Chiral effects on lepton transport in core-collapse supernovae

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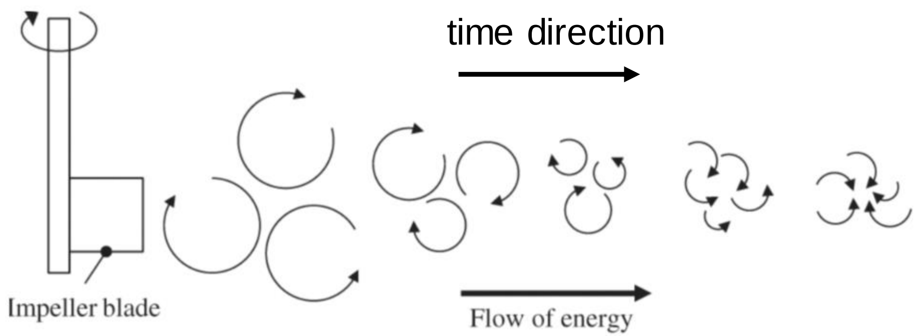
Motivations

- Supernova explosions are one of most complicated astrophysical processes : neutrino heating, different types of instabilities, magnetic fields, rotations, etc.
- 2-D simulations : easier for explosions
- 3-D simulations : more difficult
- Neutron star (pulsar) kicks
- Origin of the strong magnetic fields in magnetars
- Chiral effects on leptons : a new microscopic mechanism that could potentially affect the evolution of supernovae

credit : RIKEN

Direct & inverse energy cascades

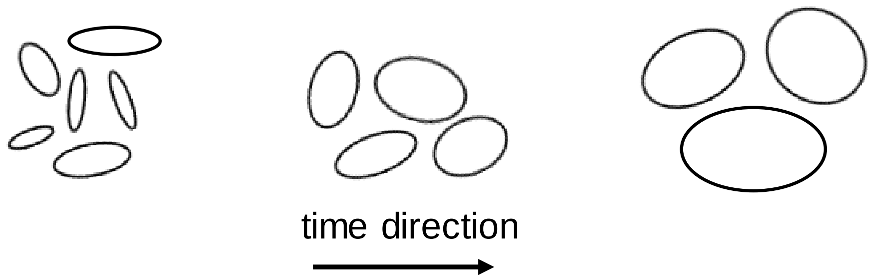
- Intuitively, why is the 3D system more difficult to achieve explosion?
- Turbulence in 3D : direct energy cascade (difficult to explode)



https://doi.org/10.1515/htmp-2016-0043

Kolmogorov 1941

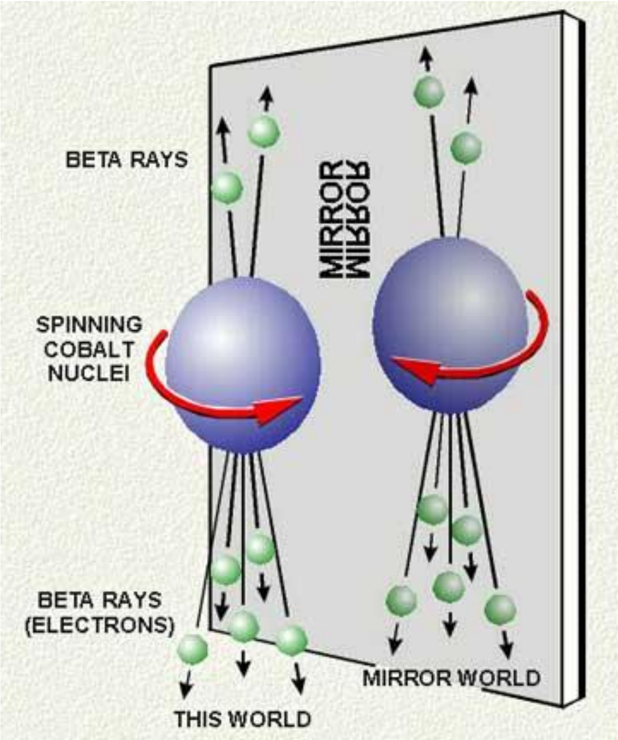
- Turbulence in 2D : inverse energy cascade (easier to explode)



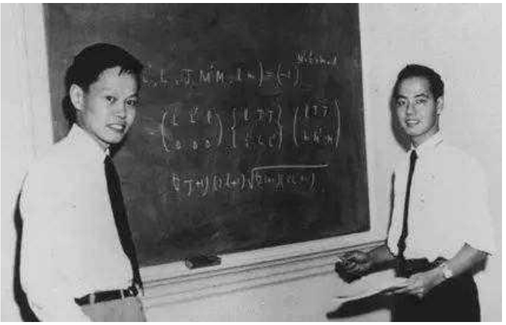
Kraichnan, Leith, Batchelor 1967-1969

- Chiral effects in 3D : inverse cascade

Parity violation & weak interaction



http://physics.nist.gov/GenInt/Parity/cover.html]



Lee & Yang

Wu, 1956

- Global parity violation in weak interaction
- Weak-interaction processes between leptons and nucleons are ubiquitous in CCSN.
- What will be the transport properties for (massless) fermions under parity (chirality) violation?

Chiral anomaly and helicity conservation

- Chiral anomaly : $\partial_\mu J_5^\mu = \frac{\mathbf{E}\cdot\mathbf{B}}{2\pi^2}$ (for approximate massless fermions) S. Adler, J. Bell, R. Jackiw, 69 K. Fujikawa, 79

$$\partial_\mu J^\mu = 0 \qquad J^\mu = J_{\mathrm{R}}^\mu + J_{\mathrm{L}}^\mu, \quad J_5^\mu = J_{\mathrm{R}}^\mu - J_{\mathrm{L}}^\mu.$$

- Helicity conservation : $\frac{dH_{\mathrm{tot}}}{dt} = 0, \qquad H_{\mathrm{tot}} \equiv N_5 + \frac{H_{\mathrm{mag}}}{4\pi^2},$

$$N_5 \equiv \int d^3x\, n_5, \qquad H_{\mathrm{mag}} \equiv \int d^3x\, \boldsymbol{A}\cdot\boldsymbol{B}.$$

exchange btw the axial charge & magnetic helicity

- Chiral magnetic effect (CME) : $J^\mu = \xi_B B^\mu, \quad \xi_B = \frac{\mu_5}{2\pi^2}. \quad (\mu_5 = \mu_R - \mu_L)$

A. Vilenkin, PRD 22, 3080 (1980)

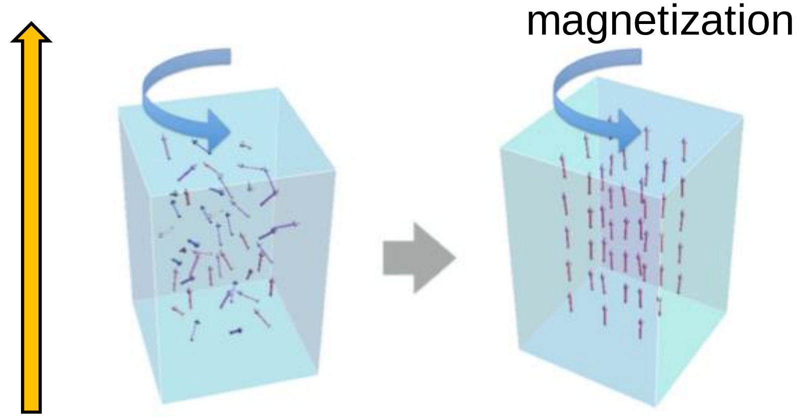
K. Fukushima, D. Kharzeev, H. Warringa, PRD78, 074033 (2008)

Chiral plasma instability

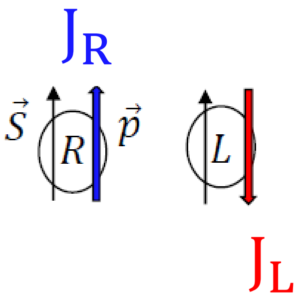
- An intuitive picture of CME :

magnetic field: B

massless pts :
chirality=helicity



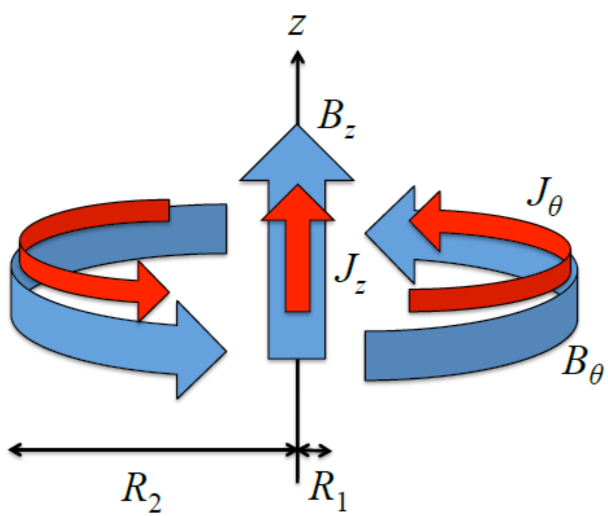
M. Matsuo et al, fphy.2015.00054



$J_V = \mathrm{J_R} + \mathrm{J_L} \neq 0$
when $n_R - n_L = n_5 \neq 0$

- Chiral plasma instability (CPI) :

M. Joyce and M. E. Shaposhnikov, PRL **79**, 1193 (1997)
Y. Akamatsu and N. Yamamoto, PRL 111, 052002 (2013)



Y. Akamatsu and N. Yamamoto, PRD 90 (2014) 12, 125031

Long-wavelength unstable modes

- Anomalous Maxwell's eq. : $\partial_t \boldsymbol{B} = -\boldsymbol{\nabla} \times \boldsymbol{E}, \quad \boldsymbol{\nabla} \times B = \eta^{-1} \boldsymbol{E} + \boxed{\xi_B \boldsymbol{B}}$.

CME

$$\Rightarrow \frac{\partial \boldsymbol{B}}{\partial t} = \boxed{\eta \boldsymbol{\nabla}^2 \boldsymbol{B}} + \boxed{\eta \, \boldsymbol{\nabla} \times (\xi_B \boldsymbol{B})}$$

diffusion CME (instability)

- Unstable modes at long wavelength : $\delta B \propto \mathrm{e}^{\sigma t + \mathrm{i} \boldsymbol{k} \cdot \boldsymbol{x}}$ ($\sigma > 0$: magnetic fields growing in time)

M. Joyce and M. E. Shaposhnikov, PRL **79**, 1193 (1997).

Y. Akamatsu and N. Yamamoto, PRL 111, 052002 (2013).

$$\sigma = \eta k (\xi_B - k)$$ (for small k, long wavelength)

❖ σ becomes maximum when $k = \xi_B / 2 \equiv k_{\mathrm{CPI}}$

$$\Rightarrow \tau_{\mathrm{CPI}} = \frac{4}{\eta \xi_B^2}, \quad \lambda_{\mathrm{CPI}} \equiv \frac{2\pi}{k_{\mathrm{CPI}}} = \frac{4\pi}{\xi_B}$$

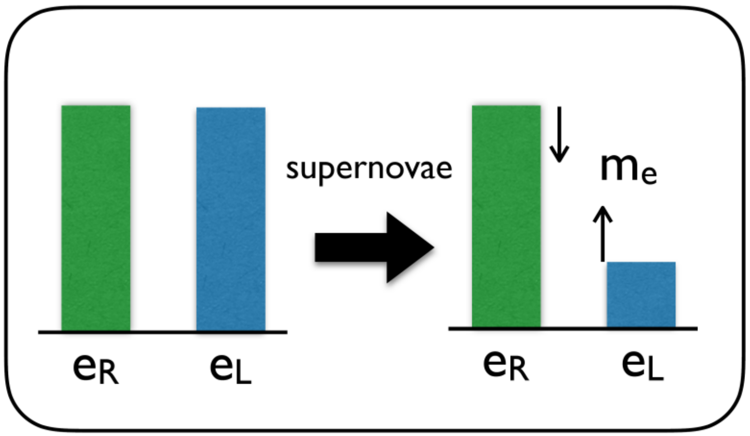
Generation of chirality imbalance

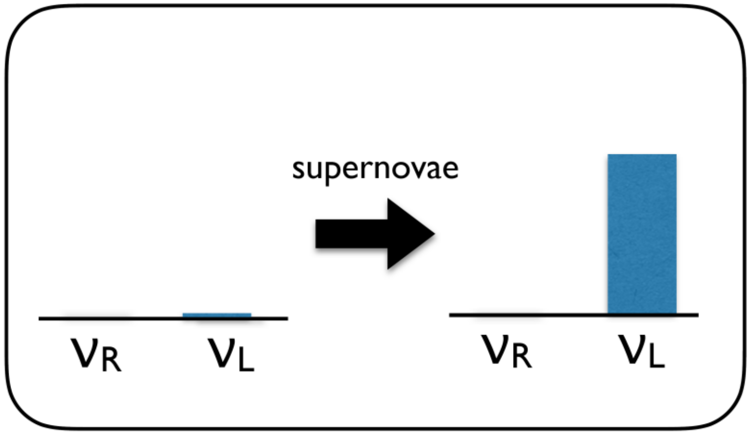
- Electron capture process in supernovae :

A. Ohnishi, N. Yamamoto, 2014, arXiv:1402.4760

an innate lefthander

$$p + e^{\mathrm{L}} \to n + \nu_e^{\mathrm{L}}$$





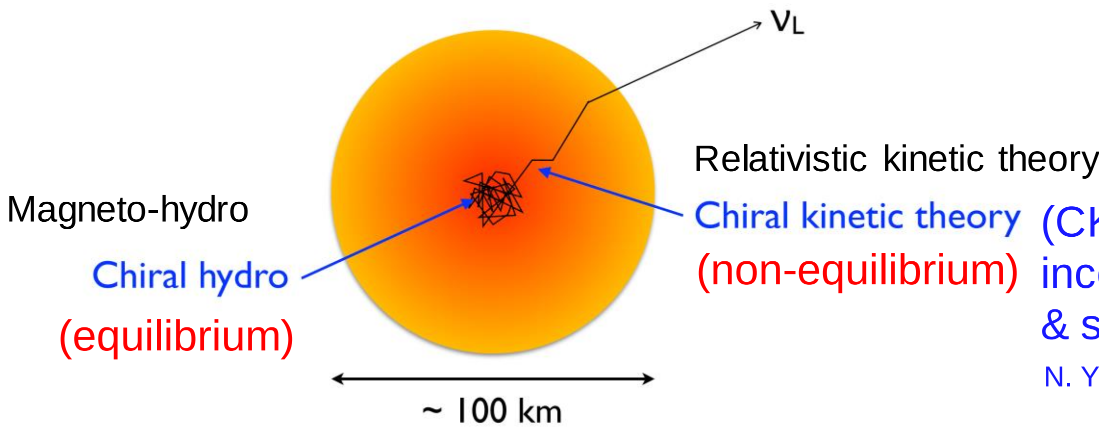
N. Yamamoto's talk, NTU, Taiwan, 18

- Back-reaction from non-equilibrium neutrinos.

Chiral radiation hydrodynamics

- Matter (e, N) in equilibrium + radiation (ν) out of equilibrium

S. W. Bruenn, Astrophys. Jl. Suppl. 58 (1985) 771.



(CKT) : quantum kinetic theory incorporating chiral anomaly & spin-orbit int.

N. Yamamoto, DY, APJ 895 (2020) 1, 56.

- Near the core : ChMHD (e, N, ν)
- Away from the core : ChMHD (e, N) + chiral kinetic theory (ν) $\Longrightarrow$ chiral radiation hydrodynamics

$$\boxed{\nabla_\mu T^{\mu\nu}_{\rm rad}} + \boxed{\nabla_\mu T^{\mu\nu}_{\rm mat}} = 0$$

(neutrinos) (electrons, nucleons : equilibrium)

Chiral radiation transport equation

- CKT for neutrinos : $\Box_{\rm i} f_q^{(\nu)} = \frac{1}{E_{\rm i}}\left[(1-f_q^{(\nu)})\Gamma_q^{<} - f_q^{(\nu)}\Gamma_q^{>}\right]$

Boltzmann eq. in the inertial frame

collision term with quantum corrections

$\Box_{\rm i} \equiv q\cdot D/E_{\rm i}$

N. Yamamoto & DY, APJ 895 (2020), 1

- Neutrino absorption : $\bar{\Gamma}_q^{({\rm ab})\lessgtr} \approx \bar{\Gamma}_q^{(0)\lessgtr} + \hbar\bar{\Gamma}_q^{(\omega)\lessgtr}(q\cdot\omega) + \hbar\bar{\Gamma}_q^{(B)\lessgtr}(q\cdot B)$

$\nu_{\rm L}^{\rm e}(q) + {\rm n}(k) \rightleftharpoons {\rm e}_{\rm L}(q') + {\rm p}(k')$

Fermi's EFT for weak int.
isoenergetic approx.:
NR approx., $M_{\rm n} \approx M_{\rm p} \approx M$
small-energy transf.

vorticity & magnetic field corrections :
breaking spherical symmetry & axisymmetry

analytic expressions :

$$\bar{\Gamma}_q^{(0)>} \approx \frac{1}{\pi\hbar^4c^4}(g_{\rm V}^2+3g_{\rm A}^2)G_{\rm F}^2(q\cdot u)^3(1-f_{0,q}^{(\rm e)})\left(1-\frac{3q\cdot u}{Mc^2}\right)\frac{n_{\rm n}-n_{\rm p}}{1-{\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}},$$

$$\bar{\Gamma}_q^{(B)>} \approx \frac{1}{2\pi\hbar^4c^4M}(g_{\rm V}^2+3g_{\rm A}^2)G_{\rm F}^2(q\cdot u)(1-f_{0,q}^{(\rm e)})\left(1-\frac{8q\cdot u}{3Mc^2}\right)\frac{n_{\rm n}-n_{\rm p}}{1-{\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}},$$

$$\bar{\Gamma}_q^{(\omega)>} \approx \frac{1}{2\pi\hbar^4c^4}(g_{\rm V}^2+3g_{\rm A}^2)G_{\rm F}^2(q\cdot u)^2(1-f_{0,q}^{(\rm e)})\left(\frac{2}{E_{\rm i}}+\beta f_{0,q}^{(\rm e)}\right)\frac{n_{\rm n}-n_{\rm p}}{1-{\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}}$$

$\bar{\Gamma}_q^{(0)>}$: S. Reddy, M. Prakash, J. M. Lattimer, PRD58:013009,1998

Neutrino flux driven by magnetic fields

- Considering neutrinos near equilibrium with the neutrino absorption on nucleons $\nu_{\rm L}^{\rm e}(q) + {\rm n}(k) \rightleftharpoons {\rm e_L}(q') + {\rm p}(k')$:   N. Yamamoto & DY, PRD 104, 123019 (2021)

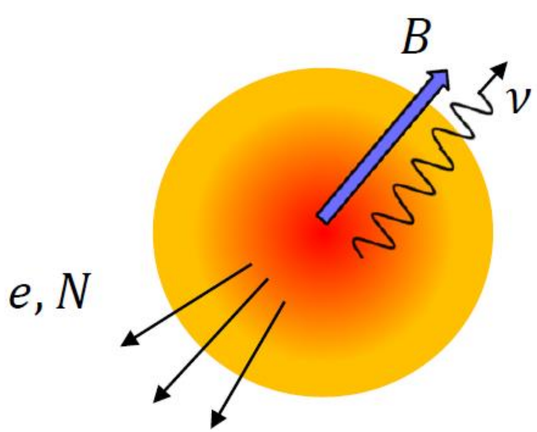
$$\Delta j_\nu^i = -\kappa(\boldsymbol{\nabla}\cdot\boldsymbol{v})B^i, \qquad \kappa = \frac{1}{72\pi M G_{\rm F}^2(g_{\rm V}^2+3g_{\rm A}^2)}\frac{{\rm e}^{2\beta(\mu_{\rm n}-\mu_{\rm p})}}{n_{\rm n}-n_{\rm p}}$$

$$\Delta T_\nu^{i0} = \mu_\nu \Delta j_\nu^i.$$

- The momentum kick from neutrinos : $\Delta T_{\rm e}^{i0} = -\Delta T_\nu^{i0}, \quad \Delta T_{\rm e}^{i0} = \mu_{\rm e}\Delta j_{\rm e}^i$

$$\Rightarrow \Delta J_{\rm e}^i = \xi_B B^i,$$ (effective CME)

$$\xi_B = -\kappa(\boldsymbol{\nabla}\cdot\boldsymbol{v})\frac{\mu_\nu}{\mu_{\rm e}}.$$



Taking $n_{\rm n}-n_{\rm p} \sim 0.1\,{\rm fm}^{-3}$, $\mu_{\rm n}-\mu_{\rm p} \sim 100\,{\rm MeV}$, $\mu_\nu \sim \mu_{\rm e} \sim 100\,{\rm MeV}$, $T \sim 10\,{\rm MeV}$, $L \sim 10\,{\rm km}$, $|\boldsymbol{v}| \sim 0.01$, we have $\xi_B \sim 10\,{\rm MeV}$. (approx. upper bound)

The implications to pulsar kicks

- Pulsar kicks : fast-moving neutron stars (v~200–500 km/s)

- The source of momentum asymmetry?

- One hypothesis : neutrino momentum flux driven by magnetic fields

$$v_{\rm kick} \lesssim \frac{\delta T^{i0}_{(\nu)B}}{\rho_{\rm core}} \sim \left(\frac{B}{10^{13}\ {\rm G}} \right)\ {\rm km/s} \Longrightarrow B \sim 10^{15-16}\ {\rm G}$$

for $v_{\rm kick} \sim 10^2\ {\rm km/s}$

(in proto-neutron stars)

- Similar estimations with different theoretical setups :

e.g. A. Vilenkin, Astrophys. J. 451, 700 (1995).
D. Lai and Y.-Z. Qian, Astrophys. J. 505, 844 (1998), astro-ph/9802345.
P. Arras and D. Lai, Phys. Rev. D 60, 043001 (1999), astro-ph/9811371
M. Kaminski, C. F. Uhlemann, M. Bleicher, and J. Schaffner-Bielich, Phys. Lett. B 760, 170 (2016), 1410.3833

- A more practical estimation entails fully non-equilibrium contributions.

Chiral magnetohydrodynamics

- Chiral magetohydrodynamics (MHD) equations:

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0\,, \qquad (17)$$

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot \left[\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \left(P + \frac{B^2}{2}\right)\mathbf{I}\right] = \boldsymbol{S}\,, \qquad (18)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{1}{\Gamma - 1}P + \frac{B^2}{2}\right) + \boldsymbol{\nabla} \cdot \left[\left(\frac{1}{2}\rho v^2 + \frac{\Gamma}{\Gamma - 1}P\right)\boldsymbol{v} + \boldsymbol{E} \times \boldsymbol{B}\right] = -\boldsymbol{S} \cdot \boldsymbol{v} - \Delta \boldsymbol{J} \cdot \boldsymbol{E}\,, \qquad (19)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta\, \boldsymbol{\nabla} \times (\xi_B \boldsymbol{B})\,, \qquad (20)$$

$$\frac{\partial n_{5,\mathrm{eff}}}{\partial t} = \frac{1}{2\pi^2}\boldsymbol{E} \cdot \boldsymbol{B}\,, \qquad (21)$$

$$\boldsymbol{S} = \rho\nu \boldsymbol{\nabla}^2 \boldsymbol{v} + \frac{1}{3}\rho\nu \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{v})$$

J. Matsumoto, N. Yamamoto, DY, PRD 105 (2022) 12, 123029
See also Y. Masada, K. Kotake, T. Takiwaki, and N. Yamamoto, PRD 98, 083018 (2018)

Time evolution of the magnetic field

- Numerical simulations :

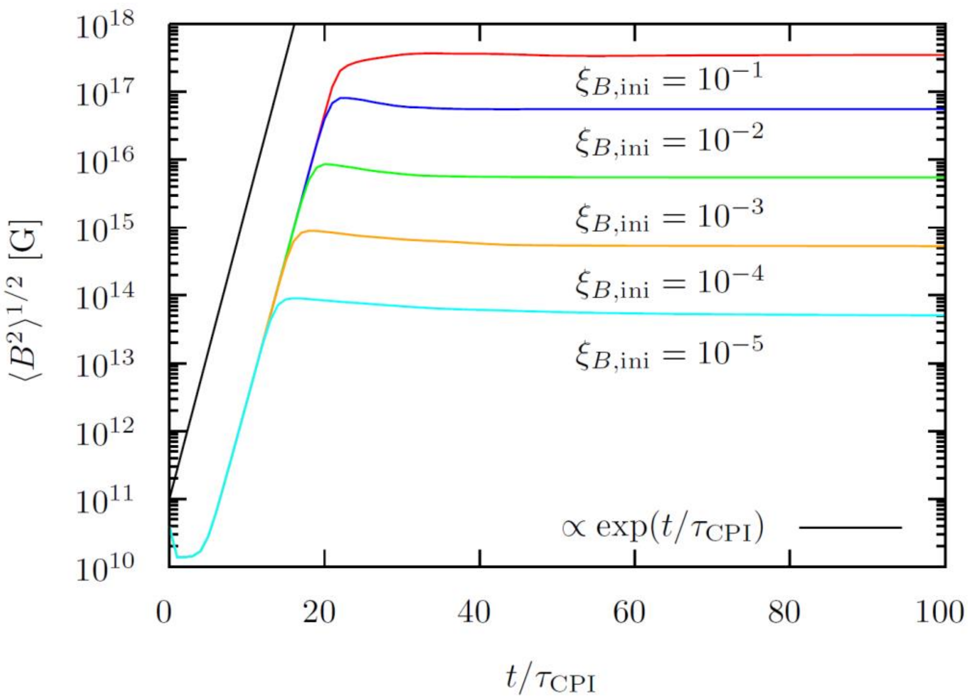
resistivity : $\eta = 1$ viscosity : $\nu = 0.01$

in the units of $100\,\mathrm{MeV} = 1$

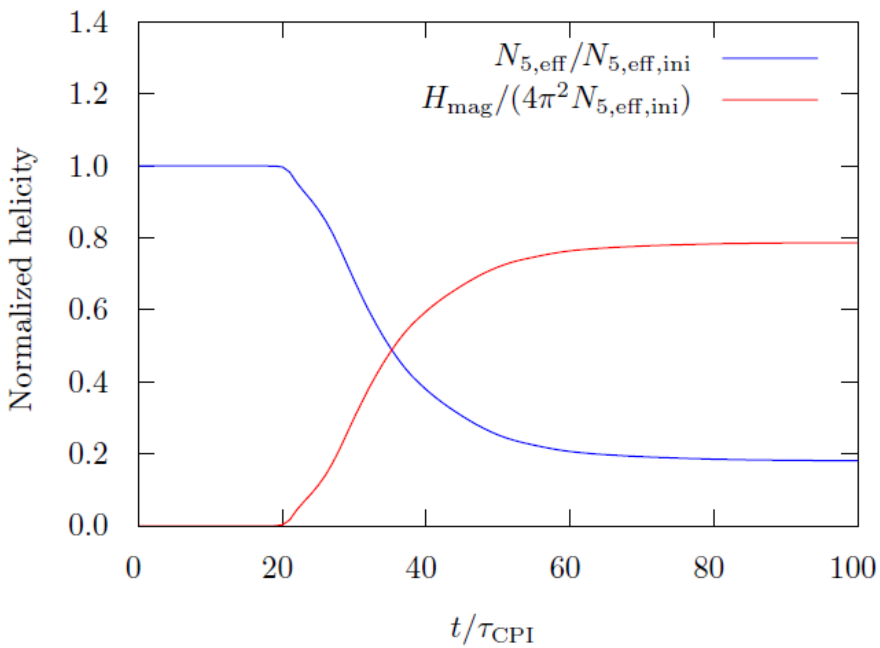
J. Matsumoto, N. Yamamoto, DY, PRD 105 (2022) 12, 123029

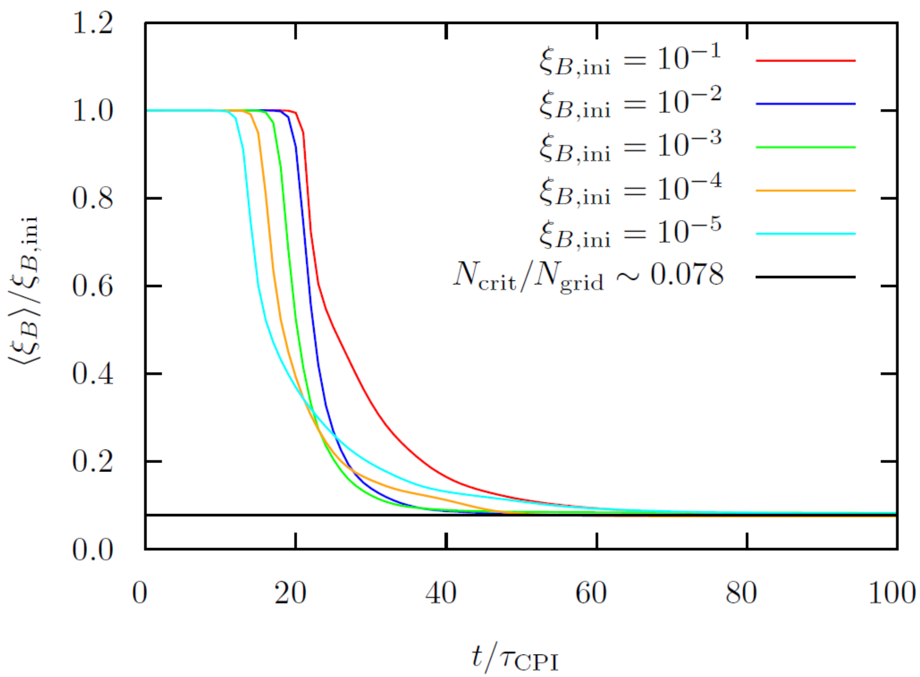
TABLE I. Summary of the simulation runs.

Name	L	$\xi_{B,\mathrm{ini}}$	τ_{CPI}
Model 1	8×10^2	10^{-1}	4×10^2
Model 2	8×10^3	10^{-2}	4×10^4
Model 3	8×10^4	10^{-3}	4×10^6
Model 4	8×10^5	10^{-4}	4×10^8
Model 5	8×10^6	10^{-5}	4×10^{10}



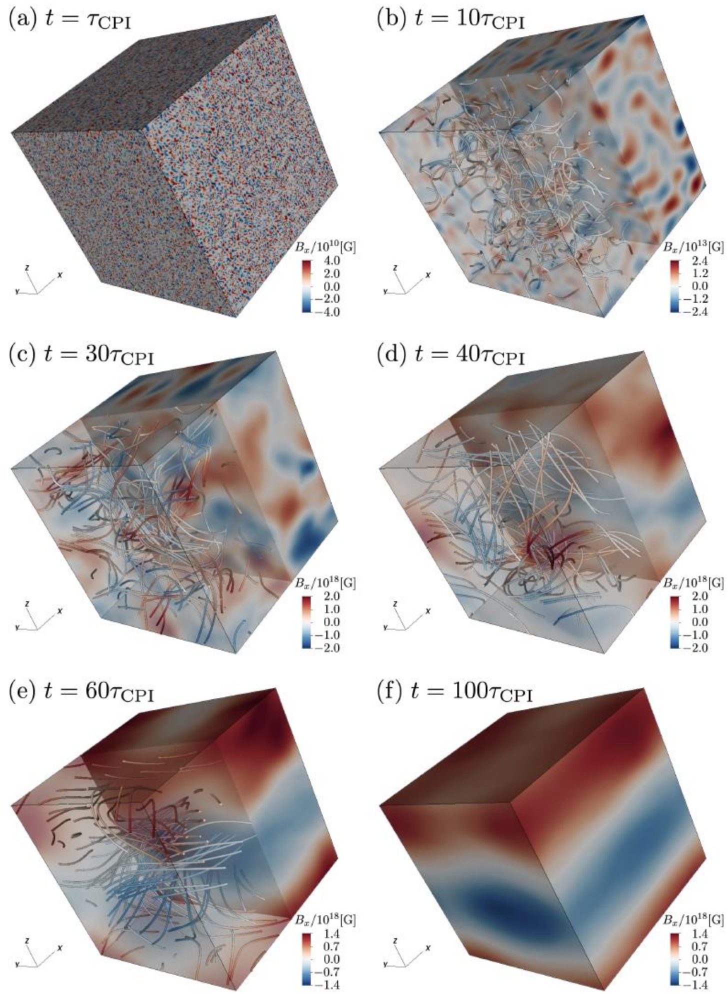
Helicity evolution



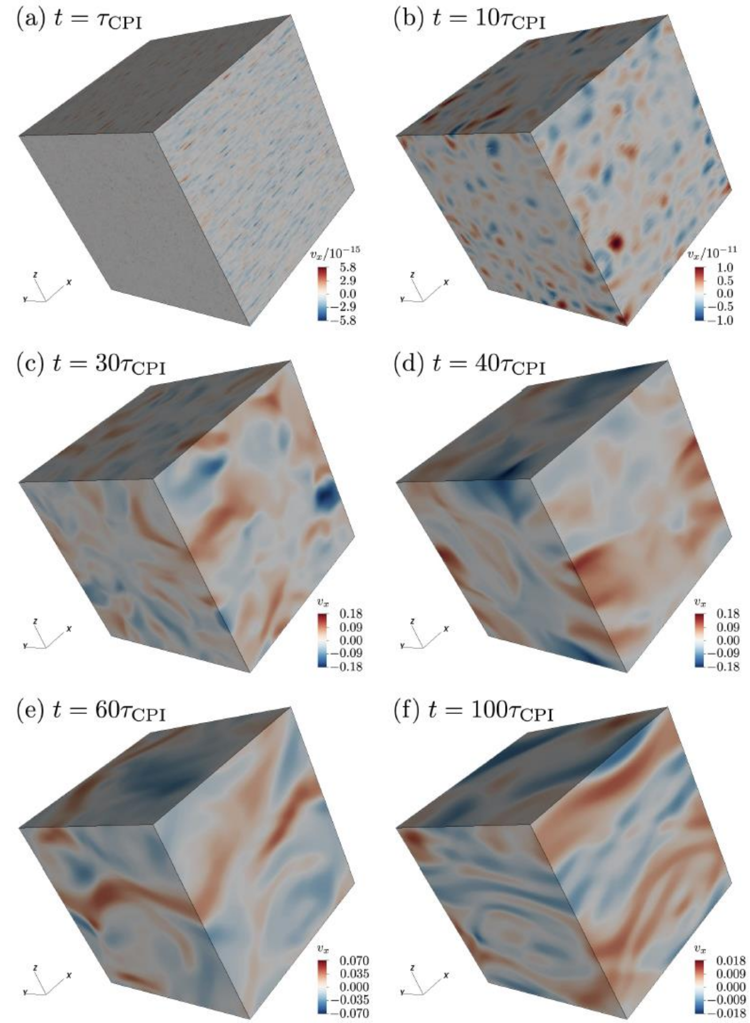


Inverse cascade

- Inverse cascade



magnetic field



fluid velocity

Effective CME from non-equilibrium neutrinos

- Effective CME from neutrino radiation :

N. Yamamoto, DY, arXiv:2211.14465

$$j_B^\mu \approx \hbar e^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^3} \frac{\delta(q^2)}{q_0} \left(B^\mu q \cdot \partial_q - q \cdot B \partial_{\bar{q}}^\mu\right) \delta f_{\mathrm{W}}^{(\mathrm{e})}$$

- Semi-analytic form of non-equilibrium neutrinos :

M. T. Keil, G. G. Raffelt, H.-T. Janka, APJ 590, 971 (2003)

$$f^{(\nu)}(q_0) = \left(\frac{q_0}{\bar{\epsilon}}\right)^\alpha \mathrm{e}^{-(\alpha+1)q_0/\bar{\epsilon}} \Longrightarrow \delta f_{\mathrm{W}}^{(\mathrm{e})}(q,x) \approx -\frac{x_0}{q_0} F_{\mathrm{W}}(q_0)$$

$$F_{\mathrm{W}} \approx \frac{(q\cdot u)^3}{\pi}(g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2) G_{\mathrm{F}}^2 (n_{\mathrm{p}} - n_{\mathrm{n}}) \left[\frac{\bar{f}_q^{(\mathrm{e})}(1-f_q^{(\nu)})}{1-\mathrm{e}^{\beta(\mu_{\mathrm{n}}-\mu_{\mathrm{p}})}} + \frac{(1-\bar{f}_q^{(\mathrm{e})}) f_q^{(\nu)}}{1-\mathrm{e}^{\beta(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})}}\right]$$

- Approximate upper bounds (in the gain region) :

$$\xi_B^{\mathrm{tot}} \approx -0.5\,\mathrm{MeV} \quad \text{for } x_0 = 0.1\,\mathrm{s}$$

- ❑ Kick velocity : $v_{\mathrm{kick}} \sim \frac{|T^{i0}_{B,\mathrm{tot}}|}{\rho_{\mathrm{core}}} \approx \left(\frac{eB}{10^{13\text{-}14}\,\mathrm{G}}\right) \mathrm{km/s}\,.$

} same order of magnitude as the estimations from near-equilibrium neutrinos

Summary & outlook

- ✓ The chiral effect for leptons due to “parity violation” could qualitatively affect the supernova evolution.

- ✓ Back-reaction on the matter sector from the magnetic-field induced neutrino flux could generate an “effective CME”, which further results in the “CPI”.

- ✓ The evolution of matter dictated by chiral MHD follows the “inverse cascade” led by CPI and generates a “strong and stable magnetic field” in late times.

(Review to appear in PPNP: K. Kamada, N. Yamamoto, DY, Chiral Effects in Astrophysics and Cosmology, arXiv:2207.09184)

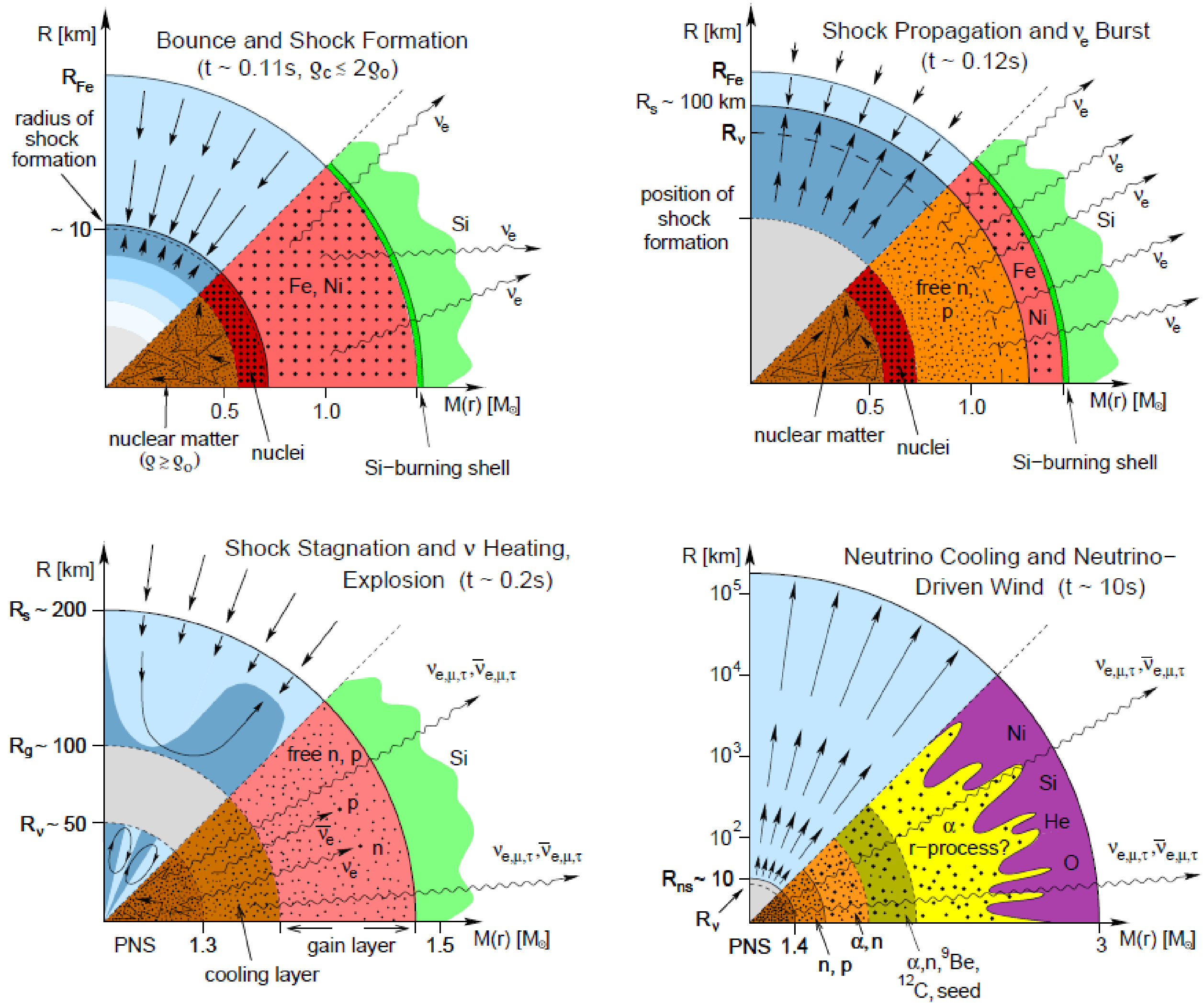
- ✓ Chiral effects of electrons from non-equilibrium neutrino radiation is found to be of the same order of magnitude as those from near-equilibrium neutrinos.

(N. Yamamoto, DY, arXiv:2211.14465)

- ❖ It is important to finally conduct the full simulations from chiral radiation hydrodynamics.
- ❖ A consistent framework incorporating chiral transport & flavor oscillations of neutrinos?

Thank you!

Core-collapse supernovae (CCSN)



H.-Th. Janka et al., astro-ph/0612072

Chirality flipping

- Chirality flipping : $\Gamma_{\mathrm{f}} \simeq \frac{\alpha^2 m_{\mathrm{e}}^2}{3\pi \mu_{\mathrm{e}}} \left(\ln \frac{4\mu_{\mathrm{e}}^2}{q_{\mathrm{D}}^2} - 1 \right)$,

$$\mu_{\mathrm{e}} \sim 100\ \mathrm{MeV\ and}\ T \sim 30\ \mathrm{MeV}, \quad \Gamma_{\mathrm{f}} \sim 10^{14}\ \mathrm{s}^{-1}.$$

D. Grabowska, D.B. Kaplan, S. Reddy, PRD 91 (8) (2015) 085035

- Axial-charge evolution : $\partial_t n_5 = \Gamma_{\mathrm{w}}(n_{\mathrm{e}} - n_5) - (\Gamma_{\mathrm{CPI}} + \Gamma_{\mathrm{f}})n_5$

depletion rate of the electron fraction due to electron capture

$\Gamma_{\mathrm{w}} \simeq 1\ \mathrm{s}^{-1}$

- Chirality imbalance in a steady state (quasi-equilibrium) :

$$n_5 = \frac{\Gamma_{\mathrm{w}}}{\Gamma_{\mathrm{f}}} n_{\mathrm{e}} \sim 10^{-14} n_{\mathrm{e}} \Rightarrow \mu_5 \sim 10^{-7}\ \mathrm{eV}$$

(could be larger for the non-equilibrium state)

CKT with collisions

- CKT with collisions $(\partial_\rho n^\mu = 0)$: (for right-handed fermions)

Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017), PRD 97, 016004 (2018)

$$\delta\left(q^2 - \hbar\frac{B\cdot q}{q\cdot n}\right)\left\{\left[q\cdot\Delta + \hbar\frac{S^{\mu\nu}_{(n)}E_\mu}{(q\cdot n)}\Delta_\nu + \hbar S^{\mu\nu}_{(n)}(\partial_\mu F_{\rho\nu})\partial^\rho_q\right]f^{(n)}_q - \tilde{\mathcal{C}}\right\} = 0,$$

magnetic-moment coupling

spin tensor : $S^{\mu\nu}_{(n)} = \frac{\epsilon^{\mu\nu\alpha\beta}}{2(q\cdot n)}q_\alpha n_\beta$

($F^{\mu\nu} = 0$: the quantum corrections only appear in collisions)

- Quantum corrections on the collision term :

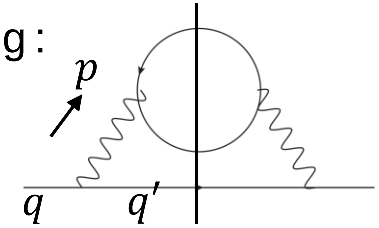
$$\tilde{\mathcal{C}} = q\cdot\mathcal{C} + \hbar\frac{S^{\mu\nu}_{(n)}E_\mu}{(q\cdot n)}\mathcal{C}_\nu + \hbar S^{\alpha\beta}_{(n)}\left((1-f^{(n)}_q)\Delta_\alpha\Sigma^<_\beta - f^{(n)}_q\Delta_\alpha\Sigma^>_\beta\right),$$

induced by inhomogeneity of the medium

$$\mathcal{C}_\beta = \Sigma^<_\beta(1-f^{(n)}_q) - \Sigma^>_\beta f^{(n)}_q.$$

also include hbar corrections

2-2 scattering :



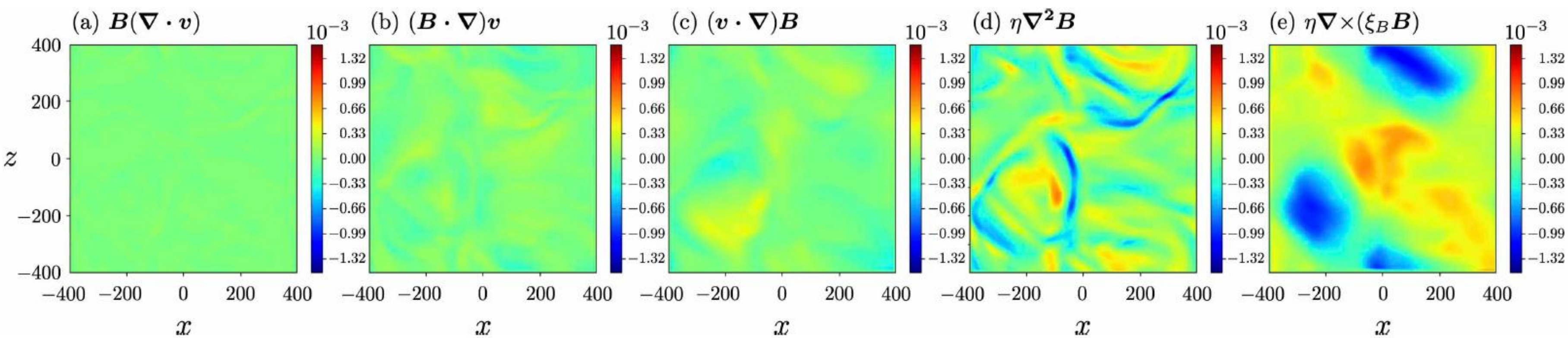
- EM tensor : $T^{\mu\nu} = \int_q 4\pi\delta(q^2)\left(q^\mu q^\nu f_q + \hbar q^{\{\mu}S^{\nu\}\rho}_q D_\rho f_q\right),\ D_\mu f_q \equiv \Delta_\mu f_q - \mathcal{C}_\mu.$

Dominance of the CPI

- Anatomy of the induction equation :

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta \, \boldsymbol{\nabla} \times (\xi_B \boldsymbol{B}) \; ,$$

$$\boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) = \boldsymbol{B}(\boldsymbol{\nabla} \cdot \boldsymbol{v}) + (\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{B}$$



- CPI dominates under the condition : $|\boldsymbol{v}| \ll \eta |\xi_B|$

Chiral kinetic equation for electrons

- Chiral kinetic equation for left-handed electrons near equilibrium:

N. Yamamoto, DY, arXiv:2211.14465

$$f_{\rm L}^{(\rm e)} = \bar{f}_{\rm L}^{(\rm e)} + \delta f_{\rm L}^{(\rm e)}$$

⇨ $$\Box_q f_{\rm L}^{(\rm e)} \approx -q\cdot n\hat{\tau}_{\rm EM}^{-1}\delta f_{\rm L}^{(\rm e)} - F_{\rm W},$$

$$\Box_q f_{\chi}^{(\rm e)} = \left(q^\mu + \chi\hbar\frac{S_q^{\mu\nu}eF_{\mu\rho}n^\rho}{q\cdot n}\right)\Delta_\mu f_{\chi}^{(\rm e)},$$

$$\Delta_\mu = D_\mu + eF_{\lambda\mu}\partial_q^\lambda$$

$\chi = \pm 1$ for R/L.

❖ collision term with neutrinos : $F_{\rm W} = \bar{f}_{\rm L}^{(\rm e)}\Gamma_{\rm W}^{>} - (1-\bar{f}_{\rm L}^{(\rm e)})\Gamma_{\rm W}^{<}$

- Modified relaxation-time approx. : $f_{\rm L}^{(\rm e)} = \bar{f}_{\rm L}^{(\rm e)} + \delta f_{\rm LEM}^{(\rm e)} + \delta f_{\rm LW}^{(\rm e)},$

$$\mathcal{O}(\delta f_{\rm LEM}^{(\rm e)}) \approx \mathcal{O}(\tau_{\rm EM}/L)$$

$$\mathcal{O}(\delta f_{\rm LW}^{(\rm e)}) \approx \mathcal{O}(\tilde{\epsilon}^4 G_F^2)$$

Effective CME from neutrino radiation

- Kinetic equation breaks into : $\Box_q \bar{f}_{\rm L}^{(\rm e)} \approx -q\cdot n \hat{\tau}_{\rm EM}^{-1}\delta f_{\rm LEM}^{(\rm e)},$

$$\Box_q \delta f_{\rm W}^{(\rm e)} \approx (1-\bar{f}_{\rm L}^{(\rm e)})\Gamma_{\rm W}^{<} - \bar{f}_{\rm L}^{(\rm e)}\Gamma_{\rm W}^{>} = -F_{\rm W}.$$

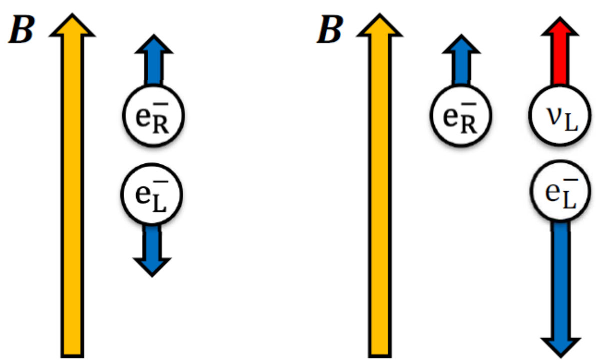
- Neutrino absorption on nucleons :

$$F_{\rm W} \approx \frac{(q\cdot u)^3}{\pi}(g_{\rm V}^2+3g_{\rm A}^2)G_{\rm F}^2(n_{\rm p}-n_{\rm n})\left[\frac{\bar{f}_q^{(\rm e)}(1-f_q^{(\nu)})}{1-{\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}} + \frac{(1-\bar{f}_q^{(\rm e)})f_q^{(\nu)}}{1-{\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}}\right]$$

- Ignoring electric fields : $q\cdot\partial\delta f_{\rm W}^{(\rm e)} \approx -F_{\rm W}$

$$\Longrightarrow \quad \delta f_{\rm W}^{(\rm e)}(q,x) = -\frac{1}{q_0}\int_0^{x_0} {\rm d}x_0' F_{\rm W}(q,x')|_{\rm c}\,,$$

$$|_{\rm c} = \{x_\perp'^{\mu} = x_\perp^{\mu},\, x_\parallel'^{\mu} = x_\parallel^{\mu} - \bar{q}^{\mu}(x_0-x_0')/q_0\}.$$



(a) $\boldsymbol{j}_B = \boldsymbol{0}$ (b) $\boldsymbol{j}_B \neq \boldsymbol{0}$

❖ Effective CME :

$$j_B^{\mu} \approx \hbar e^2 \int \frac{{\rm d}^4 q}{(2\pi)^3}\frac{\delta(q^2)}{q_0}\left(B^{\mu} q\cdot\partial_q - q\cdot B\partial_{\bar{q}}^{\mu}\right)\delta f_{\rm W}^{(\rm e)}$$