Reflected Entropy for Communicating Black Holes

Jaydeep Kumar Basak

National Sun Yat-Sen University
Taiwan

Work done in collaboration with Mir Afrasiar, Ashish Chandra and Gautam Sengupta

2205.07903, 2211.13246 and 2212.xxxx

NCTS Annual Theory Meeting 2022

14.12.2022

Outline

- **Motivation**
- **Reflected Entropy and Holography**
- **Island Proposal for Reflected Entropy**
- **Communicating Black Holes in Two Different Models**
  - Model I and Results
  - Model II and Results
- **Summary**
- **Future Directions**

Motivation

Motivation

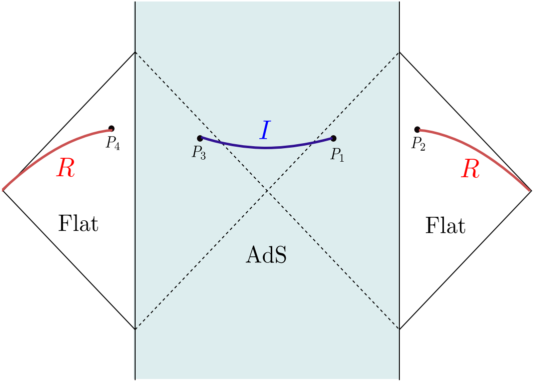
- Islands: a potential resolution of the black hole information loss paradox
- Appears after the Page time where quantum extremal surface shows a phase transition
- Imposes a bound on the growth of entanglement entropy for the radiation bath
- Result: Page curve

[Penington, Almheiri, Engelhardt, Marolf, Maxfield, Hartman, Maldacena, Shaghoulian, Tajdini, Myers, Balasubhramanian, Takayanagi, Raju, Geng, Karch, Randall, Mahajan, Sybesma, Ugajin, Hollowood, Prem Kumar,..........]

Motivation

[Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini: 19]

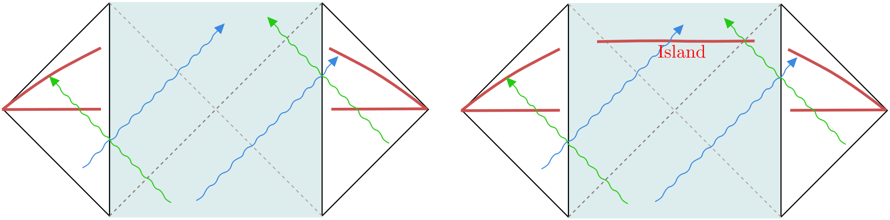
Eternal Black Hole: Island



$$S(\mathrm{R}) = \min_{\mathrm{I}} \left\{ \mathrm{ext}_{\mathrm{I}} \left[\frac{\mathrm{Area}[\partial\, \mathrm{I}]}{4G_N} + S^{eff}[\mathrm{R} \cup \mathrm{I}] \right] \right\}$$

[Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini: 19]

Eternal Black Hole: Island



(a) (b)

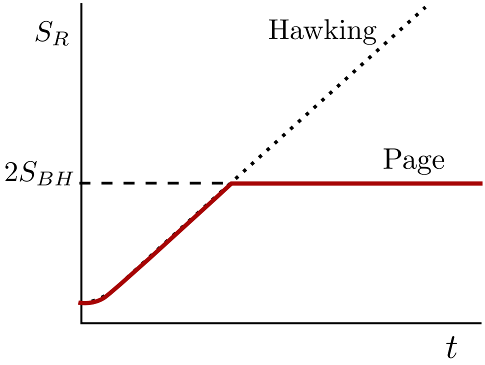
Figure 1: (a) Increasing entropy without any island (b) Island collect the partners of the bath Hawking radiation

$$S(\mathrm{R}) = \min_{\mathrm{I}} \left\{ \mathrm{ext}_{\mathrm{I}} \left[\frac{\mathrm{Area}[\partial\, \mathrm{I}]}{4G_N} + S^{eff}[\mathrm{R} \cup \mathrm{I}] \right] \right\}$$

[Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini: 19]

Eternal Black Hole: Page curve

$$S(\mathrm{R}) = \min_{\mathrm{I}} \left\{ \mathrm{ext}_{\mathrm{I}} \left[\frac{\mathrm{Area}[\partial\, \mathrm{I}]}{4G_N} + S^{eff}[\mathrm{R} \cup \mathrm{I}] \right] \right\}$$



Motivation

- **Goal**: Understand the structure of entanglement in the radiation region.
- It can be analyzed by considering the two subsystems in bath which construct a mixed state.
- Attempts has been made in this direction utilizing island construction of various measures.
  - Reflected entropy [Chandrasekaran, Miyaji, Rath : 20] [Li, Chu, Zhou: 20]
  - Entanglement Negativity [JKB, Basu, Malvimat, Parihar, Sengupta : 20, 21] [Shapourian, Liu, Kudler − Flam, Vishwanath : 20] [Vardhan, Kudler − Flam, Shapourian, Liu : 21]
- Different models can be studied to obtain more intricate phase structure of mixed state entanglement
  - Random tensor network [Akers, Faulkner, Lin, Rath : 21, 22]
  - Moving mirror [JKB, Basu, Malvimat, Parihar, Sengupta : 22]
  - Communicating black holes [Afrasiar, JKB, Chandra, Sengupta : 22]

Reflected Entropy and Holography

Reflected Entropy

[Dutta, Faulkner : 19]

- *Purification:* A bipartite quantum system $A \cup B$ in a mixed state ρ_{AB} is prepared by embedding the system $A \cup B$ in a larger tripartite system $A \cup B \cup C$.
- Reflected entropy $S_R(A:B)$ of a bipartite system AB involves the canonical purification of the given mixed state ρ_{AB} by doubling its Hilbert space to define a pure state $\left|\sqrt{\rho_{AB}}\right\rangle_{ABA^*B^*}$ such that $\rho_{AB} = \mathrm{Tr}_{A^*B^*} \left|\sqrt{\rho_{AB}}\right\rangle \left\langle\sqrt{\rho_{AB}}\right|$

ρ_{AB} $\quad\quad$ $|\sqrt{\rho_{AB}}>_{AA^*BB^*}$

$$\rho_{AB} = Tr_{A^*B^*}(|\sqrt{\rho_{AB}}><\sqrt{\rho_{AB}}|)$$

- Reflected entropy is defined as the von Neumann entropy of the reduced density matrix $\rho_{AA^*} = \mathrm{Tr}_{BB^*} \left|\sqrt{\rho_{AB}}\right\rangle \left\langle\sqrt{\rho_{AB}}\right|$,

$$S_R(A:B) \equiv S_{vN}(\rho_{AA^*})_{\sqrt{\rho_{AB}}} .$$

- $A \cup B$ in a pure state : $S_R(A:B) = 2S_{vN}(A) = 2S_{vN}(B)$

Replica Trick for Reflected Entropy

- In CFT reflected entropy is generalized by two replica indices m and n.

$$\rho_{AB}^{m} = \sum_{a,i,j} p_a^m \sqrt{l_a^i l_a^j} \left|i_a\right\rangle_A \left|i_a\right\rangle_B \left\langle j_a\right|_A \left\langle j_a\right|_B$$

- By interpreting $\left\langle j_a\right|_A \left\langle j_a\right|_B$ as states $\left|j_a\right\rangle_{A^*} \left|j_a\right\rangle_{B^*}$ on Hilbert spaces $\mathcal{H}_A^*$ and $\mathcal{H}_B^*$, one can define a state $\left|\rho_{AB}^{m/2}\right\rangle$ as

$$\left|\rho_{AB}^{m/2}\right\rangle = \sum_{a,i,j} p_a^{m/2} \sqrt{l_a^i l_a^j} \left|i_a\right\rangle_A \left|i_a\right\rangle_B \left|j_a\right\rangle_{A^*} \left|j_a\right\rangle_{B^*}$$

  The normalized form of this state is a purification of ρ_{AB}^m.

- The Rényi reflected entropy of the reduced density matrix may be obtained as

$$S_n(AA^*) = \frac{1}{1-n} \log \left[\frac{\mathrm{Tr}_{AA^*}\left(\mathrm{Tr}_{BB^*}\left|\rho_{AB}^{m/2}\right\rangle\left\langle\rho_{AB}^{m/2}\right|\right)^n}{\left(\mathrm{Tr}_{AA^*}\left(\mathrm{Tr}_{BB^*}\left|\rho_{AB}^{m/2}\right\rangle\left\langle\rho_{AB}^{m/2}\right|\right)\right)^n} \right] = \frac{1}{1-n} \log \frac{Z_{n,m}}{(Z_{1,m})^n}$$

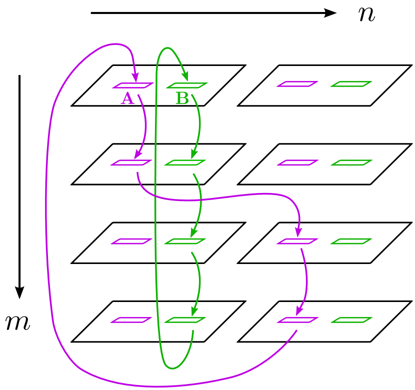


Figure 2: Structure of the replica manifold for the Rényi reflected entropy between subsystems A and B. The sewing of the individual replicas along the subsystems A and B are denoted by magenta and green arrows corresponding to the twist fields σ_{g_A} and σ_{g_B}, respectively.

- This partition function can subsequently be expressed in terms of the correlation function of the twist operators σ_{g_A} and σ_{g_B} inserted at the end points of the intervals $A \equiv [z_1, z_2]$ and $B \equiv [z_3, z_4]$ as

$$\frac{Z_{n,m}}{(Z_{1,m})^n} = \frac{\left\langle \sigma_{g_A}(z_1)\sigma_{g_A^{-1}}(z_2)\sigma_{g_B}(z_3)\sigma_{g_B^{-1}}(z_4)\right\rangle_{CFT^{\otimes mn}}}{\left(\left\langle \sigma_{g_m}(z_1)\sigma_{g_m^{-1}}(z_2)\sigma_{g_m}(z_3)\sigma_{g_m^{-1}}(z_4)\right\rangle_{CFT^{\otimes m}}\right)^n}.$$

where the conformal dimensions of the twist fields σ_{g_A}, σ_{g_B}, σ_{g_m} and $\sigma_{g_B g_A{}^{-1}}$ are given as,

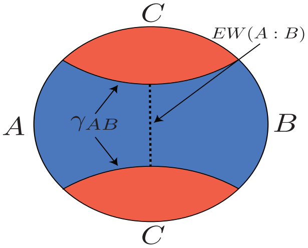
$$h_{g_A} = h_{g_B} = \frac{nc}{24}\left(m - \frac{1}{m}\right), \qquad h_{g_m} = \frac{c}{24}\left(m - \frac{1}{m}\right), \qquad h_{g_B g_A{}^{-1}} = \frac{2c}{24}\left(n - \frac{1}{n}\right).$$

- The reflected entropy for such bipartite states may finally be obtained in the replica limit $n \to 1$ and $m \to 1$ as

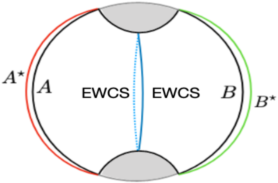
$$S_R(A:B) = \lim_{n,m\to 1} S_n(AA^*).$$

Holographic Reflected Entropy

- Entanglement wedge cross section (EWCS) : codimension two surface with minimized area dividing the wedge for $A \cup B$.
- For a disconnected wedge, $EWCS = 0$



(a) Entanglement Wedge Cross Section Picture credits [Akers,Rath: 19]



(b) Gravity dual of the canonically purified state. Picture credits [Dutta, Faulkner: 19].

- Reflected entropy is twice the area of EWCS [Dutta, Faulkner: 19]

$$S_R(A : B) = 2\text{Area}(EWCS)$$

Island Proposal

[Almheiri, Engelhardt, Marolf, Maxfield: 19],
[Almheiri, Mahajan, Maldacena, Zhao: 19], [Penington: 19]

- The fine grained entropy of a region in QFT coupled to semi-classical gravity is given by the island formula,

$$S(\mathrm{A}) = \min_{Is(A)} \left\{ \mathrm{ext}_{Is(A)} \left[\frac{\mathrm{Area}[\partial Is(A)]}{4G_N} + S^{eff}[\mathrm{A} \cup Is(A)] \right] \right\}.$$

$Is(A)$−island corresponding to the subsystem A

S^{eff}−effective von-Neumann entropy of quantum matter fields.

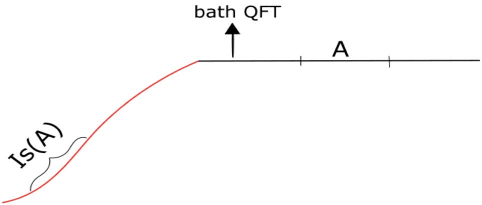


Figure 4: Entanglement entropy of single interval in a bath CFT_{1+1} coupled to gravity.

Island for Reflected Entropy

[Chandrasekaran, Miyaji, Rath : 20] [Li, Chu, Zhou: 20]

- The island formula for reflected entropy is,

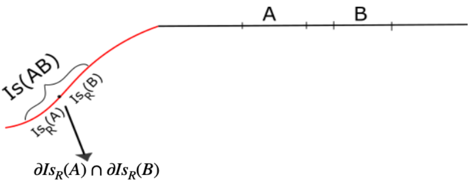
$$S_R(A:B) = \min_{Q'} \text{ext}_{Q'} \left\{ \frac{2\text{Area}(Q')}{4G_N} + S_R^{\text{eff}}(A \cup Is_R(A) : B \cup Is_R(B)) \right\}$$

where,

$Q' = \partial Is_R(A) \cap \partial Is_R(B)$

S_R^{eff} – Effective reflected entropy of bulk quantum matter fields.

$Is_R(A)$ and $Is_R(B)$ – Reflected entropy islands for A and B.



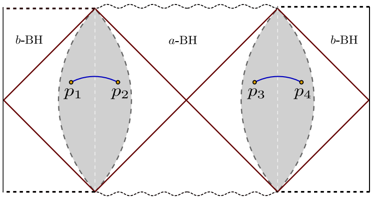
- In general $Is_R(A) \neq Is(A)$
- $Is(A \cup B) = Is_R(A) \cup Is_R(B)$

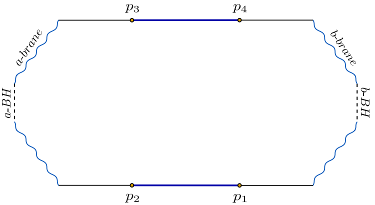
Communicating Black Holes

Model I

[Balasubramanian, Craps, Khramtsov, Shaghoulian : 21]

- Two finite sized non-gravitating reservoirs (CFT_2) coupled to two quantum dots at its boundaries
- The holographic dual of these quantum dots are Planck branes described by AdS_2 geometries.
- Two eternal JT black holes located on the Planck branes communicating via the shared reservoirs.

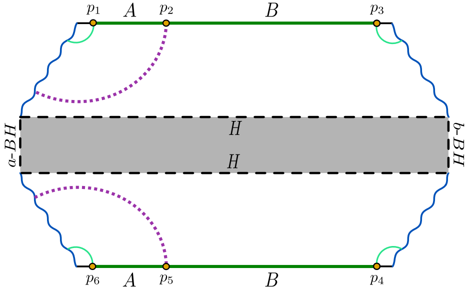




[Afrasiar, JKB, Chandra, Sengupta : 22]

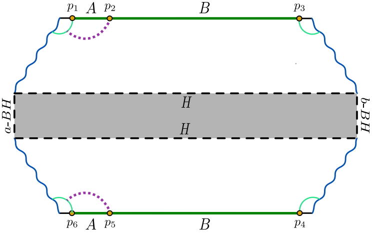
- Both the field theoretic and holographic computations satisfy the equality,

$$S_R(A:B) = 2\text{Area}(EWCS)$$

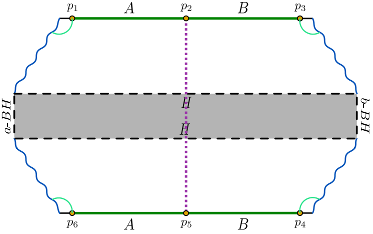


(a)

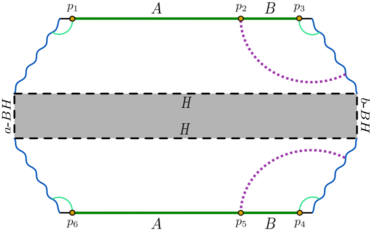
[Afrasiar, JKB, Chandra, Sengupta : 22]



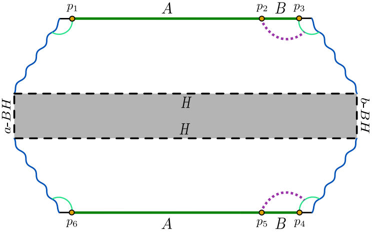
(b)



(c)



(d)



e)

Reflected Entropy

[Afrasiar, JKB, Chandra, Sengupta : 22]

- We compute the reflected entropy and mutual information between two adjacent subsystems $A = [0.01L, 0.25L]$ and $B = [0.25L, 0.99L]$ wrt time.

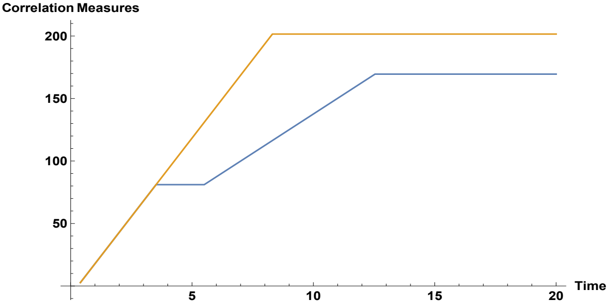


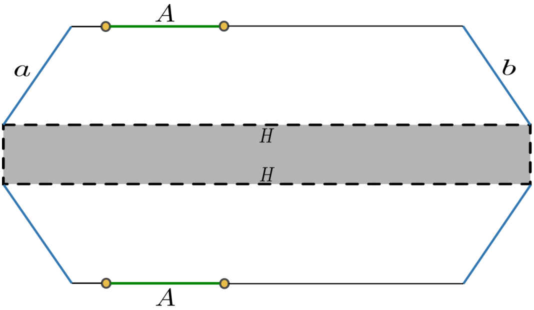
Figure 8: Reflected entropy (yellow) and mutual information (blue).

Model II

Model II

[Geng, Lüst, Mishra, Wakeham : 21]

- Thermal $BCFT_2$ with two boundaries and distinct boundary conditions are imposed on them.
- Holographic dual is defined by a wedge enclosed by two Karch-Randall (KR) branes in a bulk AdS_3 geometry.
- Holographic dual geometry is described by an eternal AdS_3 BTZ black hole which induces two dimensional black holes on the KR branes.



Reflected Entropy

[Afrasiar, JKB, Chandra, Sengupta : 22]

- We compute the reflected entropy and mutual information between two adjacent subsystems $A = [r_I + \epsilon, r]$ and $B = [r, r_O - \epsilon]$. The common point r is varied at a constant time.

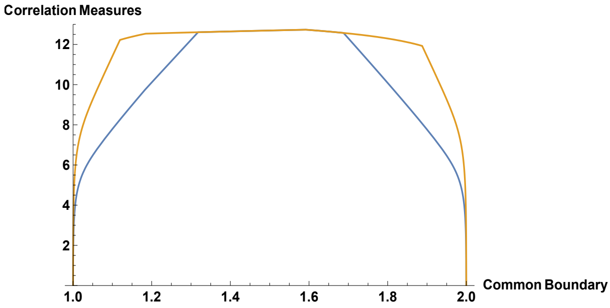


Figure 9: Reflected entropy (yellow) and mutual information (blue).

Summary

- We have investigated the reflected entropy in two different models of communicating black holes.
- The first model describes two finite sized non-gravitating reservoirs coupled to two quantum dots at their boundaries. The holographic dual of these quantum dots are described by two Plank branes with AdS_2 geometries.
- These Plank branes contain two eternal JT black holes which are in communication with each other through the shared reservoirs.
- The second model describes a braneworld geometry involving a bulk eternal AdS_3 BTZ black hole and the bulk geometry is truncated by two KR branes.
- On these branes two dimensional black holes are induced from the higher dimension which are communicating to each other through the shared baths described by thermal $BCFT_2$s with two boundaries.
- Reflected entropy shows intricate phase structure for the communication between two black holes on the branes.
- We observe the duality, $S_R = 2(EWCS)$
- All of our results satisfy the relation $I \leq S_R$.

Future Directions

- Reflected entropy with a gravitating bath.
- $S_R - I$ in various doubly holographic pictures, AdS/BCFT etc.
- Subregion complexity, multipartite correlation (entanglement of purification) etc in communicating black hole model.
- Others !!!

Thank You!!