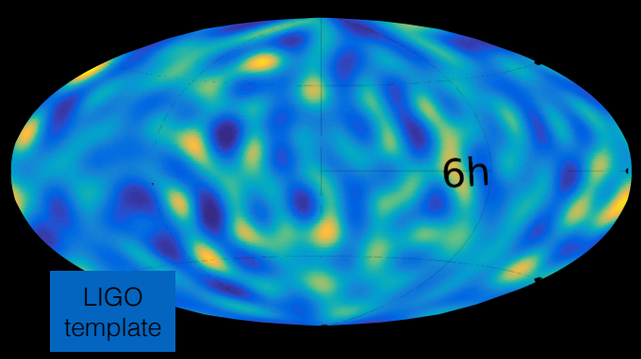
Anisotropy of the Gravitational Wave Background from Primordial Fluctuations

Yuhsin Tsai

University of Notre Dame



Highlights of 2022, NTU
12/29/2022

Yuhsin Tsai ytsai3@nd.edu

University of Notre Dame: hiring HEP students for 2024!

COLLEGE OF SCIENCE

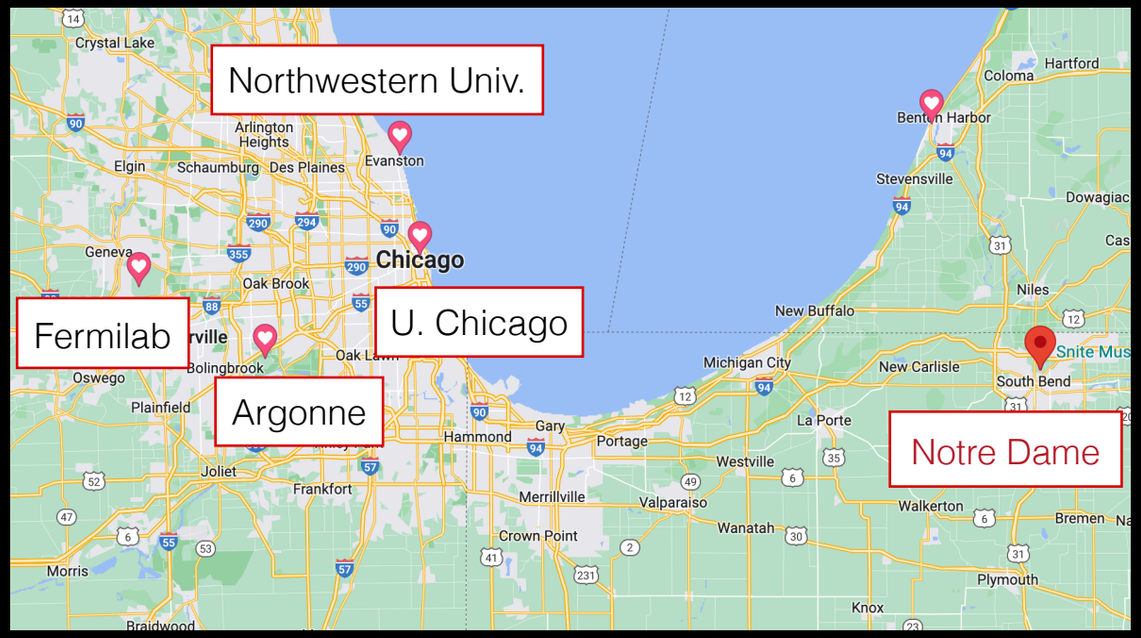
Department of Physics and Astronomy

Notre Dame HEP group

SCAN ME

Tsai's research group page

SCAN ME



The talk is based on three works

"Primordial Anisotropies in the Gravitational Wave Background from Cosmological Phase Transitions"

Michael Geller, Anson Hook, Raman Sundrum, **YT** (2018)

"Non-Gaussian Stochastic Gravitational Waves from Phase Transitions"

Soubhik Kumar, Raman Sundrum, **YT** (2021)

"Unraveling Cosmological Anisotropies within Stochastic GW Background"

Yanou Cui, Soubhik Kumar, Raman Sundrum, **YT** (hopefully soon)

Let’s focus on the anisotropy of stochastic gravitational wave background (GWB)

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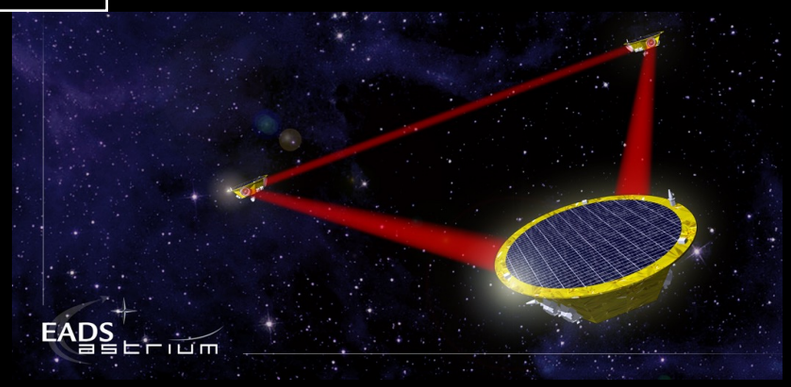
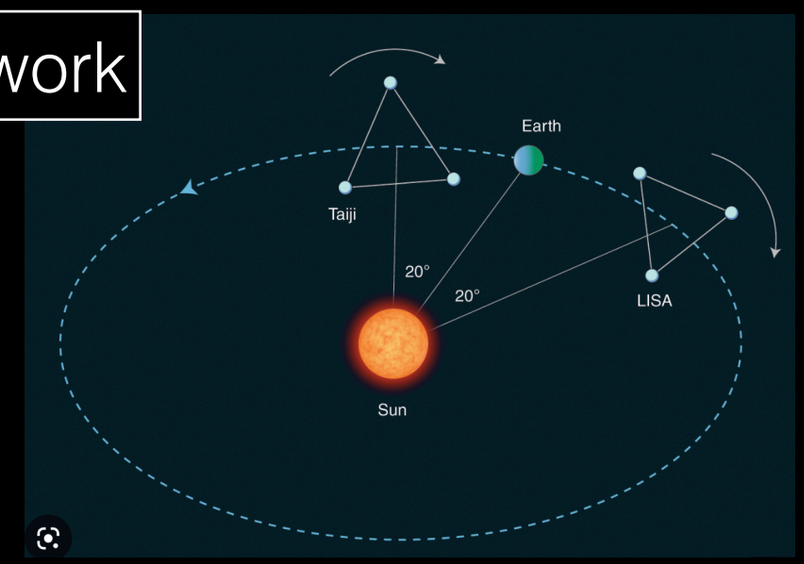
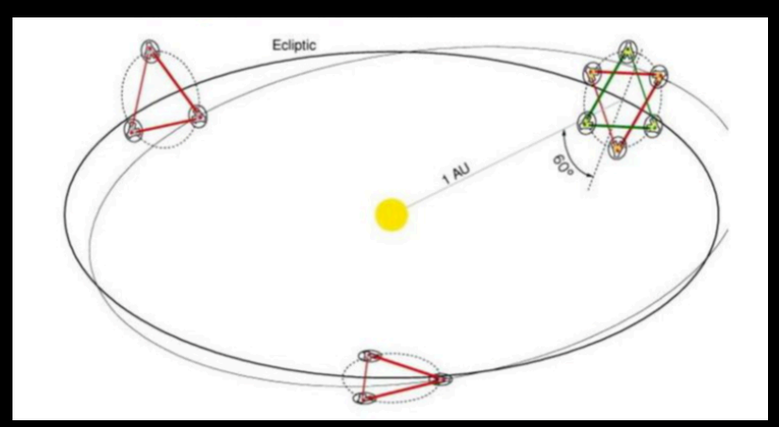
LISA



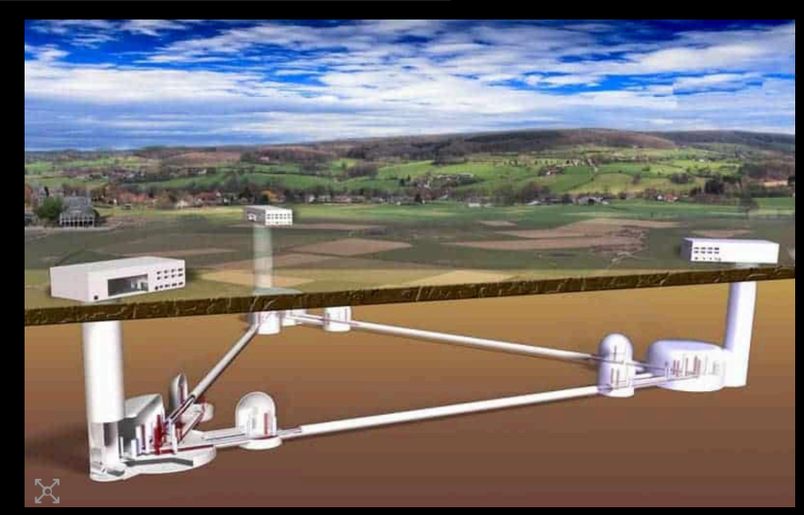
LISA+Taiji network



Big-bang observer (BBO)
similar to DECIGO

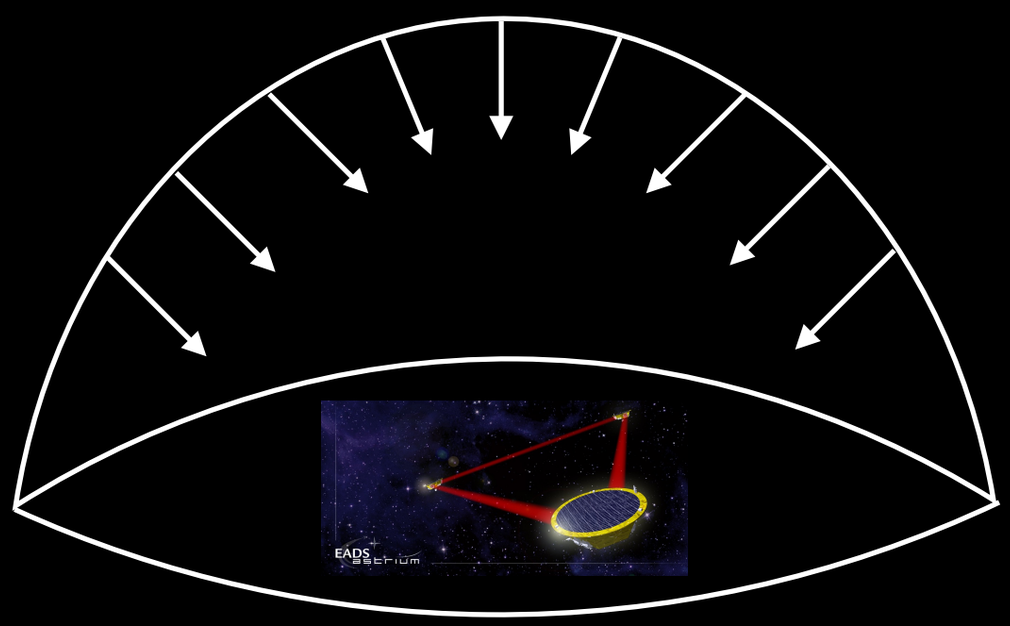


Einstein telescope (ET)

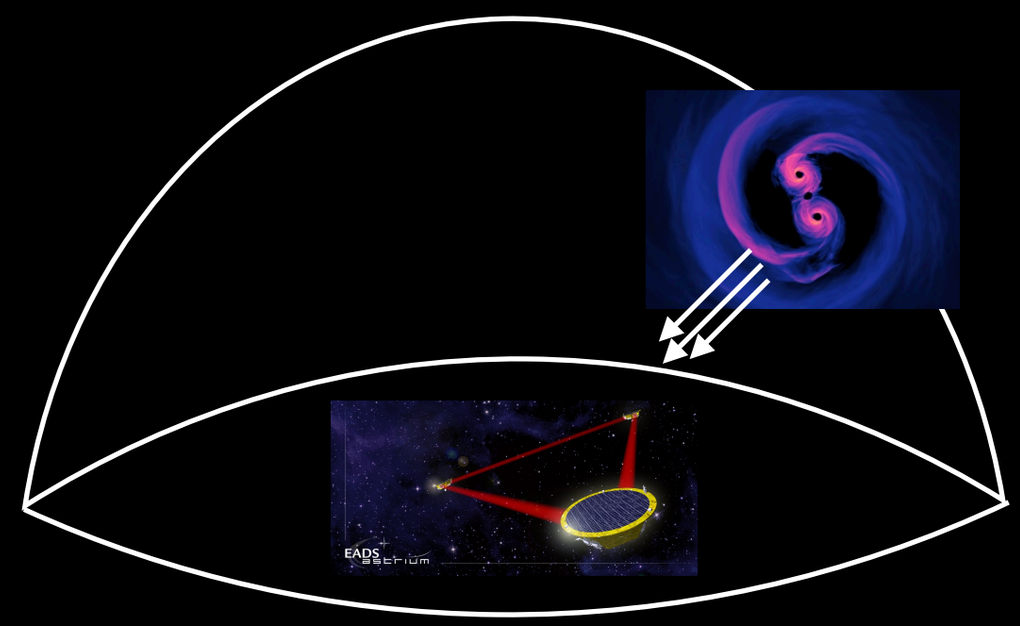


Let's focus on the anisotropy of stochastic gravitational wave background (GWB)

Stochastic signal
(cannot identify individual sources)



Localized objects
(like black hole mergers)



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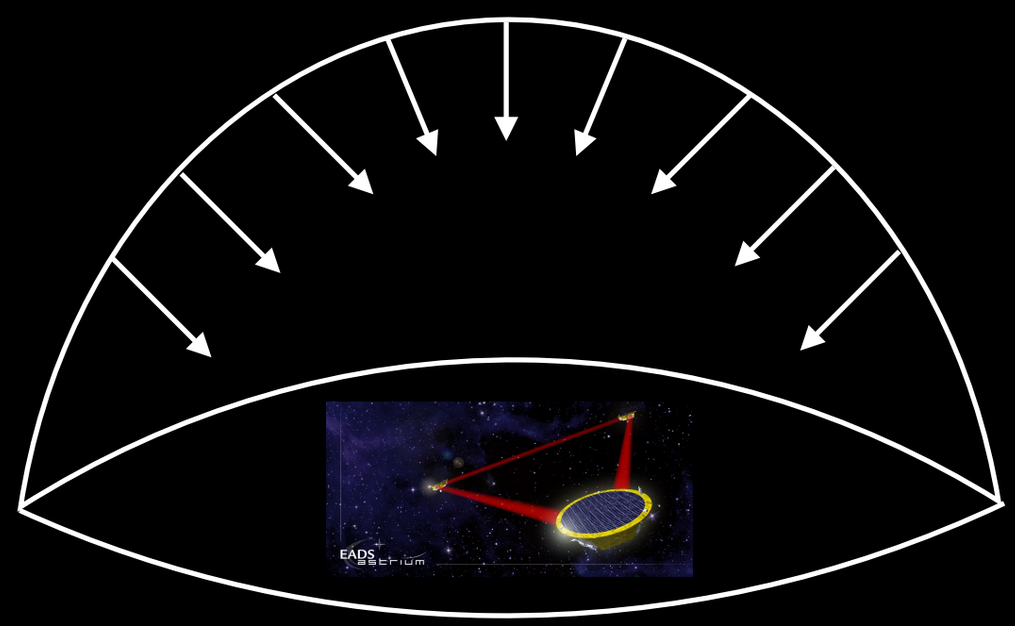
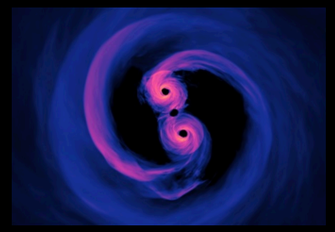
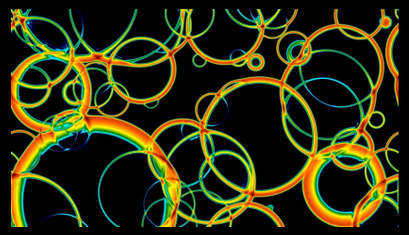


- un-resolved astro sources
  - mainly far away black hole or neutron star mergers

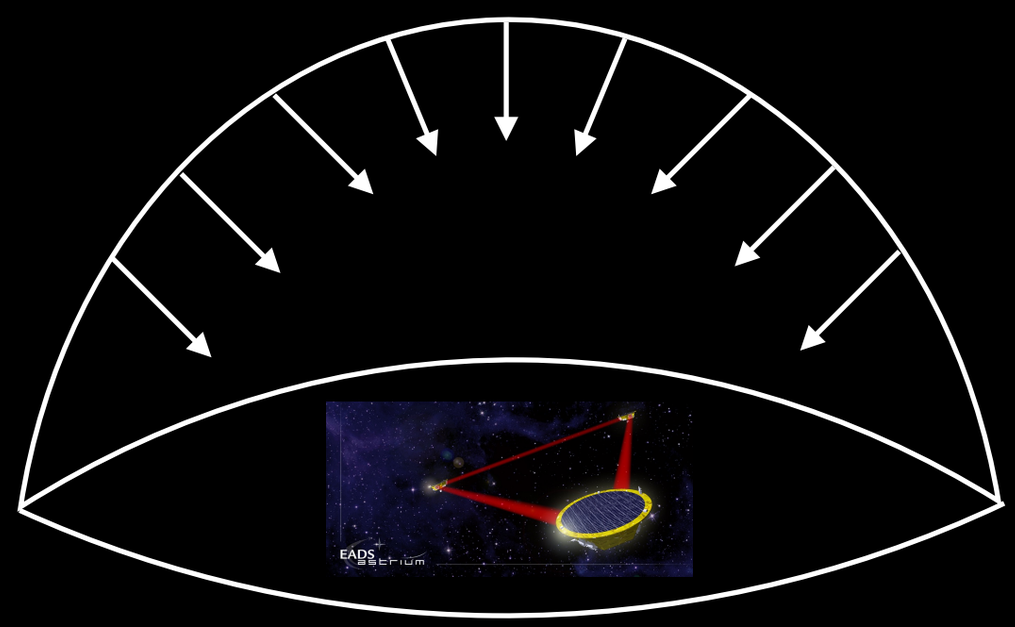


- cosmological origin
  - violate preheating / phase transition
  - topological defects: cosmic string
  - inflationary dynamics
  - primordial black holes
  - …

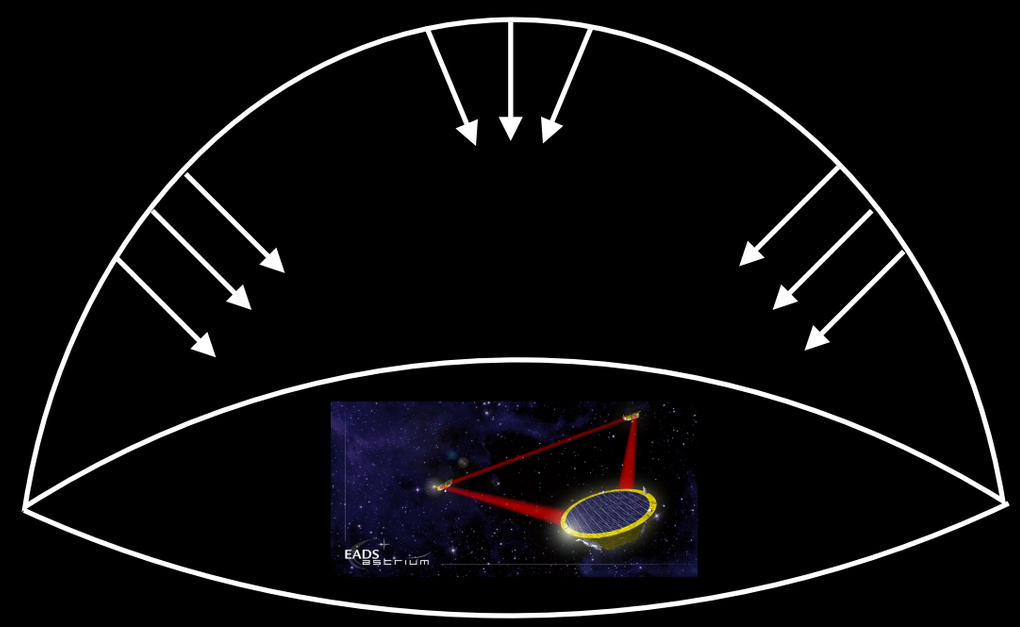


Let's focus on the anisotropy of stochastic gravitational wave background (GWB)

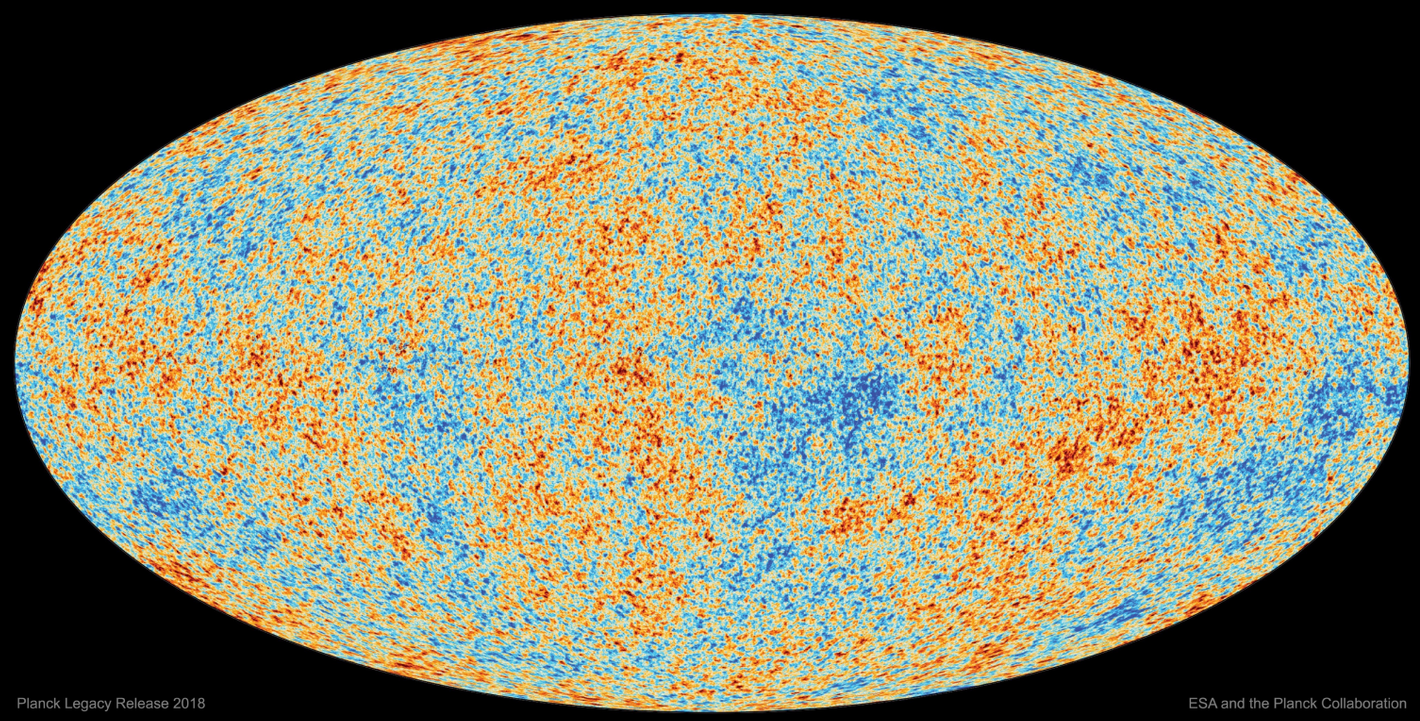
Isotropic signal



Anisotropic signal



Cosmic Microwave Background (CMB)

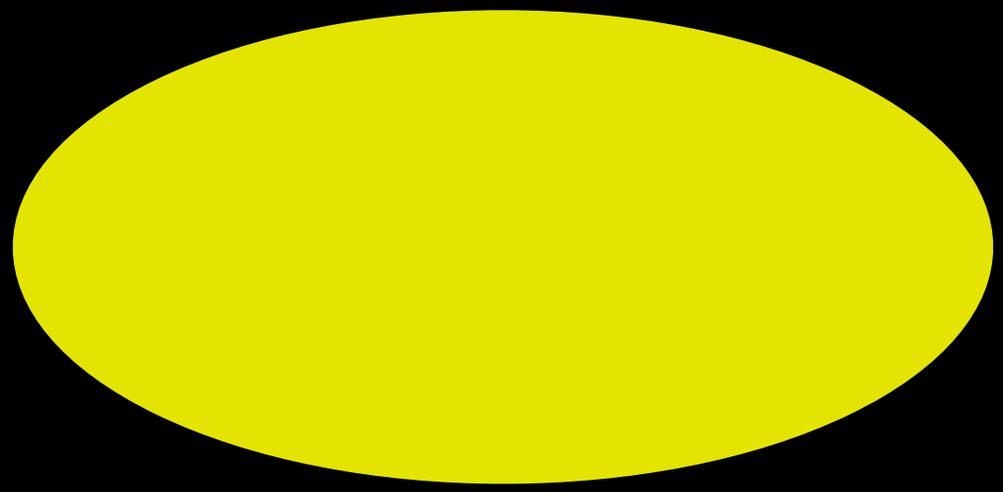


- an-isotropic with fluctuation $\Delta T/T \sim 10^{-4}$
- fluctuation is very Gaussian $\langle \Delta T \Delta T \rangle \neq 0$

$$\langle \Delta T^{n \geq 3} \rangle \sim 0$$

Similar to CMB, if we get to see
GWB with a cosmological origin

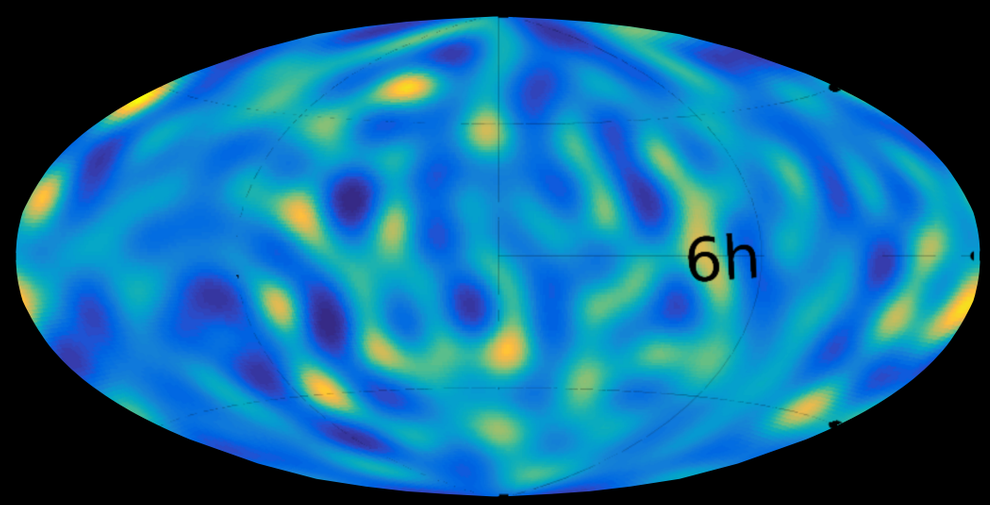
The signal in the whole sky will NOT look like this



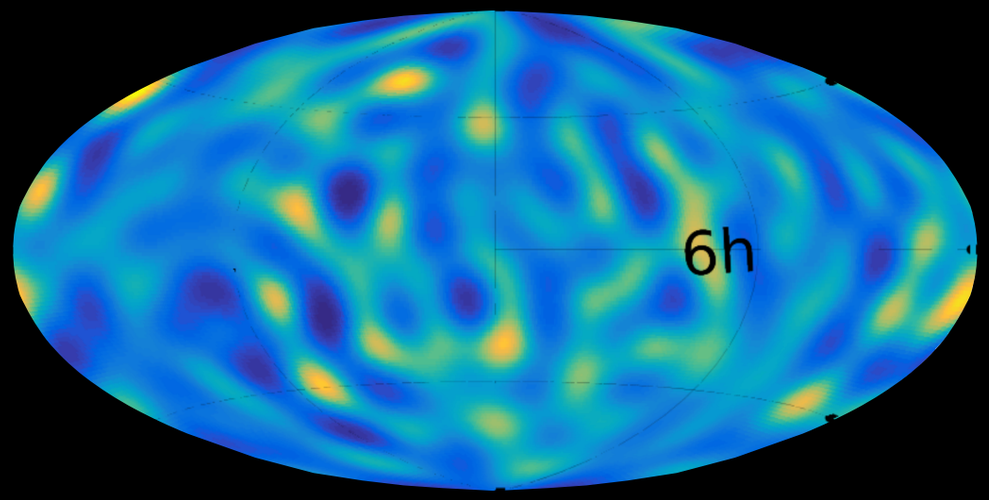
Similar to CMB, if we get to see GWB with a cosmological origin

But will look like this

(with enough detector sensitivity)



If we get to see the signal,



we can learn both about the

- Mechanism of GW production
- Energy/gravity fluctuation of the universe

deep inside the primordial dark age

(e.g., when the horizon size ~ atom for the phase transition signal)

I'm going to discuss the following questions

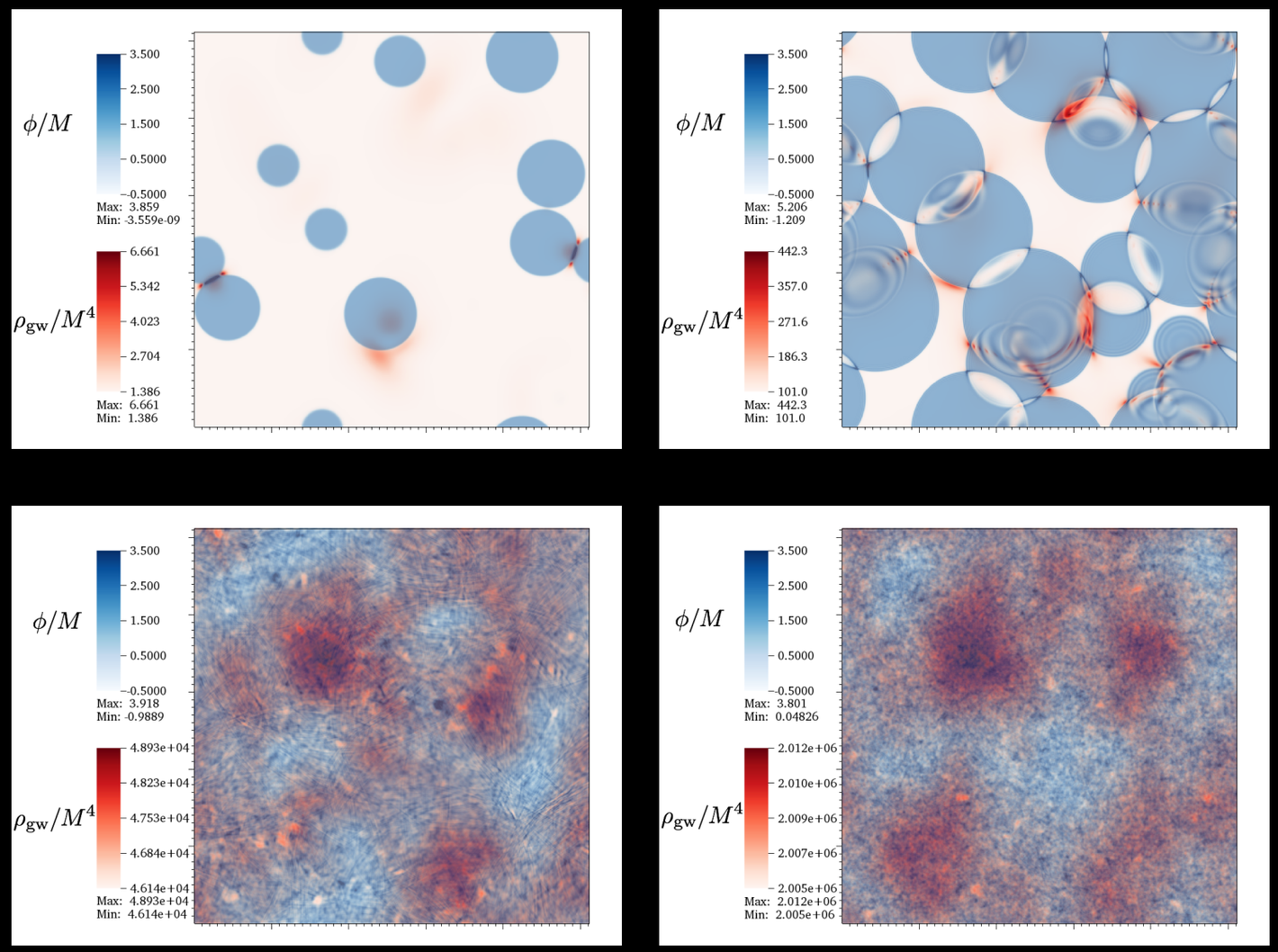
- How large can the an-isotropic GW signal be?
- What can we learn from the an-isotropic signal?
- Can the proposed GW experiment see it?

(with limited detector sensitivity + astrophysical foreground)

For a concrete example, let's focus on signals from the first order phase transition (PT)

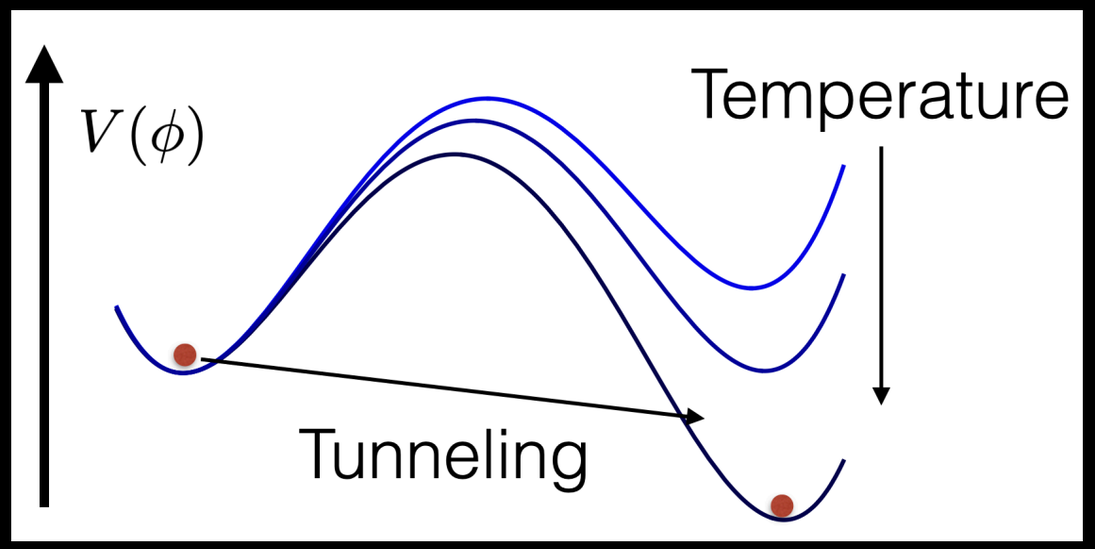
GW from the first order PT

Bubble collisions form GW waves (need quadrupole mass for GW)



Cutting, Hindmarsh, Weir (2018)

GW from the first order PT



$$\mathrm{Prob}(T) \sim \Gamma(T)\, e^{-S(T)}$$

Probability density as a function of temperature

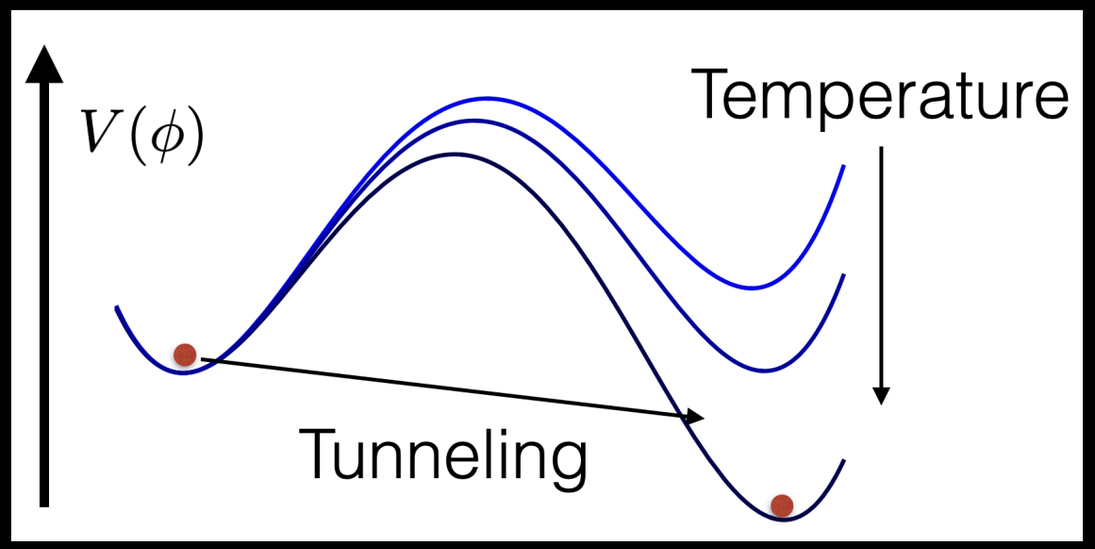
From the wall collision only

$$\rho_{\mathrm{GW}}^{\mathrm{today}} \sim 0.1 \frac{\rho_{\mathrm{PT}}^2}{\rho_{\mathrm{total}}^2} \left(\frac{\beta}{H_{\mathrm{PT}}}\right)^{-2} \rho_{\gamma}^{\mathrm{today}}$$

ρ_{PT} energy density in the sector undergoing the PT

$\Gamma(t) \approx \Gamma_* \, e^{\beta(t-t_*)}$ β sets the time duration of PT

GW from the first order PT



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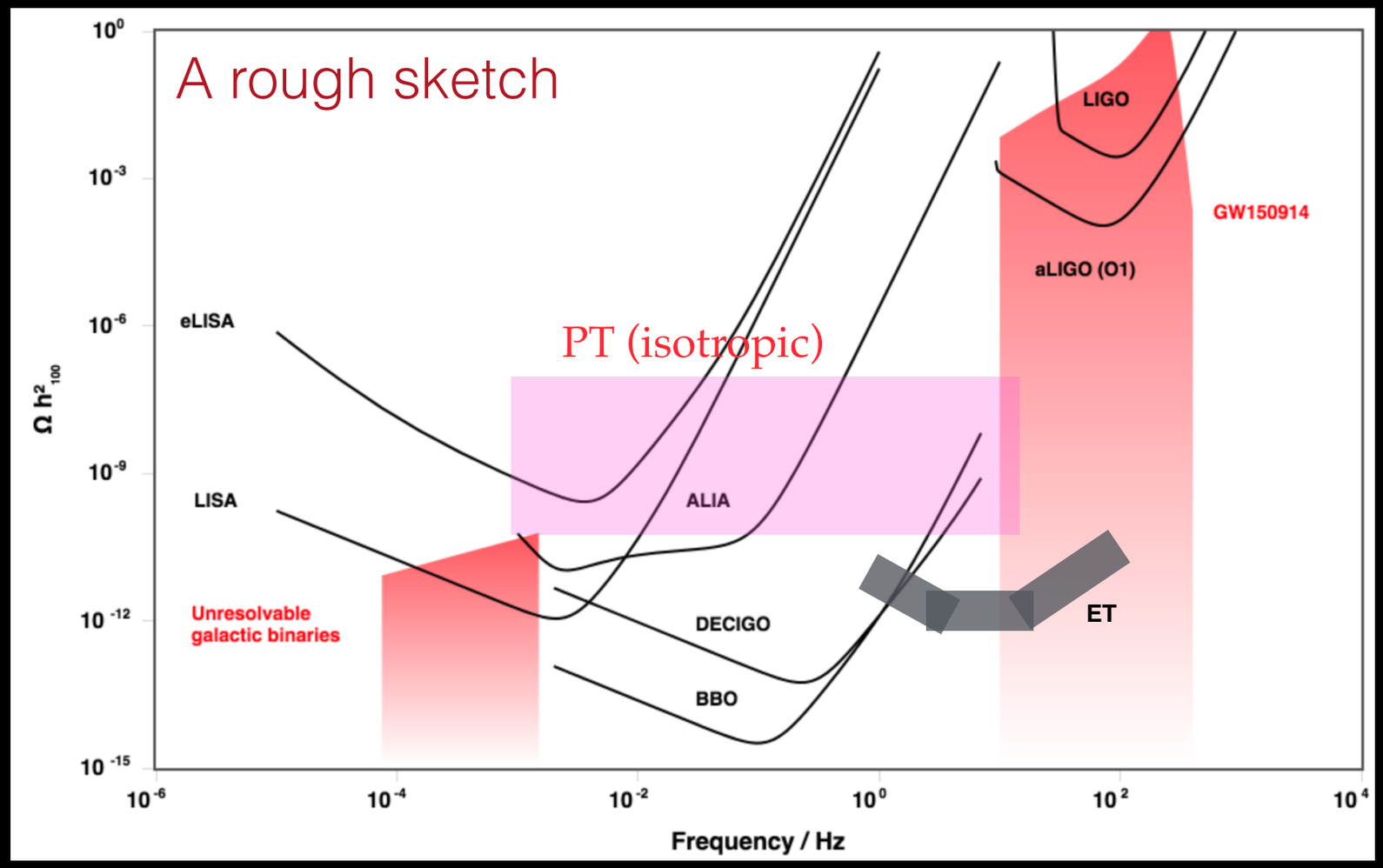
naive dimensional analysis => $(\beta/H_{\mathrm{PT}})^{-2} \sim 10^{-4}$

very strong PT scenarios => $(\beta/H_{\mathrm{PT}})^{-2} \sim 10^{-1}$ is possible

The GW energy density today (isotropic part)

$$\rho_{\rm GW}^{\rm today} \approx 10^{-5} - 10^{-2}\rho_\gamma \qquad \omega_{\rm GW}^{\rm today} = 0.03\,{\rm mHz}\left(\frac{\beta}{H_{\rm PT}}\right)\left(\frac{T_{\rm PT}}{\rm TeV}\right)$$

$\Omega_{\rm GW}h^2$



for $T_{\rm PT} = \mathcal{O}(1 - 10^6)$ TeV phase transition & $\rho_{\rm PT} \approx \rho_{\rm total}$

If Universe has "single source of quantum fluctuation"
& "single sector reheating"

all the radiation/matter perturbations come from the same adiabatic perturbation

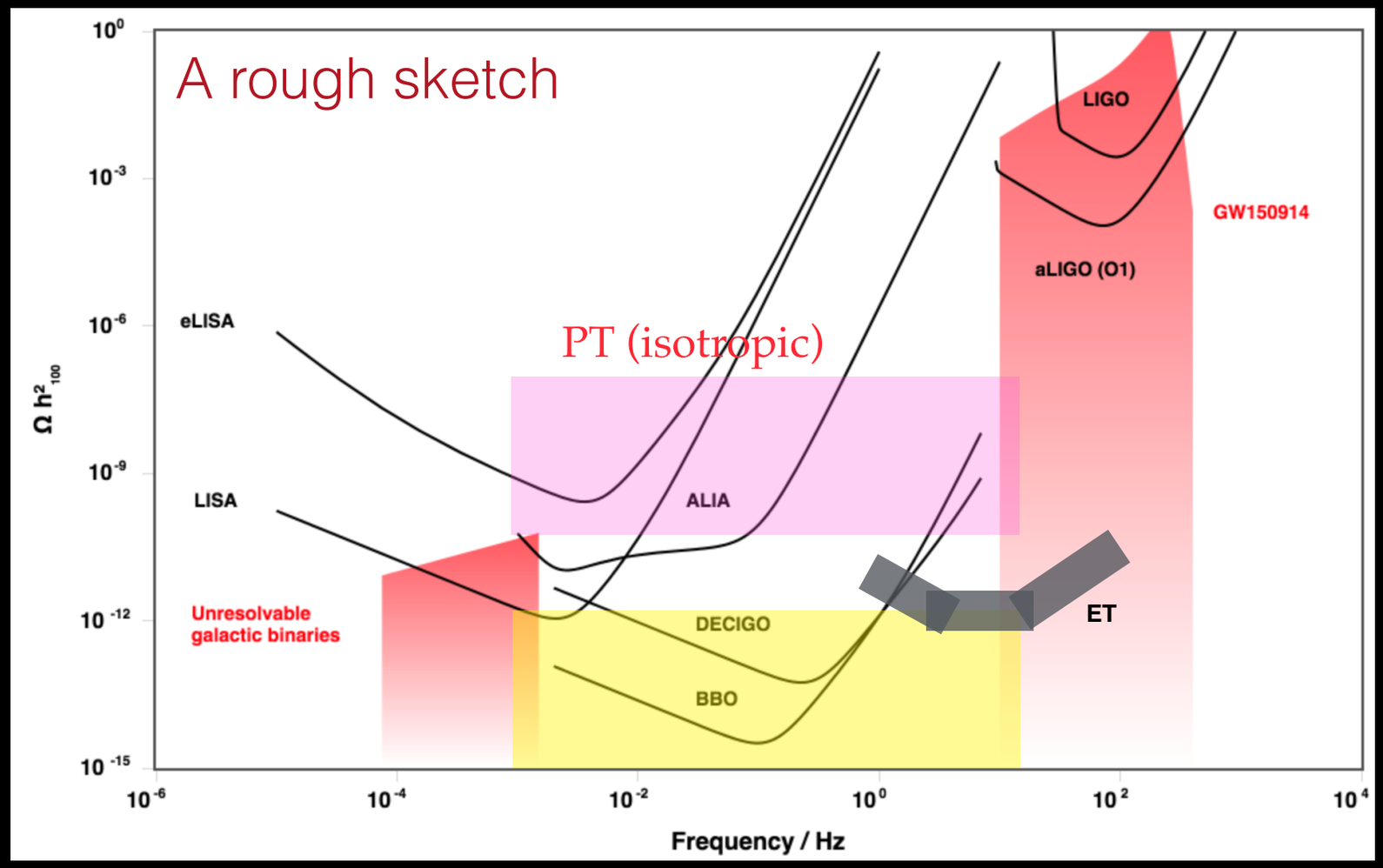
=> size of the GW anisotropy ~ CMB anisotropy

$$\delta_{\rm GW} \sim \delta_\gamma \approx 10^{-5} \qquad \delta_i \equiv \frac{\delta\rho_i}{\bar{\rho}_i}$$

The GW energy density today (anisotropic)

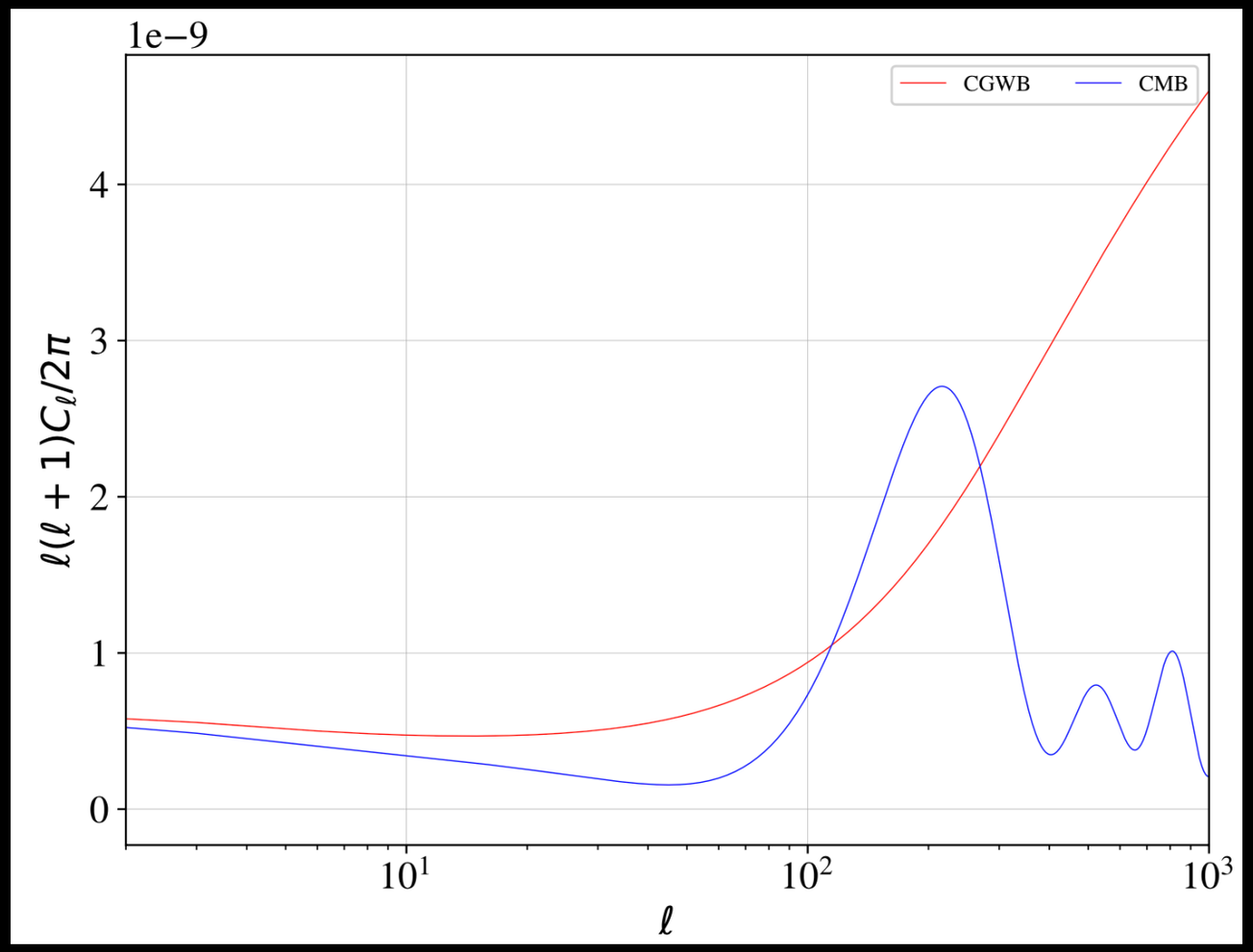
$$\rho_{\rm GW}^{\rm today} \approx 10^{-5} - 10^{-2}\rho_\gamma \qquad \delta\rho_{\rm GW}^{\rm today} \approx 10^{-10} - 10^{-7}\rho_\gamma$$

$\Omega_{\rm GW}h^2$



for $T_{\rm PT} = \mathcal{O}(1 - 10^6)$ TeV phase transition & $\rho_{\rm PT} \approx \rho_{\rm total}$

Power spectrum of the GWB fluctuation



2201.08782

Angular resolution of LISA: $\ell_{\max} \approx 10$

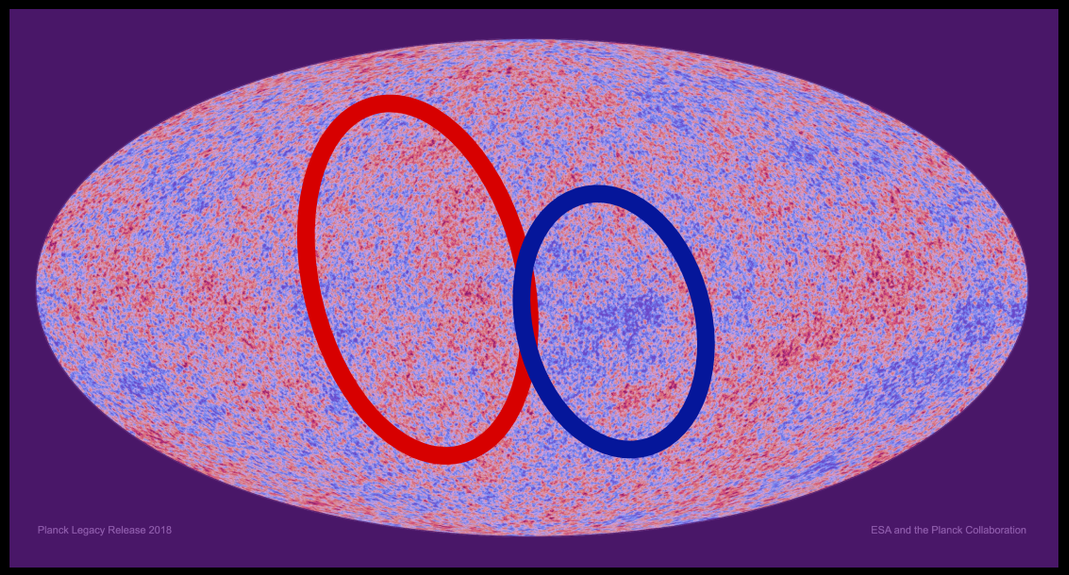
BBO: $\ell_{\max} = \mathcal{O}(100)$

If Universe comes from single reheating process

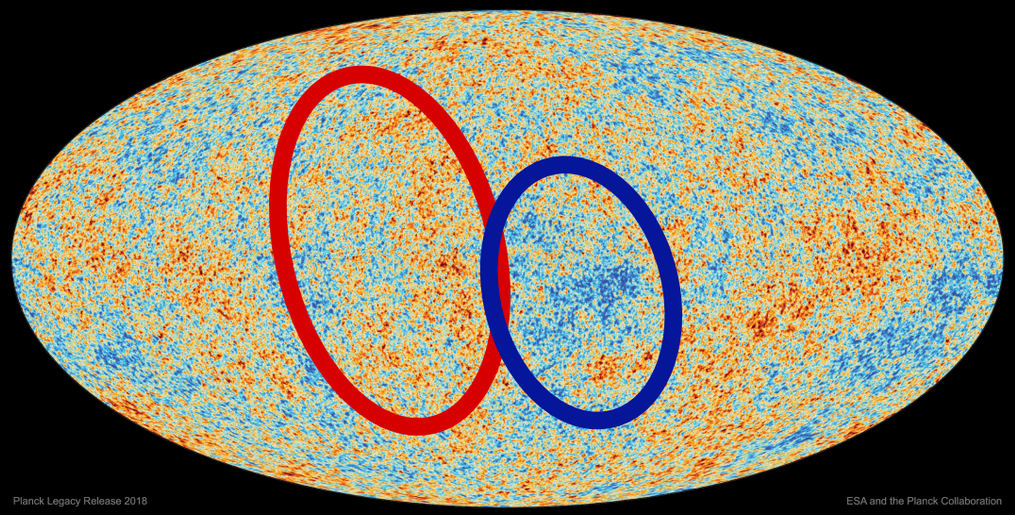
The anisotropy of GWB is totally correlated to the CMB!

Hot & cold spots of the two signals are on top of each other

GWB



CMB

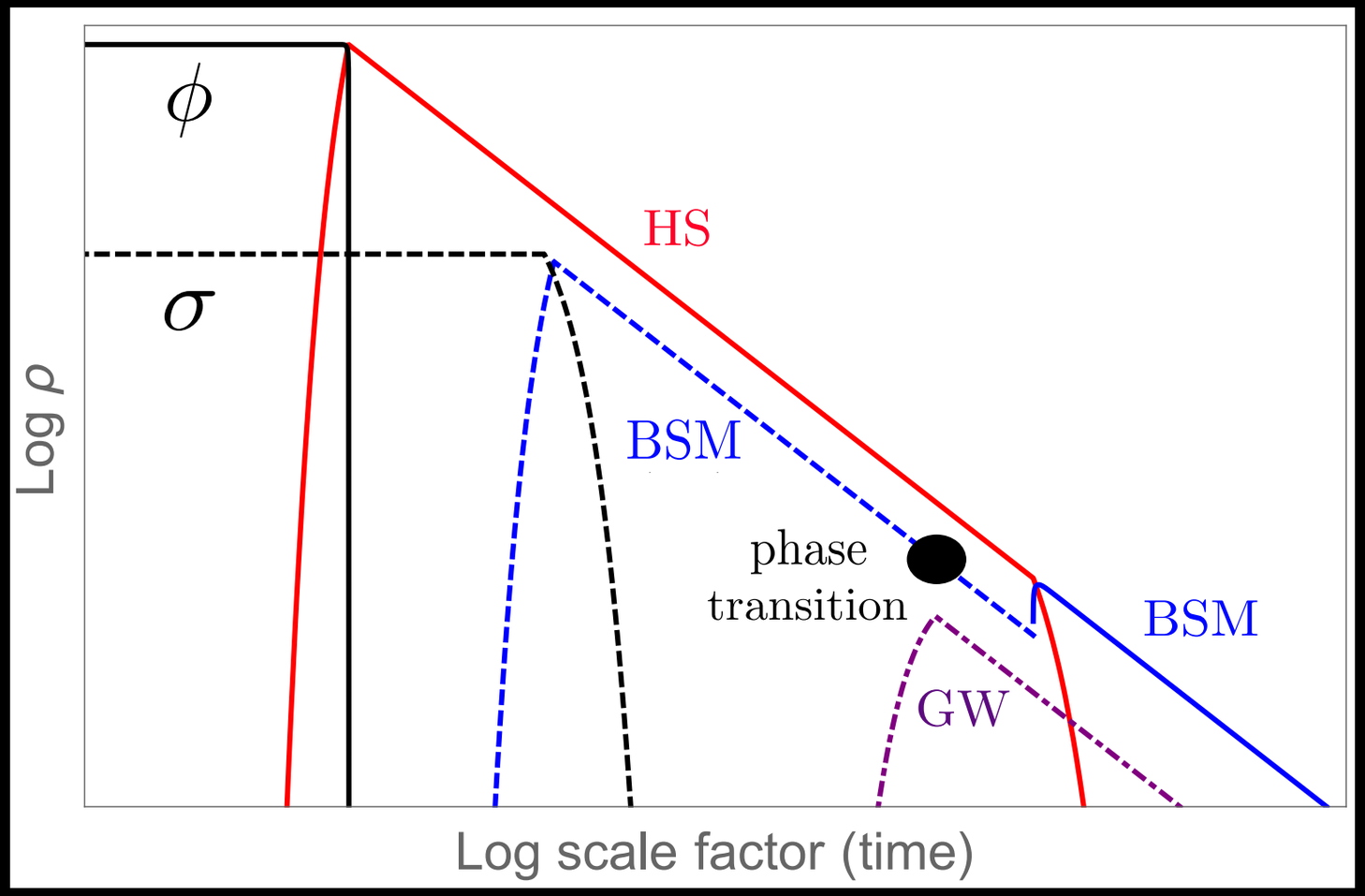


* not the real GWB, only for illustration

But Universe can have multiple reheating processes!

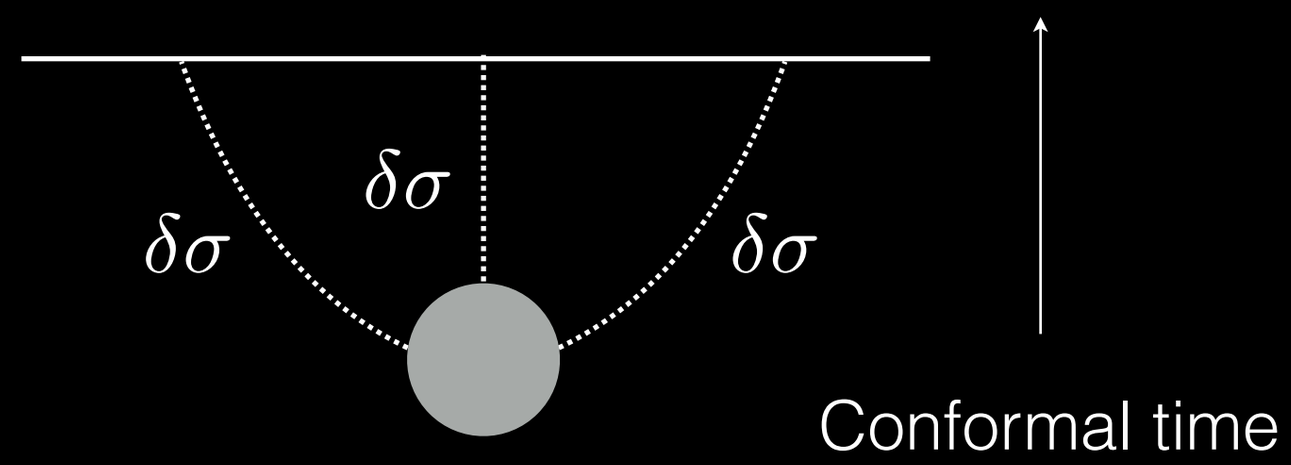
Reheating from the σ decay gives non-adiabatic perturbations

For example, we can have the following reheating process



In this case, the GW anisotropy can be un-correlated to CMB

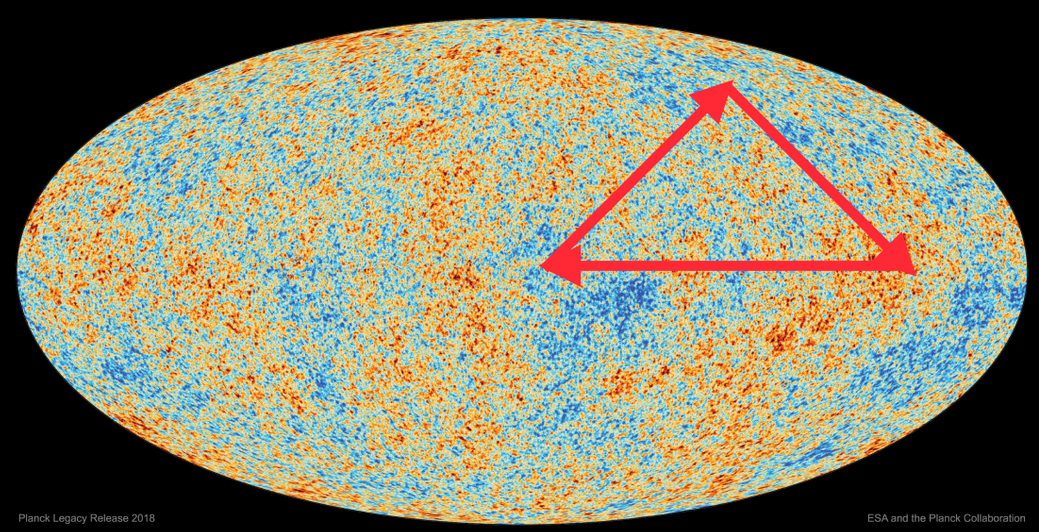
GW perturbations can even be highly non-Gaussian



The resulting GW anisotropy can even be non-Gaussian (NG) with non-vanishing $\langle \delta\rho_{\rm GW}^3 \rangle$

Measuring non-Gaussianity (the bi-spectrum)

$$F_{\rm CMB} \equiv \frac{\langle \delta_\gamma(\vec{k}_1)\delta_\gamma(\vec{k}_2)\delta_\gamma(\vec{k}_3)\rangle}{\langle \delta_\gamma(\vec{k}_1)^2\rangle\langle \delta_\gamma(\vec{k}_2)^2\rangle + \text{permutations}} \sim \delta_\gamma^{-1} \times (\text{size of NG})$$



The primordial perturbation of the CMB looks very Gaussian

(size of NG) < 10^{-3}

* have stripped off $(2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)$ from the correlators

In the multiple-reheating scenarios

Reheating from the curvaton σ decay can be non-Gaussian, giving NG isocurvature perturbations (energy fraction from curvaton $f_{\rm BSM} \ll 1$)

$$S_\sigma \equiv 3(\zeta_\sigma - \zeta_\phi) \qquad \delta_{\rm GW}^{\rm obs} \sim -\zeta_\phi + S_\sigma \qquad \delta_\gamma^{\rm obs} \sim -\zeta_\phi - f_{\rm BSM} S_\sigma$$

the 3-point function

$\langle \zeta_\phi^3 \rangle = 0$ but $\langle S_\sigma^3 \rangle \neq 0$

$$F_{\rm CMB} \sim f_{\rm BSM}^3 \left(\frac{\langle S_\sigma^2 \rangle + \ldots}{\langle \zeta_\phi^2 \rangle + \ldots} \right) F_{S_\sigma}$$

$$F_{\rm GW} \sim \left(\frac{\langle S_\sigma^2 \rangle + \ldots}{\langle \zeta_\phi^2 \rangle + \ldots} \right) F_{S_\sigma}$$

$$f_{\rm BSM} \approx \frac{\rho_\sigma}{\rho_{\rm total}}$$

* see Kumar, Sundrum, **YT** (2021) for more details

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$$F_{\rm GW} \sim \left(\frac{\langle S_\sigma^2 \rangle + \ldots}{\langle \zeta_\phi^2 \rangle + \ldots} \right) F_{S_\sigma}$$

$$F_{\rm CMB} \sim f_{\rm BSM}^3 F_{\rm GW}$$

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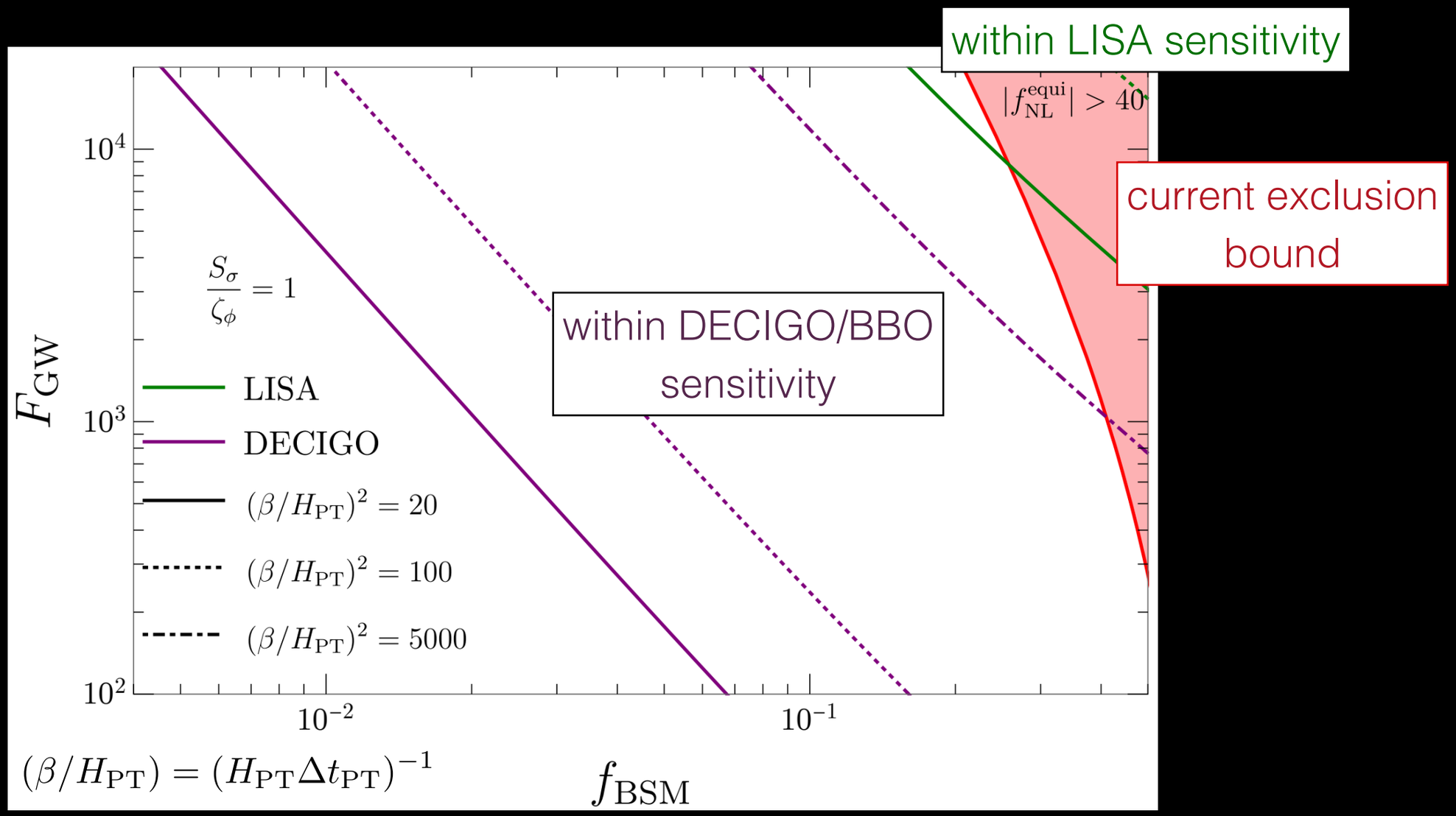
$$F_{\rm CMB} \sim f_{\rm BSM}^3 F_{\rm GW}$$

A large NG can hide in the GWB

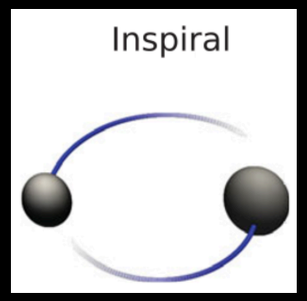


DECIGO/BBO can easily pick up the NG signal,
and LISA also has a small chance

Kumar, Sundrum, YT (2021)

Can we see the anisotropy in GW detectors?

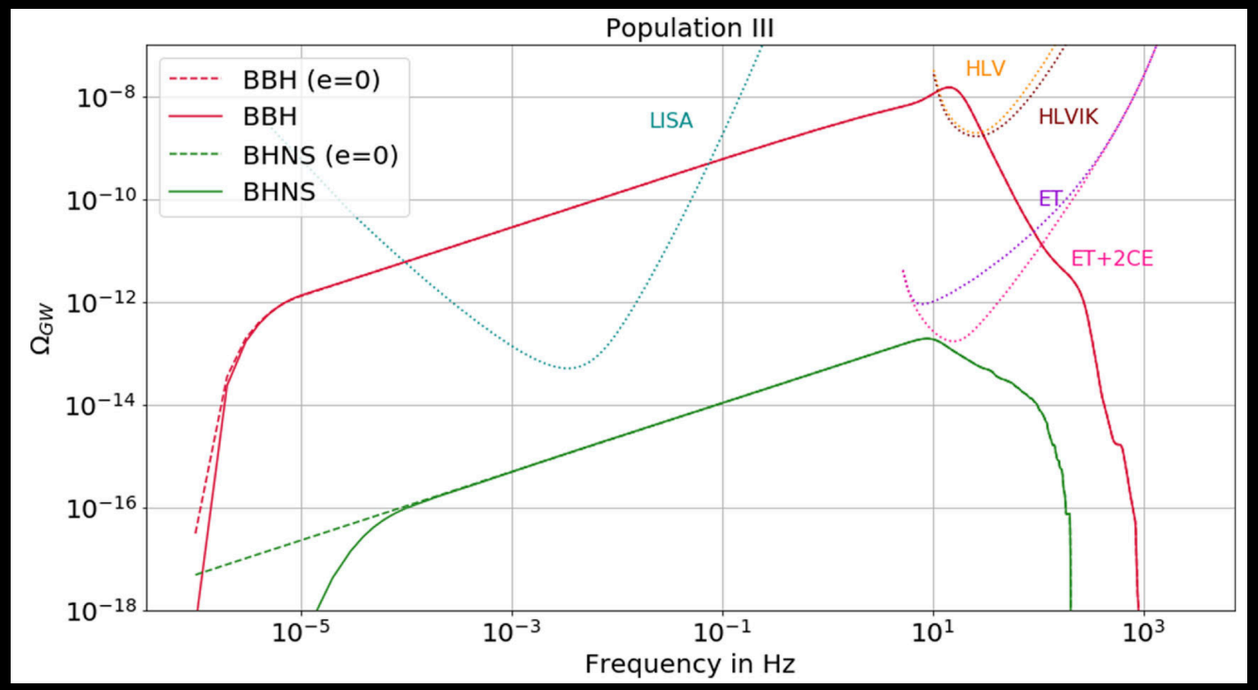
Astrophysical foreground



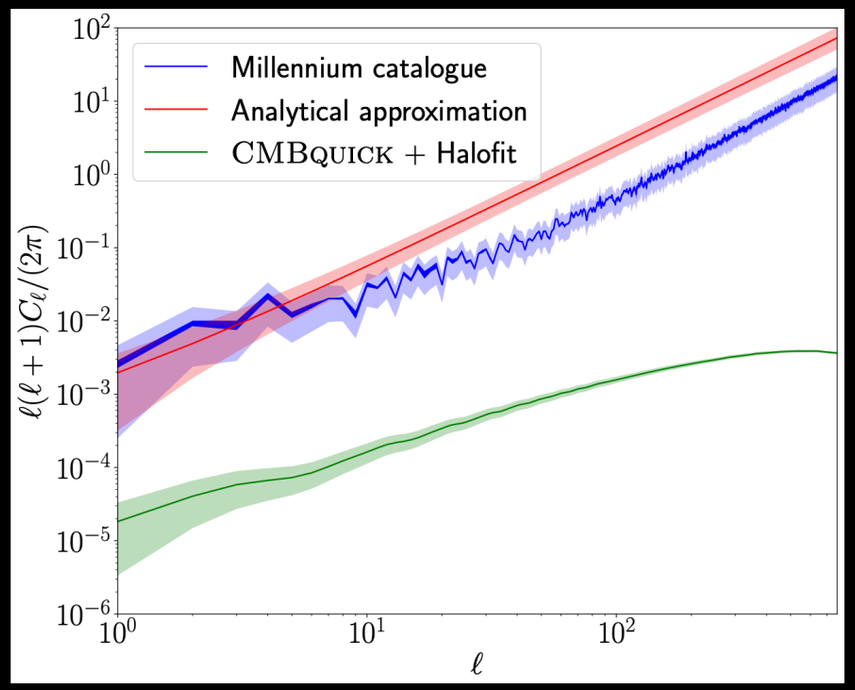
Astrophysical foreground in frequency band from LISA to peak ET mainly comes from un-resolved black hole & neutron star binaries

Expected signal $\delta\Omega_{\rm GW}^{\rm astro} \sim 10^{-11} \times 10^{-(1\sim2)} \sim 10^{-(12\sim14)}$

$\bar{\Omega}_{\rm GW}^{\rm astro}$ $\delta\rho_{\rm GW}^{\rm astro}/\bar{\rho}_{\rm GW}^{\rm astro}$

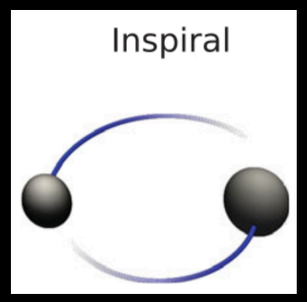


e.g., Perigois, Belczynski, Bulik, Regimbau (2021)



e.g., Jenkins, O`Shaughnessy, Sakellariadou, Wysocki (2018)

Astrophysical foreground



Parametrize the anisotropy spectrum

$$\left(\delta\Omega^{\rm astro}_{\ell,\rm GW}\right)^2 = A_a^2 \left(\frac{f}{f_0}\right)^{2\gamma} \frac{\ell^{2\delta}}{\ell(2\ell+1)}$$

$$A_a \sim 10^{-(12-14)} \qquad \gamma \approx \frac{2}{3} \qquad 2\delta \approx 1$$

with uncertainties mainly come from BH & binary formation physics

Using Fisher analysis to quantify the uncertainty of

(1) seeing the anisotropic signal

(2) distinguishing the cosmo signal from the astro background

Parametrize the total anisotropic spectrum as

$$\left(\delta\Omega^{\rm total}_{\ell,{\rm GW}}\right)^2 = \underbrace{A_c^2\, S(f,f_*)^2 \frac{\ell^{2\beta}}{\ell(\ell+1)}}_{\text{cosmo}} + \underbrace{A_a^2 \left(\frac{f}{f_0}\right)^{2\gamma} \frac{\ell^{2\delta}}{\ell(2\ell+1)}}_{\text{astro}}$$

Will know the frequency spectrum $S(f,f_*)$ well due to the large isotropic (monopole) signal

Using Fisher analysis to quantify the uncertainty of

(1) seeing the anisotropic signal

(2) distinguishing the cosmo signal from the astro background

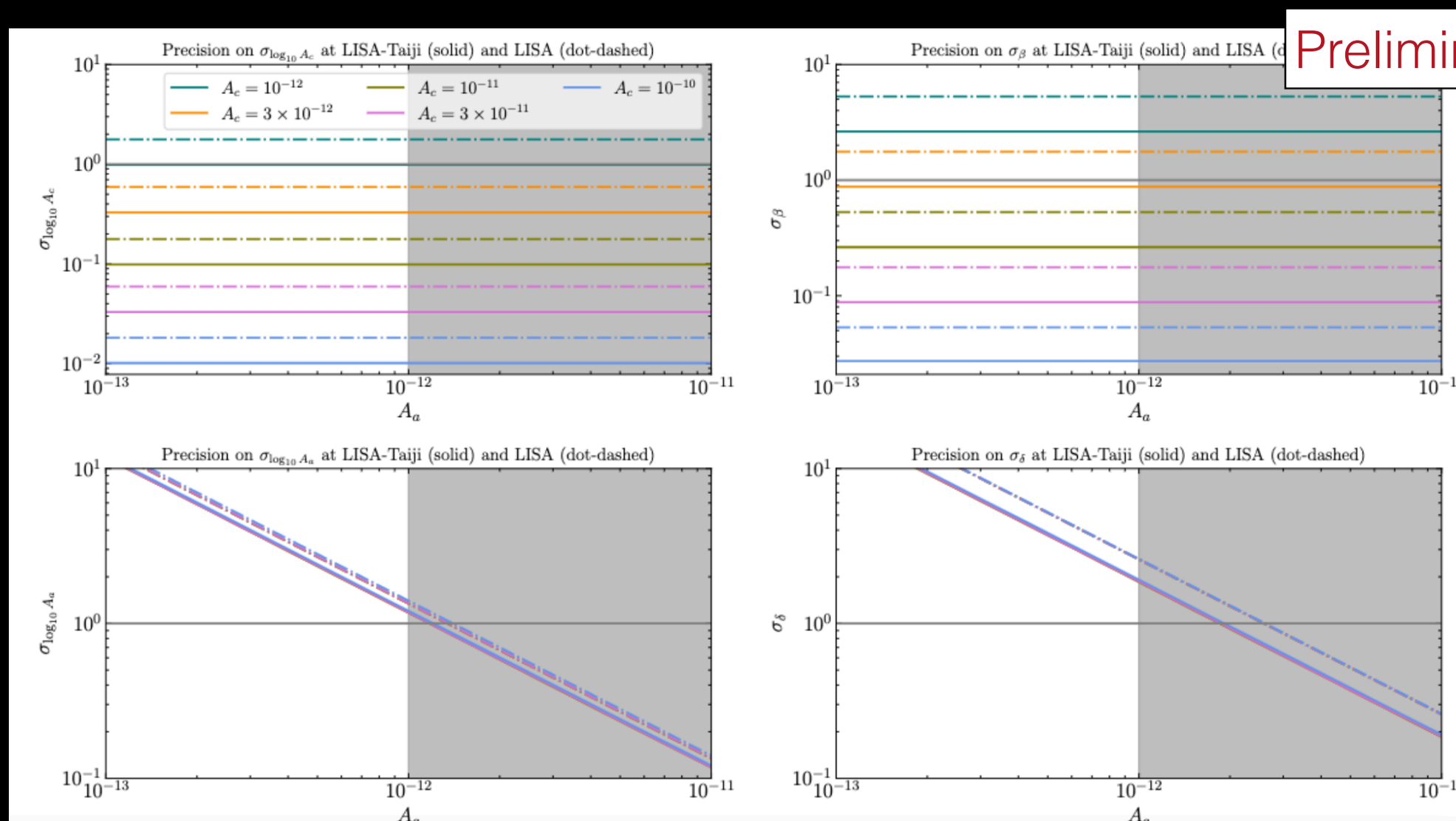
Obtain experimental sensitivity from the Fisher matrix

$$F_{pq} = \sum_{f,\ell} \left[\frac{\frac{\partial\left(\Omega^{\rm total}_{\ell,\rm GW}\right)^2}{\partial p} \frac{\partial\left(\Omega^{\rm total}_{\ell,\rm GW}\right)^2}{\partial q}}{\left(\Omega^{\rm total}_{\ell,\rm GW}\right)^2} \right] \left[\left(\Omega^{\rm total}_{\ell,\rm GW}\right)^2 + \left(\Omega^{\rm noise}_{\ell,\rm GW}\right)^2 \right]^{-1}$$

$$p, q = \{A_c, A_a, \beta, \gamma, \delta\}$$

LISA sensitivity

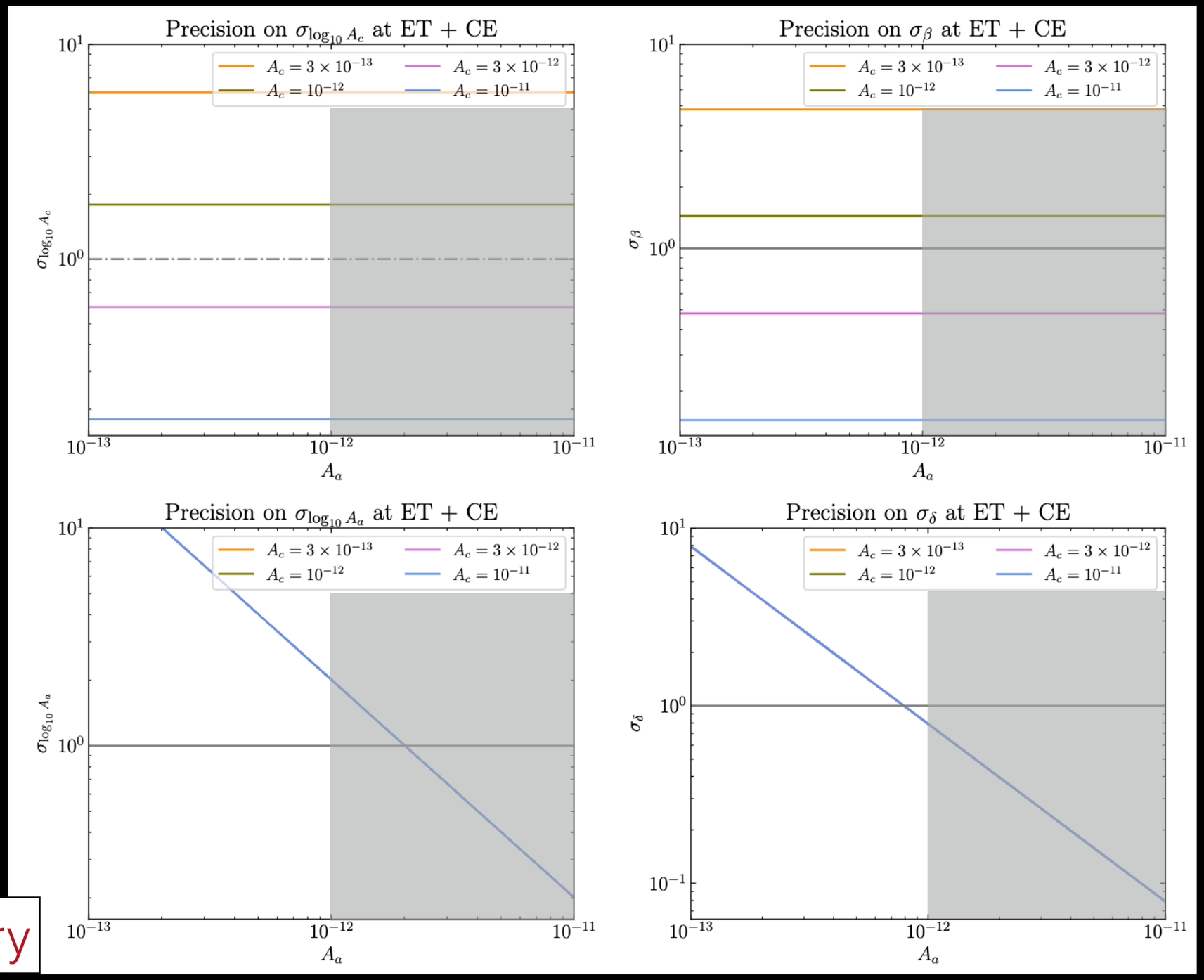
$$\left(\delta\Omega^{\rm total}_{\ell,\rm GW}\right)^2 = \underbrace{A_c^2\, S(f,f_*)^2 \frac{\ell^{2\beta}}{\ell(\ell+1)}}_{\text{cosmo}} + \underbrace{A_a^2 \left(\frac{f}{f_0}\right)^{2\gamma} \frac{\ell^{2\delta}}{\ell(2\ell+1)}}_{\text{astro}}$$



LISA & LISA+Taiji can measure the anisotropy GWB from the strongest PT to O(10)% level, no chance of seeing the astro foreground

Cui, Kumar, Sundrum, YT (soon)

ET
+
cosmic explorer



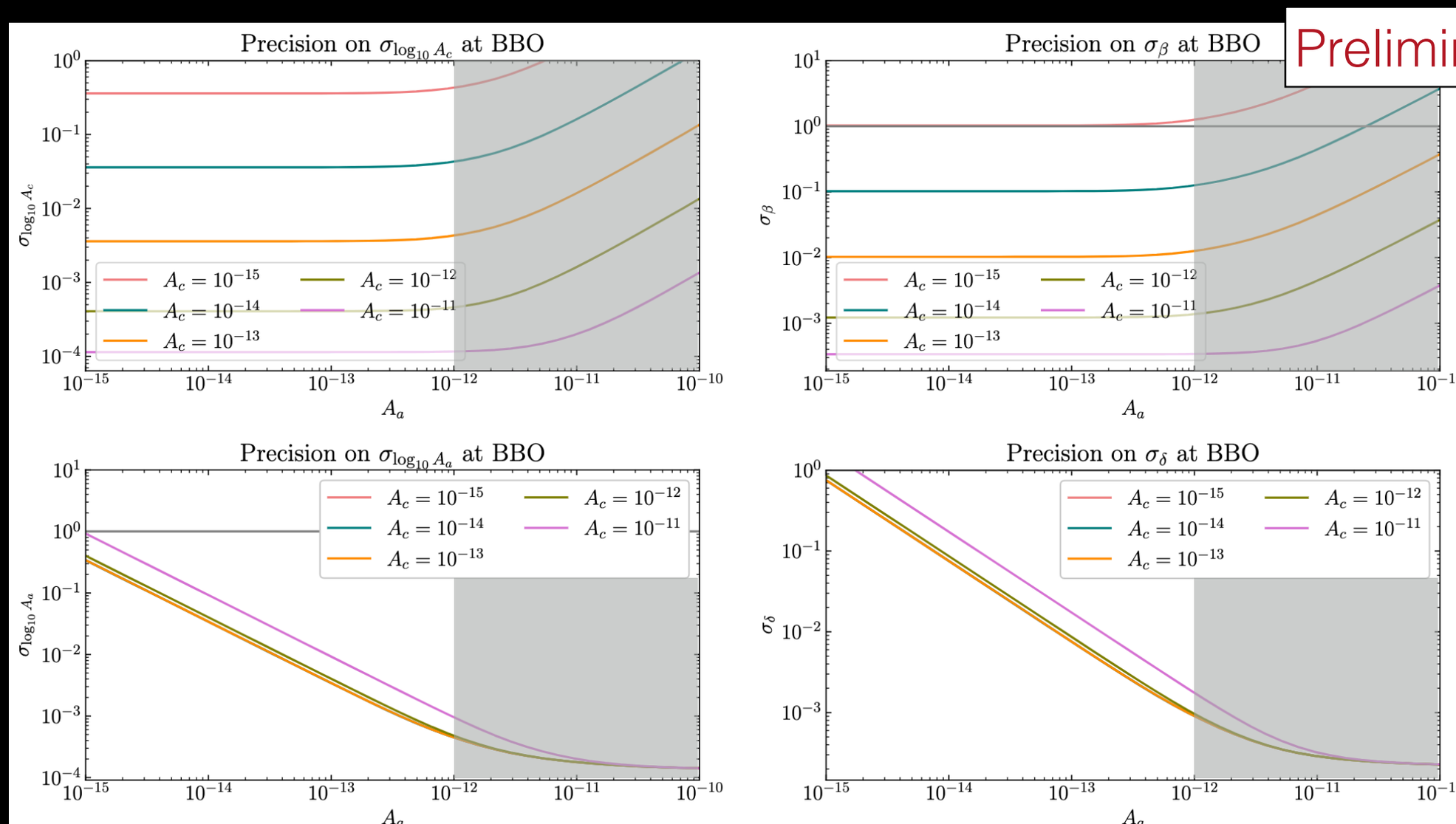
Preliminary

ET+cosmic explorer can measure the anisotropy GWB from the strongest PT to O(10)% level, no chance of seeing the astro foreground

Cui, Kumar, Sundrum, YT (soon)

BBO sensitivity

$$\left(\delta\Omega^{\rm total}_{\ell,\rm GW}\right)^2 = \underbrace{A_c^2\, S(f,f_*)^2 \frac{\ell^{2\beta}}{\ell(\ell+1)}}_{\text{cosmo}} + \underbrace{A_a^2 \left(\frac{f}{f_0}\right)^{2\gamma} \frac{\ell^{2\delta}}{\ell(2\ell+1)}}_{\text{astro}}$$



BBO can measure the anisotropy GWB from the typical strength PT to O(10)% level, measure the astro foreground well

Cui, Kumar, Sundrum, YT (soon)

Conclusion

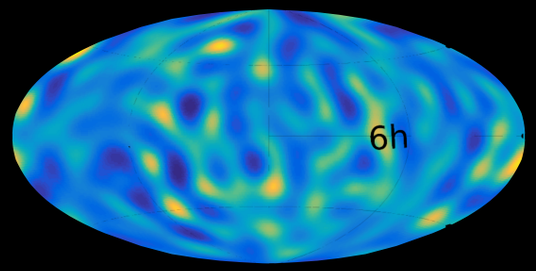
Stochastic GW background from early cosmology must be anisotropic

Large Non-Gaussian perturbations can still hide in the GWB
and can be visible in future experiments

Depending on the inflationary & reheating processes,
the GWB can be correlated or un-correlated to the CMB

For GW from strong phase transitions, the anisotropic signal
can be visible in future GW experiments: LISA(+Taiji), BBO, ET

SCAN ME

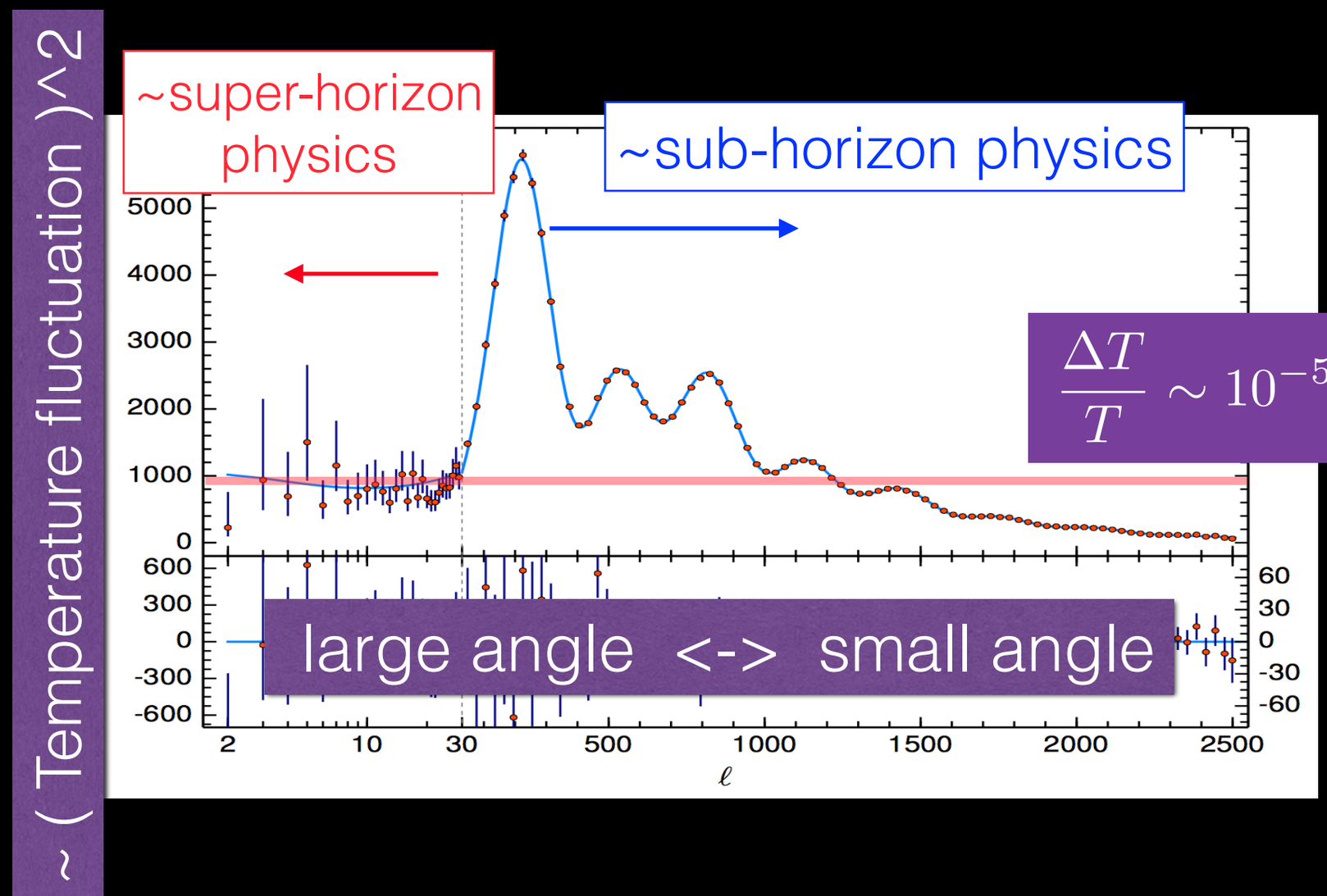


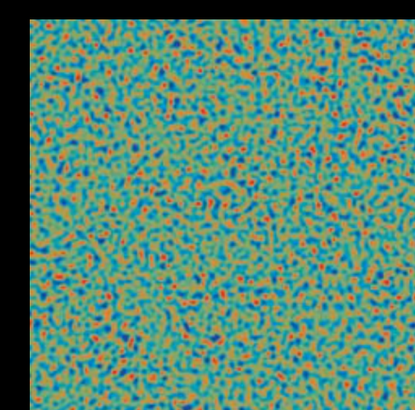
Yuhsin Tsai ytsai3@nd.edu

Backup Slides

Power spectrum of the CMB fluctuation

$\ell(\ell+1)C_\ell^{TT}/2\pi$

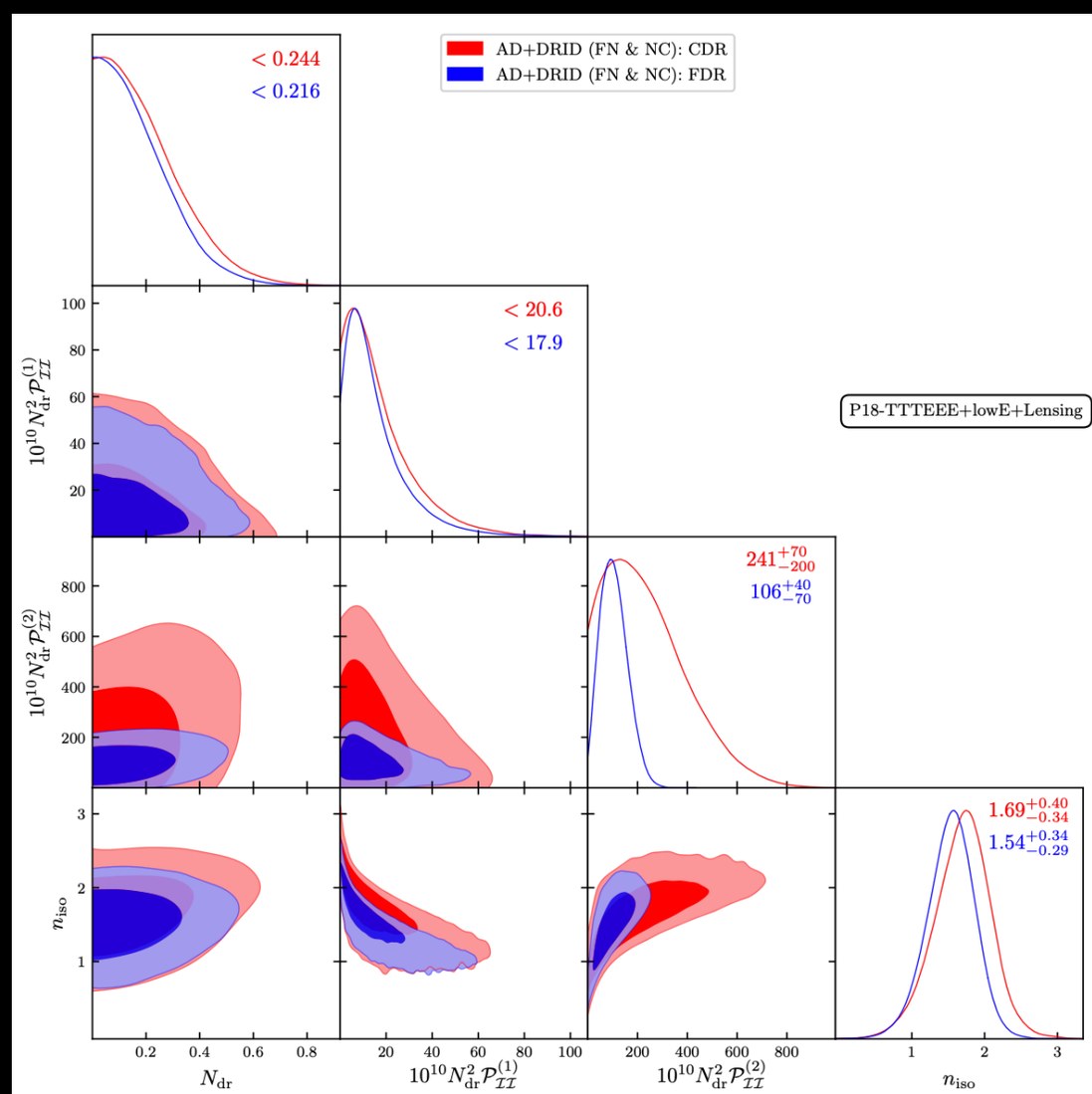




In this case, $\delta\rho_{\rm GW}$ needs to satisfy the isocurvature constraints

No previous bounds on dark radiation isocurvature (like for GW)

We calculated it ourselves with the Planck 2018 data



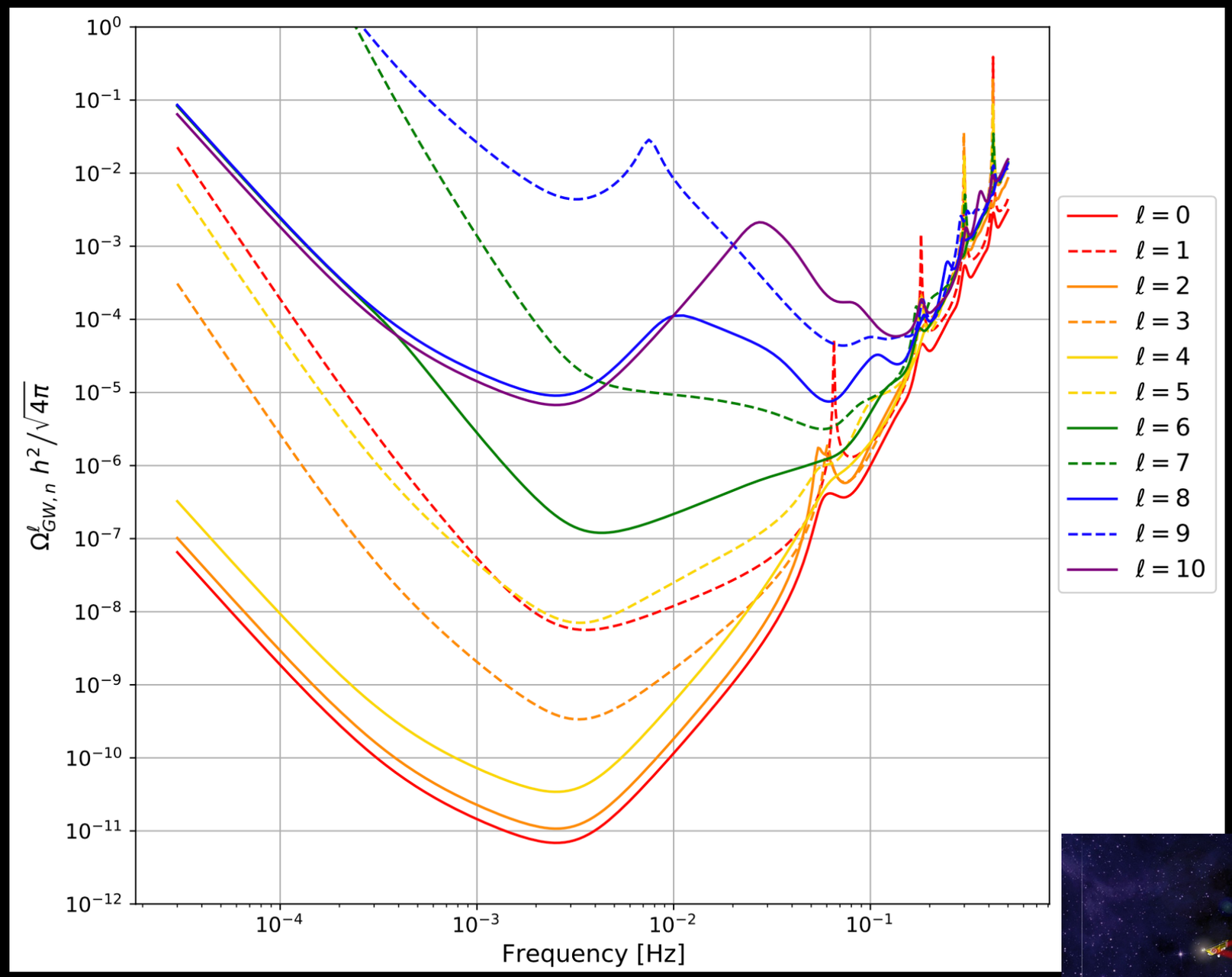
perturbation needs to be less than

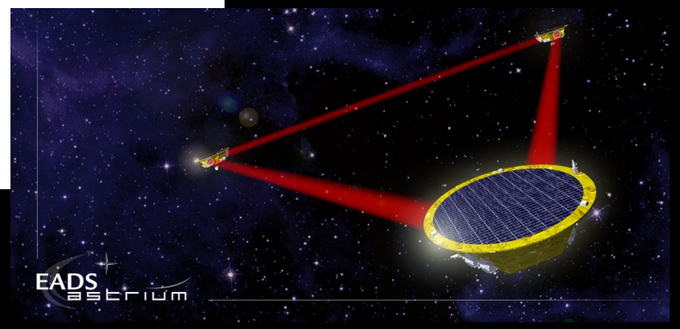
$$\frac{\delta\rho_{\rm dr}}{\bar{\rho}_{\rm dr}} < 10^{-3}\left(\frac{\Delta N_{eff}}{0.1}\right)^{-1/2}$$

Also helps to relax the H0 tension!

Subhajit Ghosh, Soubhik Kumar, YT (2022)

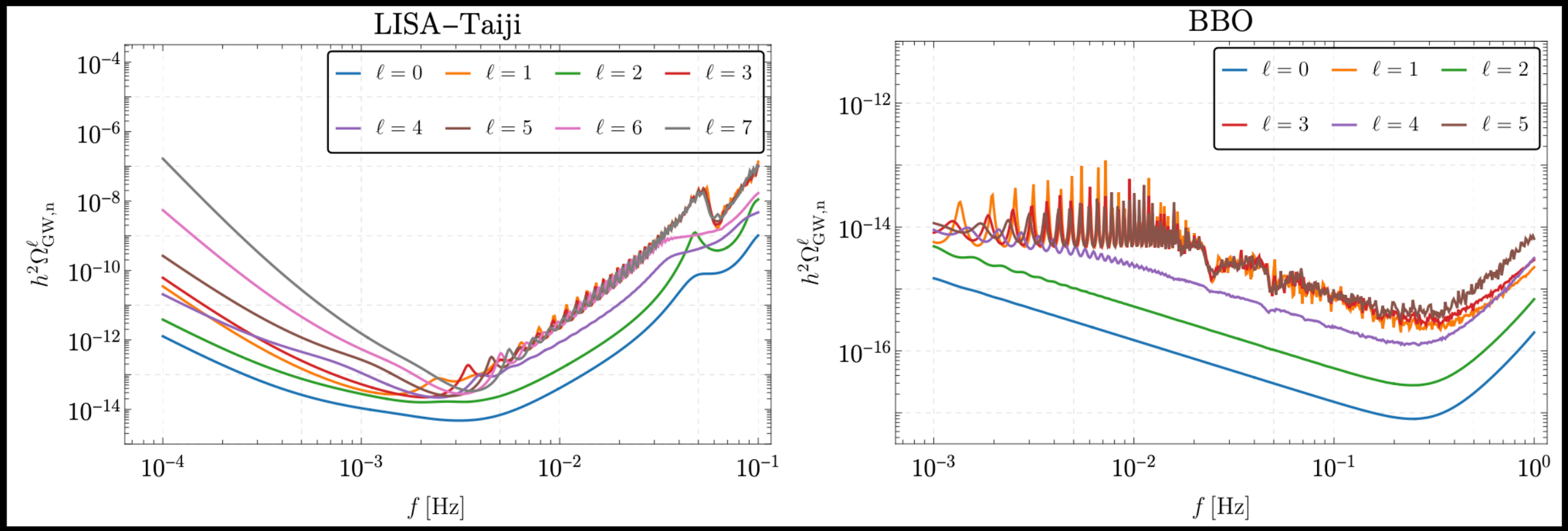
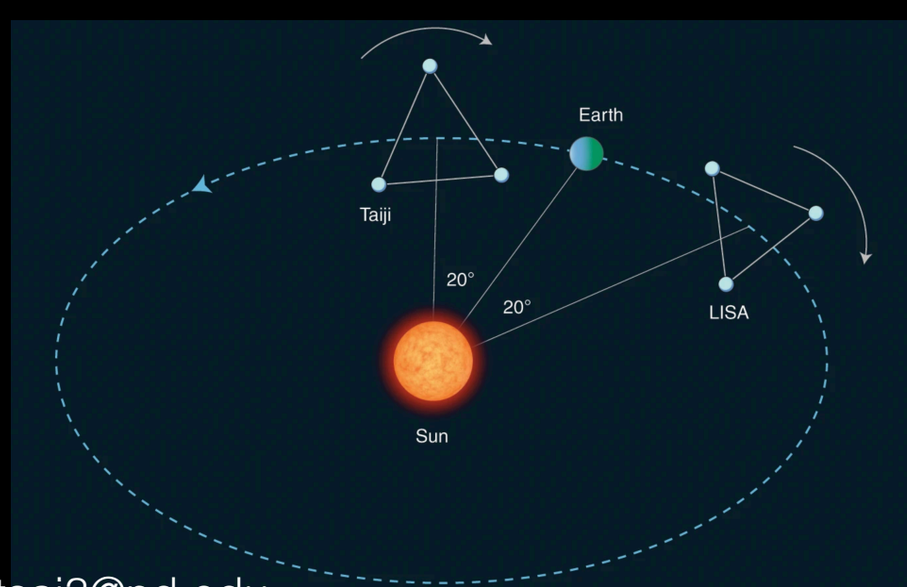
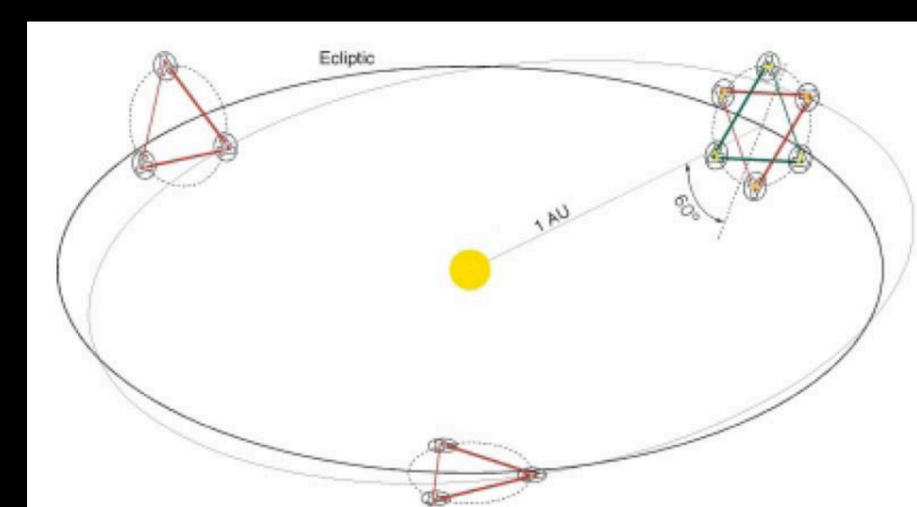
LISA sensitivity for different ℓ-modes





arXiv: 2201.08782

LISA+Taiji & BBO



Dimastrogiovanni, Fasiello, Malhotra, Tasinato (2022)