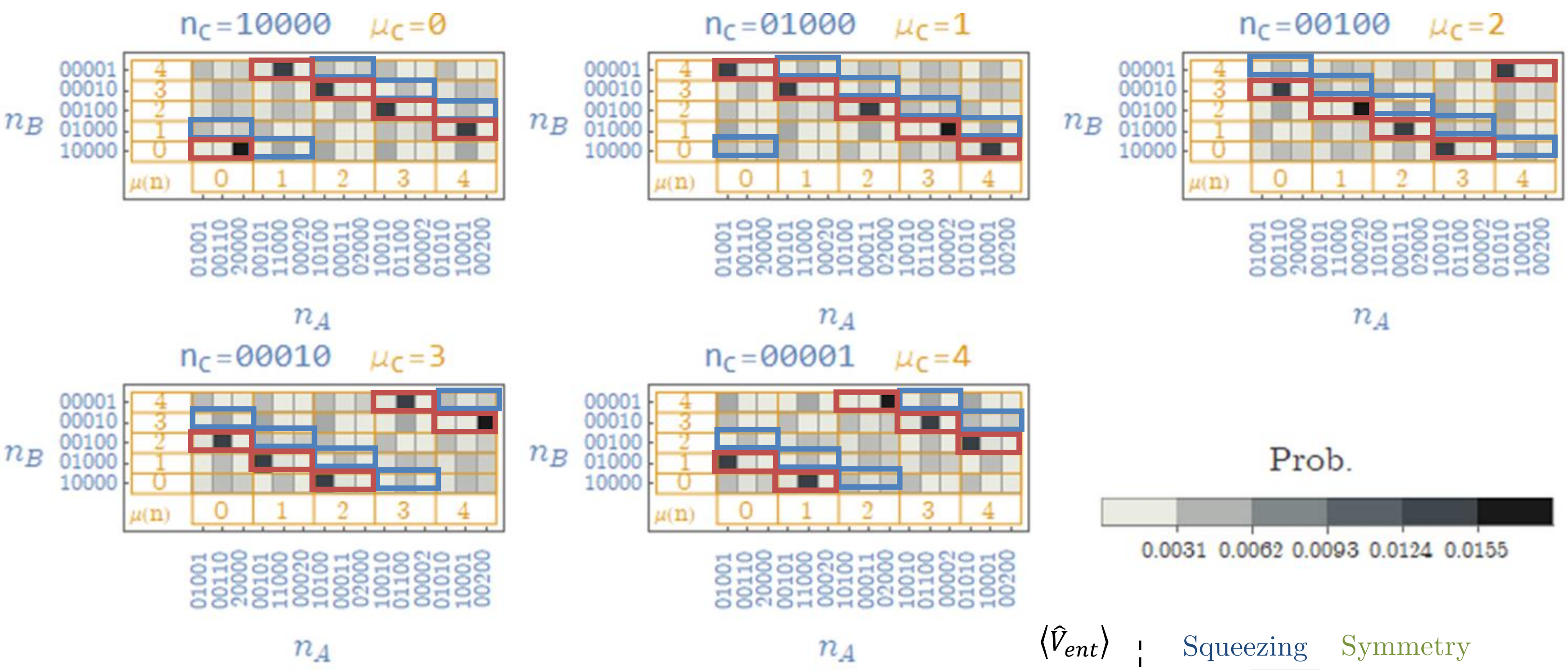


Target state: $|\psi_{k=0,\kappa}\rangle$

$$\widehat{\Lambda}_j \otimes \widehat{\Lambda}_{-j} \otimes \widehat{\Lambda}_{-j} |\psi_{k=0,\kappa}\rangle = \omega^{\kappa} |\psi_{k=0,\kappa}\rangle$$



Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement



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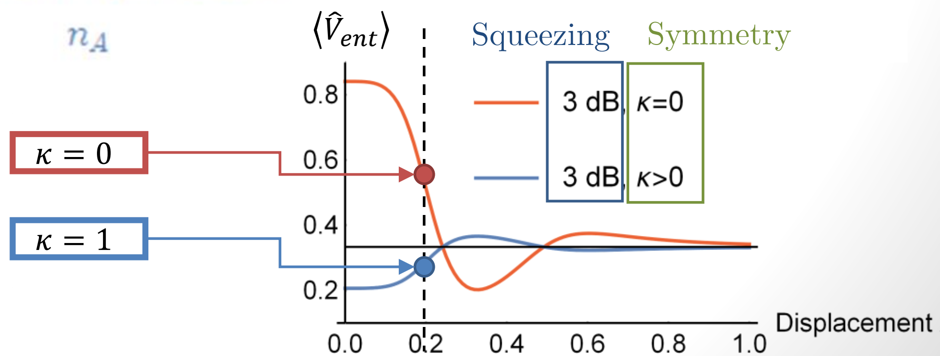
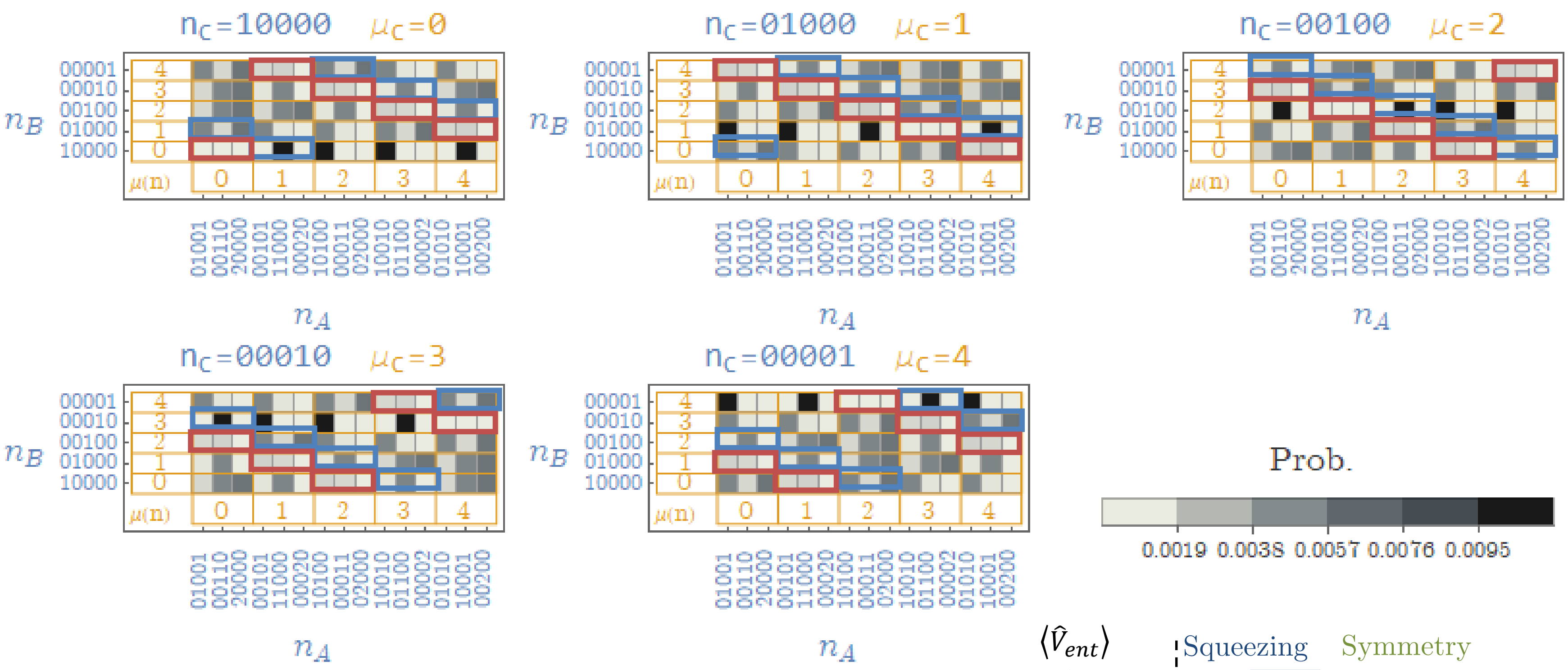


Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement



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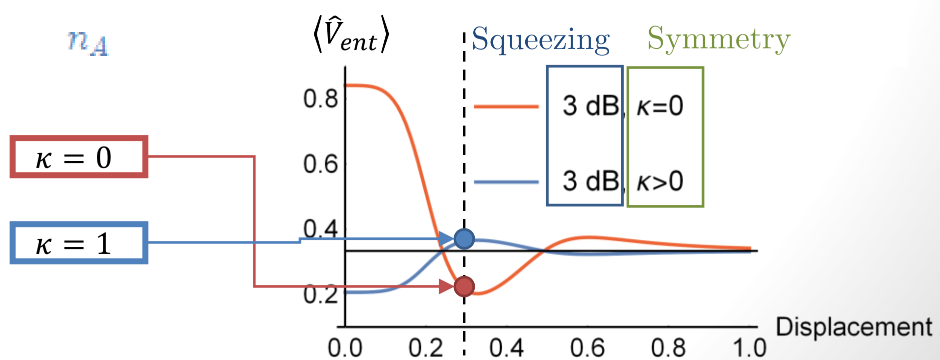
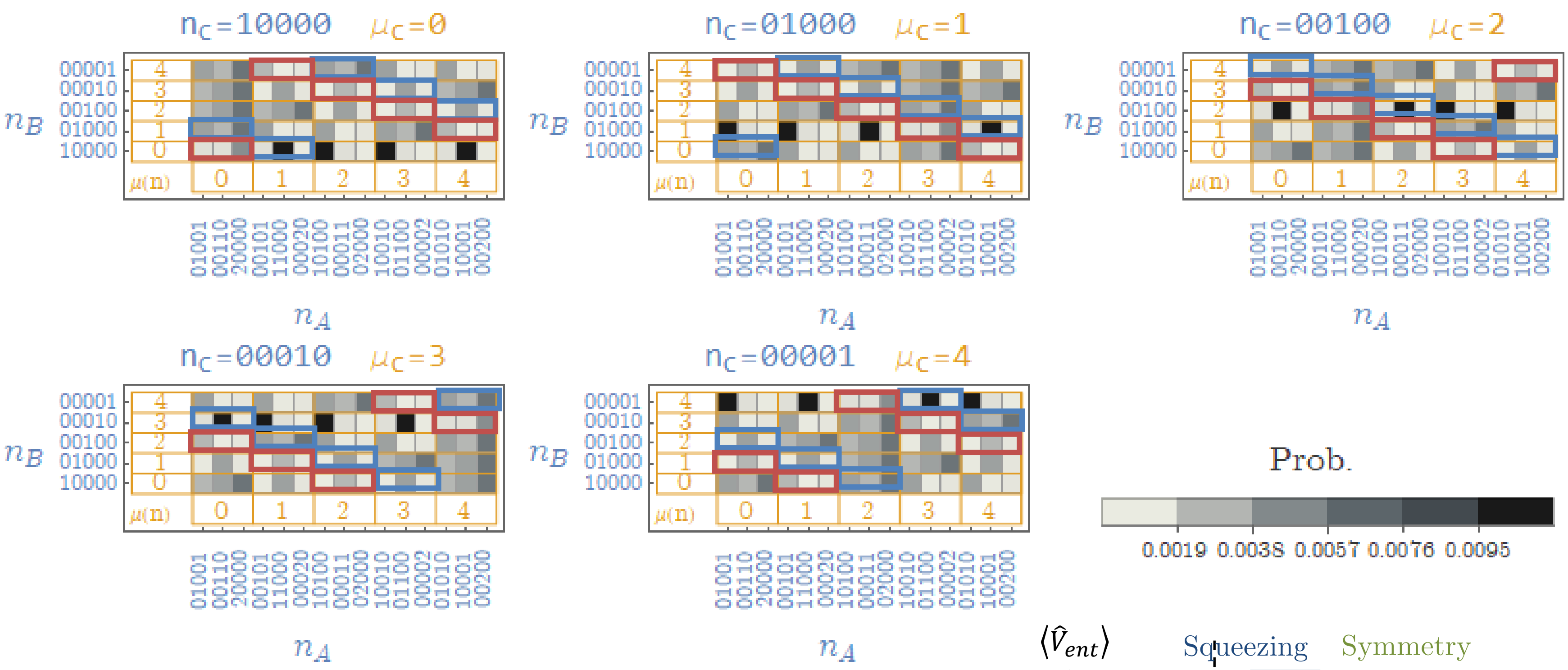
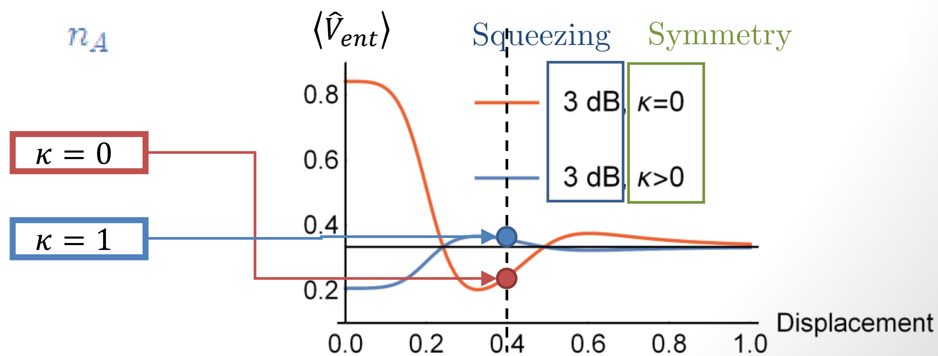


Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement

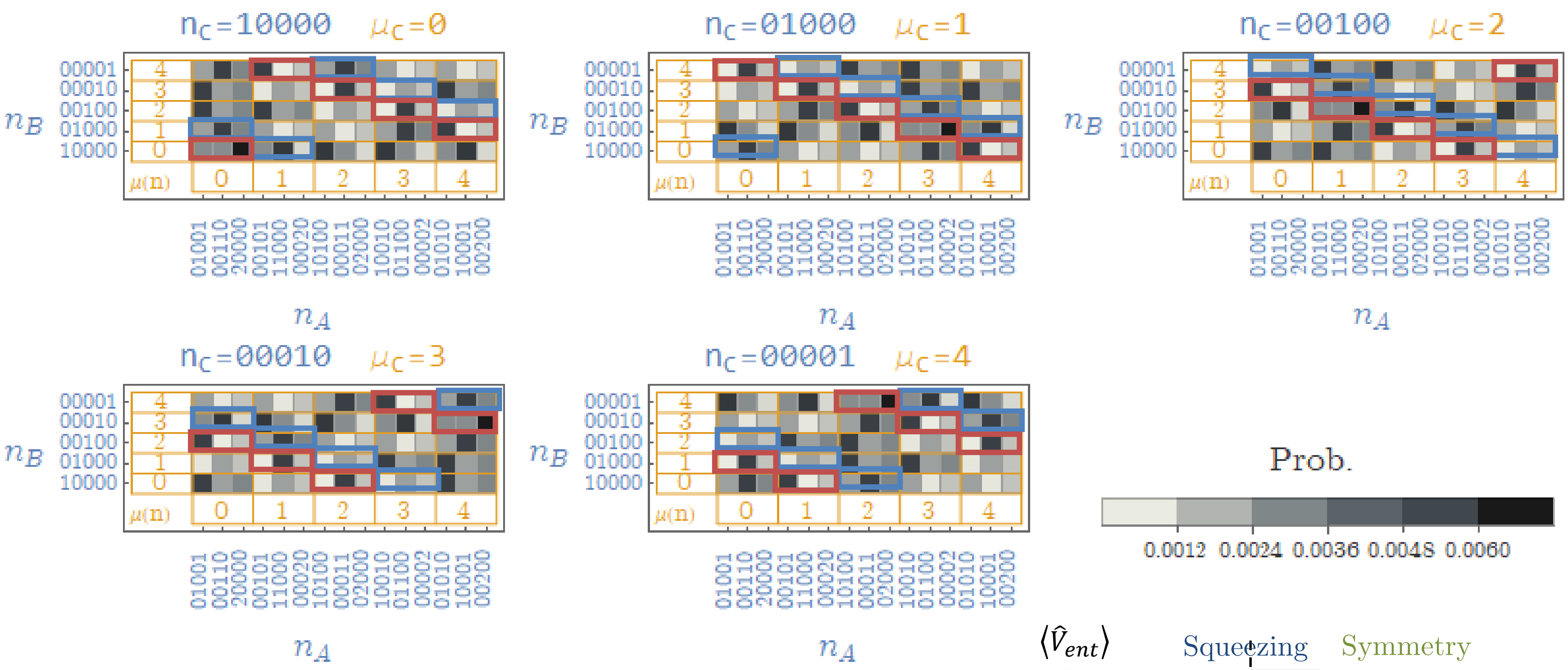


Target state: $|\psi_{k=0,\kappa}\rangle$

$$\hat{\Lambda}_j \otimes \hat{\Lambda}_{-j} \otimes \hat{\Lambda}_{-j} \, |\psi_{k=0,\kappa}\rangle = \omega^{\kappa} \, |\psi_{k=0,\kappa}\rangle$$



Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement



Target state: $|\psi_{k=0,\kappa}\rangle$

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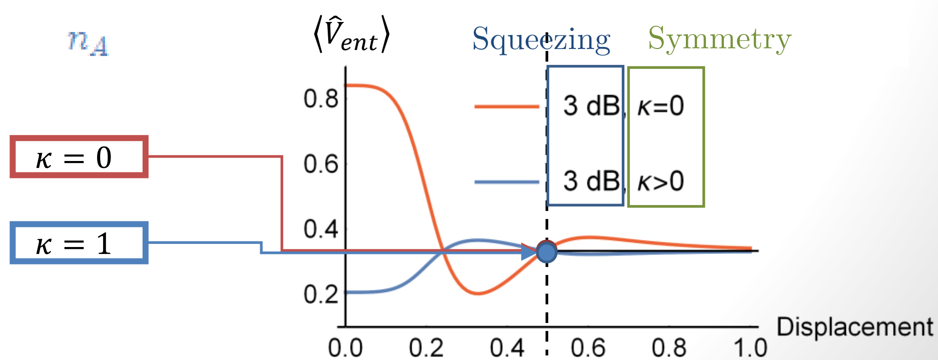
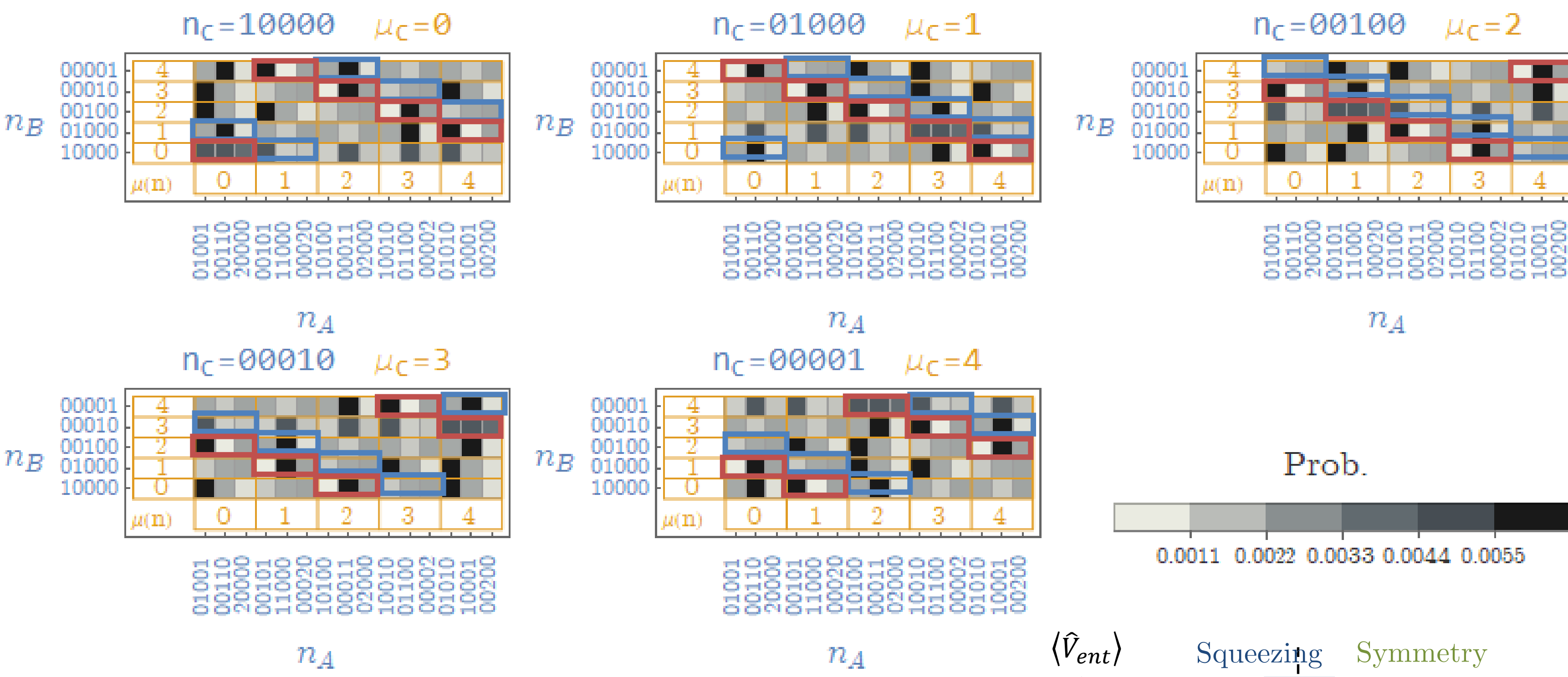


Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement



Target state: $|\psi_{k=0,\kappa}\rangle$

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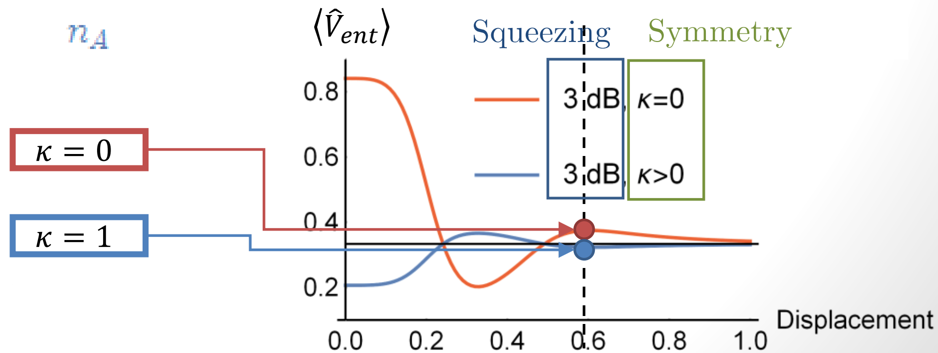
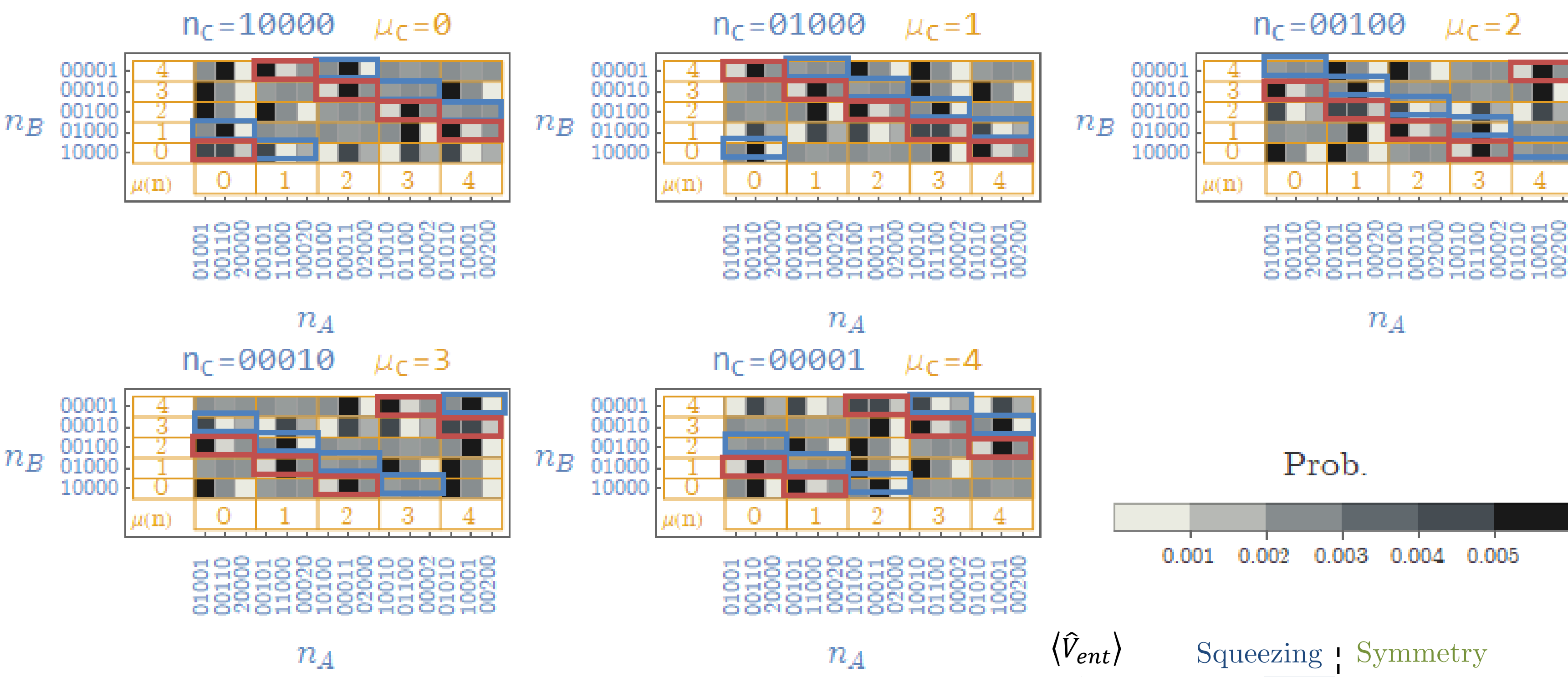


Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement



Target state: $|\psi_{k=0,\kappa}\rangle$

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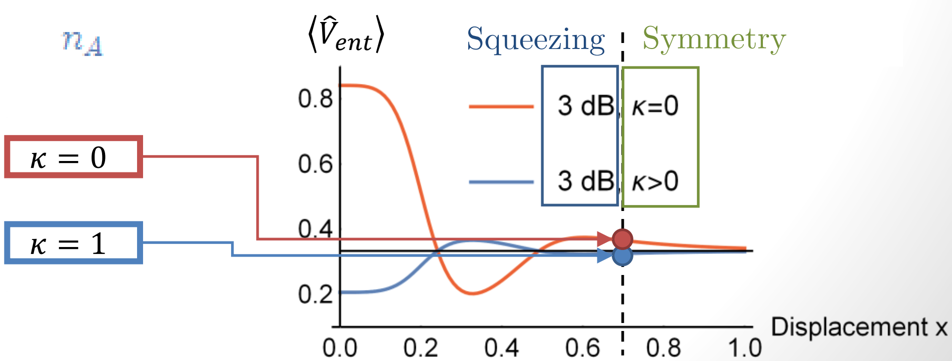
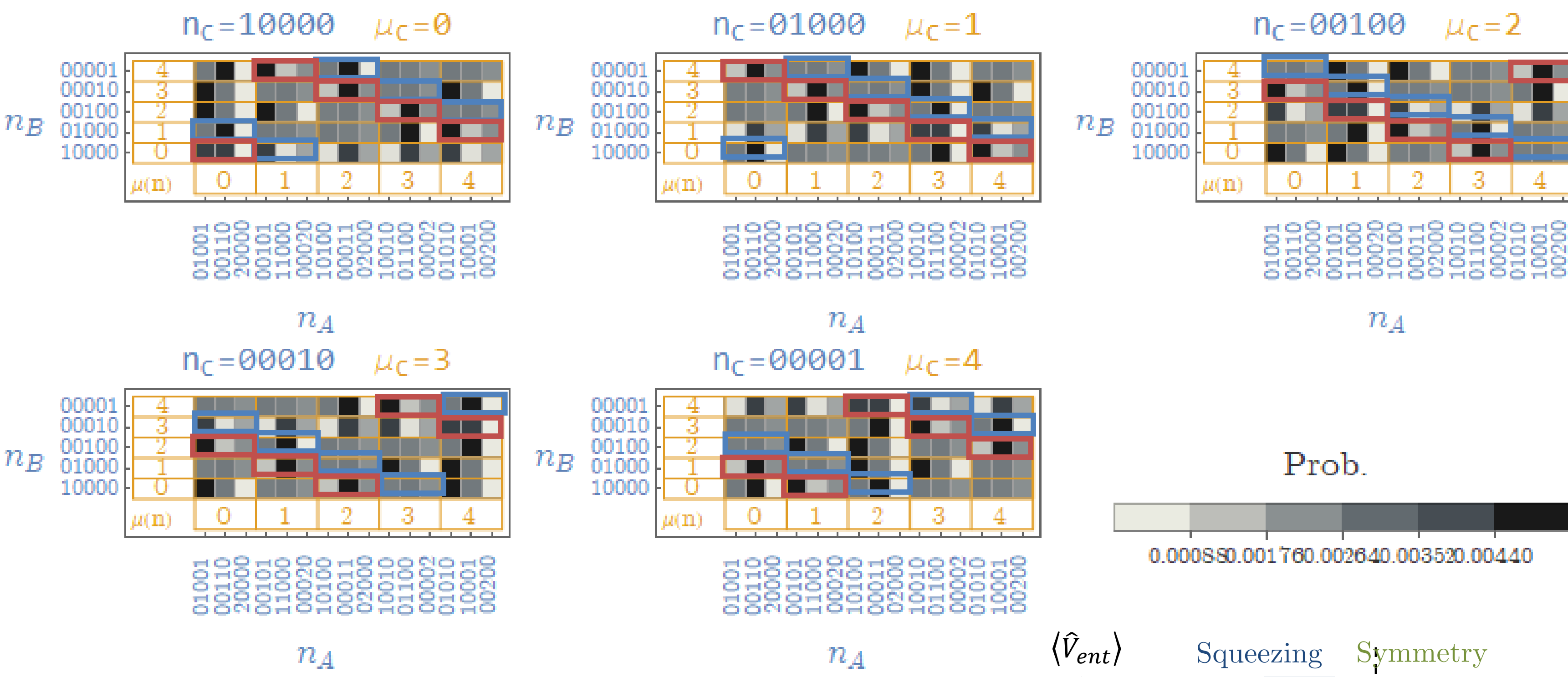


Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement



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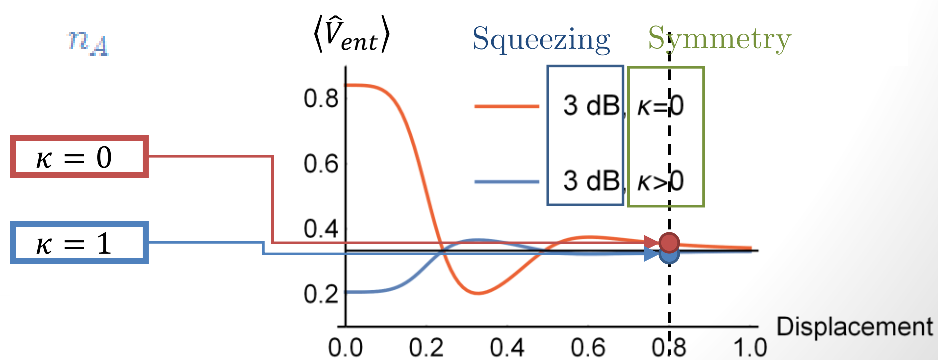
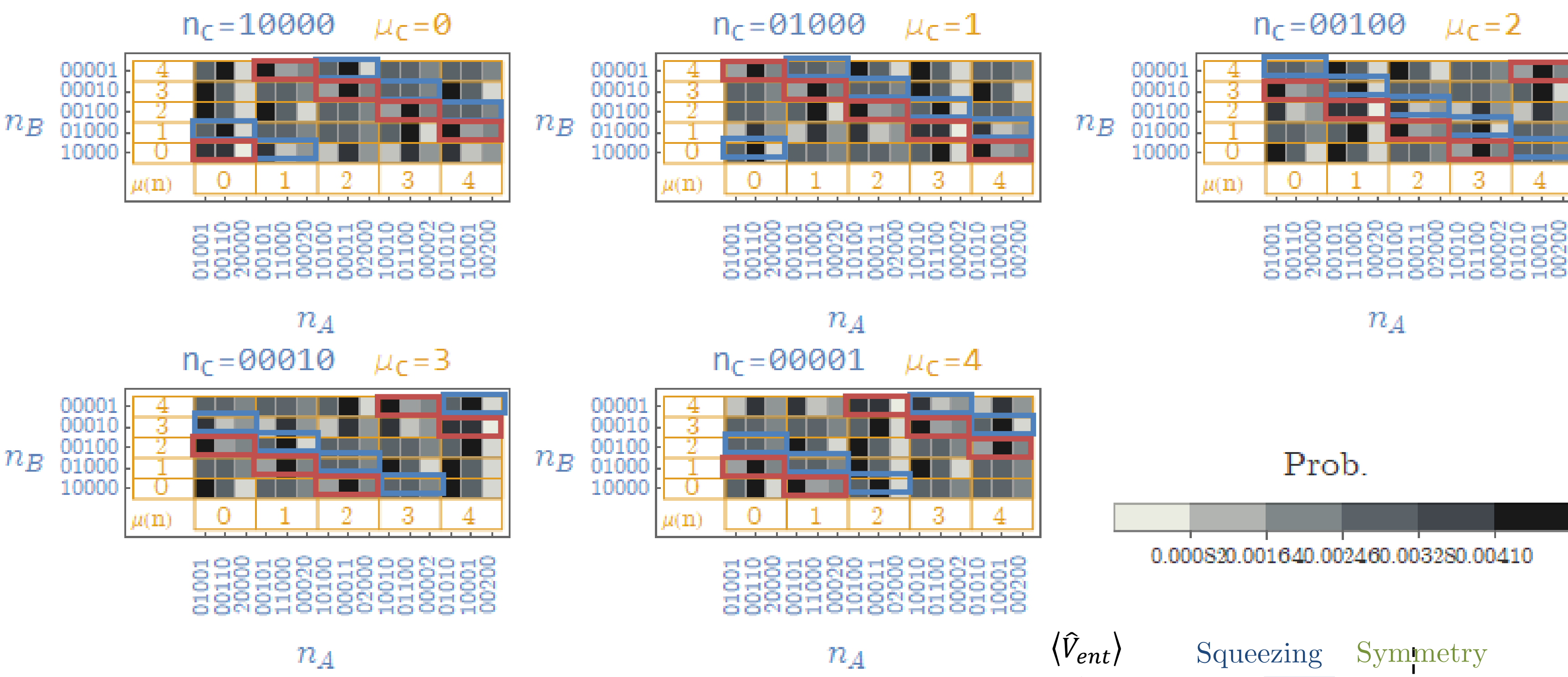
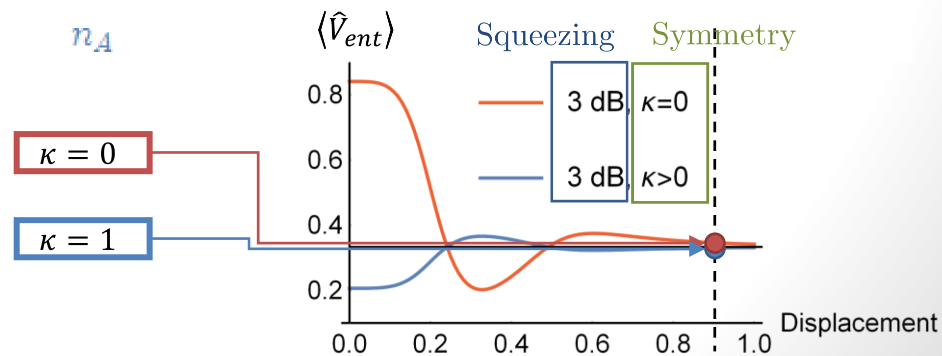


Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement

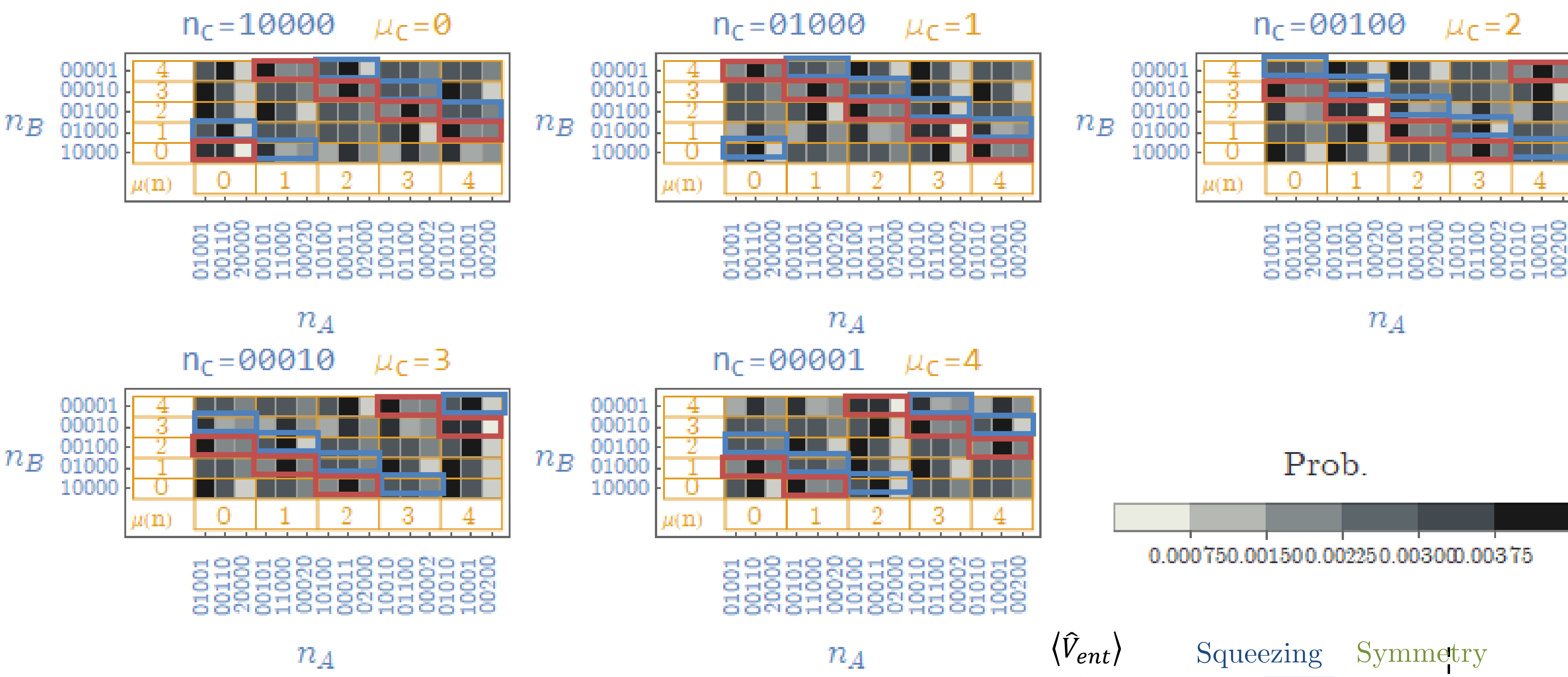


Target state: $|\psi_{k=0,\kappa}\rangle$

$$\hat{\Lambda}_j \otimes \hat{\Lambda}_{-j} \otimes \hat{\Lambda}_{-j} \, |\psi_{k=0,\kappa}\rangle = \omega^{\kappa} \, |\psi_{k=0,\kappa}\rangle$$



Simulated photon statistics of $\hat{\Lambda}_1 \otimes \hat{\Lambda}_{-1} \otimes \hat{\Lambda}_{-1}$ measurement



Target state: $|\psi_{k=0,\kappa}\rangle$

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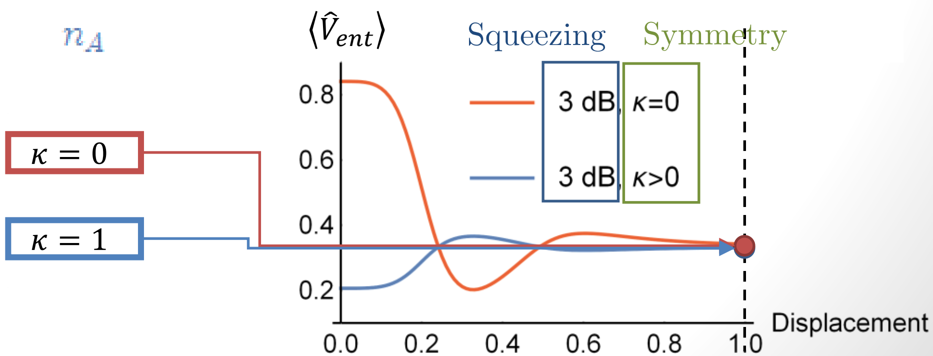
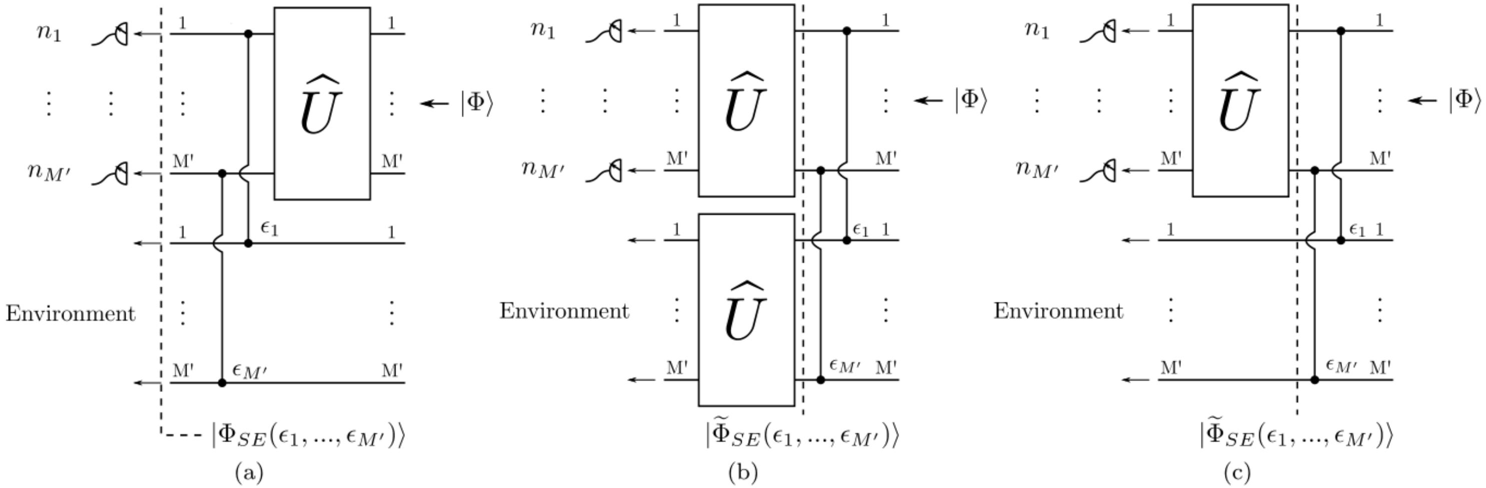
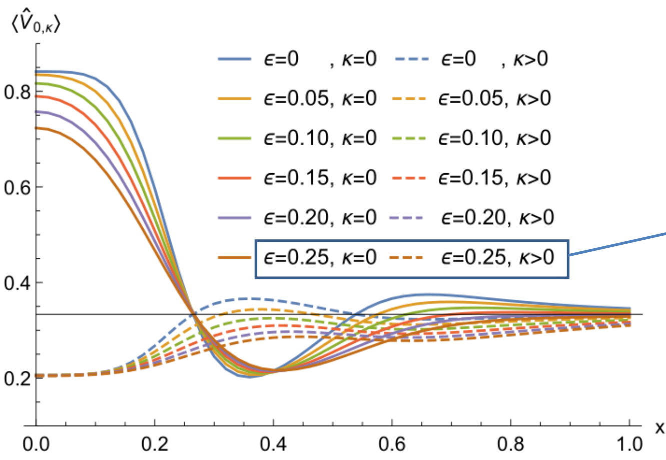


Robustness against photon losses

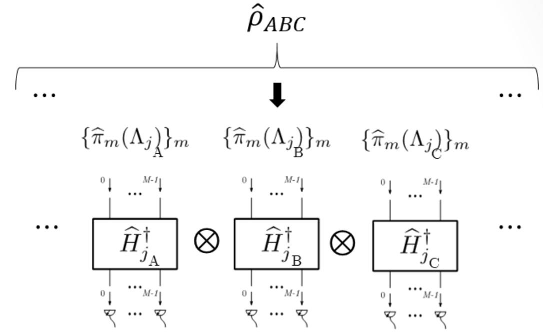




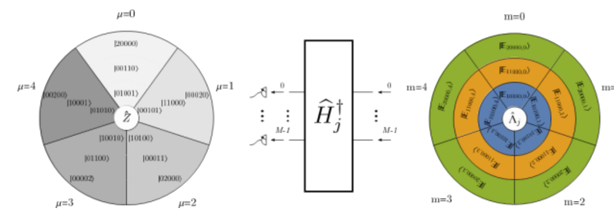
With 25% photon loss, one can still detect GME of (2,1,1)-photon state in (5,5,5)-mode LONs

Conclusion

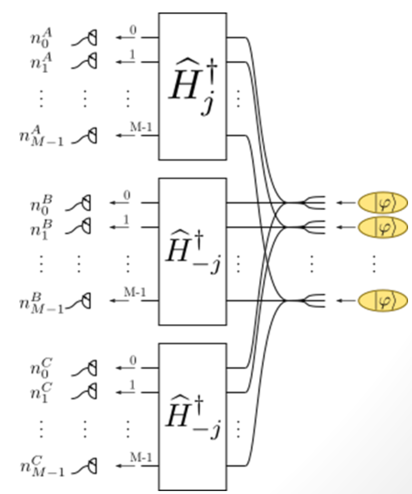
- Multipartite entanglement detection in single-photon LONs:
  - A target GM-entangled state $|\psi_{GME}\rangle$
  - A state verifier/stabilizer $\hat{V}_{GME}$ for the target state
  - Determine the bi-producible upper bound on $\hat{V}_{GME}$



- In multiphoton LONs[a], A state verifier can be constructed in Pauli measurements
  - implemented with generalized Hadamard gates,
  - which can be evaluated efficiently on classical computers.



- GME in multiphoton LONs[b]
  - GME can be generated in Gaussian boson sampling
  - One can observe the transfer of GME among fixed photon-number subspaces.



[a] J.-Y. Wu. and Mio Murao, New J. Phys. 22, 103054 (2020)
[b] J.-Y. Wu, arXiv:2203.14322

Thank you!

NJP. 22, 103054 (2020) + arXiv:2203.14322

$$\widehat{V}_{\rm GME} = u_Z \widehat{V}_Z + \sum_{j_1,\ldots,j_P} u_{j_1,\ldots,j_P} \widehat{V}_{j_1,\ldots,j_P}(\psi)$$

If $\mathrm{tr}(\widehat{V}_{\rm GME}\widehat{\rho}) > \mathcal{B}_{bi-produdible}$, then $\widehat{\rho}$ is GM-entangled.