$SU(2)$ gauge-Higgs theories on the lattice : from BSM physics to cuprate superconductors

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Outline

- ❑ $SU(2)$ gauge theory with 2 fundamental Higgs in 4d
  - ♦ Motivation – 2-Higgs Doublet Model (2HDM)
  - ♦ Preliminary results
- ❑ $SU(2)$ gauge theory with 4 adjoint Higgs in 3d
  - ♦ Motivation – Cuprate superconductors
  - ♦ Preliminary results

First results appeared in Lattice 2022 Proceedings arxiv:2210.09855 [hep-lat]

$SU(2)$ gauge theory with 2 fundamental Higgs in 4d

- Two Higgs Doublet Model (2HDM) allows for new sources of CP violation and a first-order electroweak phase transition that drives baryogenesis.
- No observed deviation from the SM — one of the Higgs doublets must mimic SM Higgs (*alignment* requirement). In supersymmetry-inspired 2HDM, masses of the scalar particles in the second Higgs doublet $\sim$ 5 TeV — cannot be discovered in direct searches at LHC (*decoupling* limit). [P. Athron et al.2017]
- Alignment without decoupling with masses of extra scalar particles $\lesssim$ 1 TeV, may be facilitated with choices of coupling and mixing strengths. Requires certain couplings amongst the scalar fields to be strong. [Hou, Kikuchi 2018]
- Combining this with the triviality property of these couplings, it is necessary to investigate the viability of this scenario by examining carefully the relationship amongst the spectrum, the cut-off scale, and the couplings. Necessary to employ non-perturbative methods $\to$ lattice gauge theory techniques.

2HDM III

$SU(2)$ fundamental Higgs doublet:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2^{(n)}(x) + i\phi_3^{(n)}(x) \\ \phi_0^{(n)}(x) + i\phi_1^{(n)}(x) \end{pmatrix}, \phi_\alpha^{(n)}(x) \in \mathbb{R}$$

Quaternion representation

$$\Phi_n(x) = \frac{1}{\sqrt{2}} \sum_{\alpha=0}^{3} \theta_\alpha \phi_\alpha^{(n)}(x)$$

$$\theta_0 = \mathbf{1}_{2\times 2}, \theta_i = \sigma_i$$

$$V_{\text{2HDM}}(\Phi_1, \Phi_2) = \mu_{11}^2 \operatorname{Tr}\left(\Phi_1^\dagger \Phi_1\right) + \mu_{22}^2 \operatorname{Tr}\left(\Phi_2^\dagger \Phi_2\right) + \left[\mu_{12}^2 \operatorname{Tr}\left(\Phi_1^\dagger \Phi_2\right) + \text{h.c.}\right]$$
$$+ \eta_1 \operatorname{Tr}\left(\Phi_1^\dagger \Phi_1\right)^2 + \eta_2 \operatorname{Tr}\left(\Phi_2^\dagger \Phi_2\right)^2 + \eta_3 \operatorname{Tr}\left(\Phi_1^\dagger \Phi_1\right) \operatorname{Tr}\left(\Phi_2^\dagger \Phi_2\right) + \eta_4 \operatorname{Tr}\left(\Phi_1^\dagger \Phi_2\right) \operatorname{Tr}\left(\Phi_2^\dagger \Phi_1\right)$$
$$+ \left[\eta_5 \operatorname{Tr}\left(\Phi_1^\dagger \Phi_2\right)^2 + \eta_6 \operatorname{Tr}\left(\Phi_1^\dagger \Phi_1\right) \operatorname{Tr}\left(\Phi_1^\dagger \Phi_2\right) + \eta_7 \operatorname{Tr}\left(\Phi_2^\dagger \Phi_2\right) \operatorname{Tr}\left(\Phi_1^\dagger \Phi_2\right) + \text{h.c.}\right].$$

$Z(2)$ symmetry ($\Phi_n \to -\Phi_n$) is relaxed in 2HDM III.
All parameters considered to be real for simplicity.

Lattice action

Fields and couplings made dimensionless with powers of lattice spacing

Reality of couplings and $\mathrm{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2^\dagger(x)\right)$ allows to remove one coupling

$$
\begin{aligned}
S_{\mathrm{2HDM}} &= S_{\mathrm{YM}}(U_\mu(x);\beta) + \sum_x \sum_{n=1}^{2} \left\{ \sum_\mu -2\kappa_n \,\mathrm{Tr}\left(\hat{\Phi}_n^\dagger(x) U_\mu(x) \hat{\Phi}_n(x+\hat{\mu})\right) \right. \\
&\left. + \mathrm{Tr}\left(\hat{\Phi}_n^\dagger(x)\hat{\Phi}_n(x)\right) + \eta_n \left[\mathrm{Tr}\left(\hat{\Phi}_n^\dagger(x)\hat{\Phi}_n(x)\right) - 1\right]^2 \right\} + 2\mu^2 \,\mathrm{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2(x)\right) \\
&+ \xi_2 \,\mathrm{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2(x)\right)^2 + \xi_1 \,\mathrm{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_1(x)\right) \mathrm{Tr}\left(\hat{\Phi}_2^\dagger(x)\hat{\Phi}_2(x)\right) \\
&+ 2\,\mathrm{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_2(x)\right) \left[\xi_3 \,\mathrm{Tr}\left(\hat{\Phi}_1^\dagger(x)\hat{\Phi}_1(x)\right) + \xi_4 \,\mathrm{Tr}\left(\hat{\Phi}_2^\dagger(x)\hat{\Phi}_2(x)\right)\right],
\end{aligned}
$$

Lattice action with **10 couplings**

Observables / Simulation

Local observables:

- Plaquette $P = \frac{1}{12V} \sum_{\Box} \mathrm{Tr}\{U_{\Box}\}$
- Higgs length $\rho^2 = \frac{1}{V} \sum_x \rho^2(x)$
- Gauge-invariant links
  $L_\phi = \frac{1}{8V} \sum_{x,\mu} \mathrm{Tr}\{\phi^\dagger(x) U_\mu(x) \phi(x+\hat{\mu})\}$
  $L_\alpha = \frac{1}{8V} \sum_{x,\mu} \mathrm{Tr}\{\alpha^\dagger(x) U_\mu(x) \alpha(x+\hat{\mu})\}$
- Susceptibilities $\langle \chi_O \rangle = \langle (O - \langle O \rangle)^2 \rangle$

Interpolators for Higgs and W boson states:

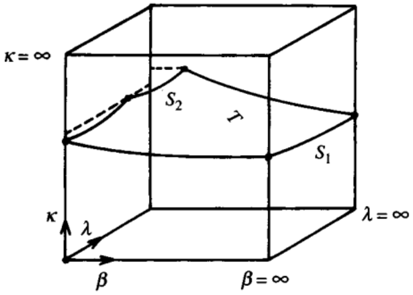
$$H(\vec{x}, t) = \sum_{\mu=1}^{3} \mathrm{Tr}\left(\Phi^\dagger(x) U_\mu(x) \Phi(x+\hat{\mu})\right)$$

$$W_{i,\mu}(\vec{x}, t) = \mathrm{Tr}\left(\Phi^\dagger(x) U_\mu(x) \Phi(x+\hat{\mu}) \tau_i\right)$$

- Hybrid Monte Carlo code developed in Julia parallelised for GPUs
- Results independently checked with Metropolis algorithm
- First study with full Higgs potential (10 real coupling constants)
- Previous study with symmetric Higgs potential and only one mixing term

$N_h = 1$ phase diagram

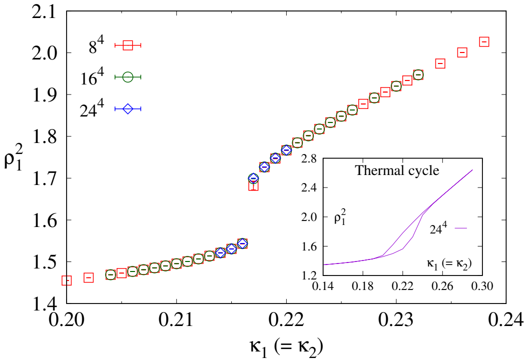
Single Higgs doublet

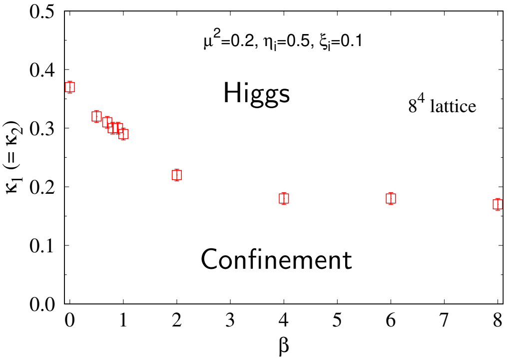


[Quantum Fields on a Lattice, Montvay & Münster, CUP 1994]

- $N_h = 1$ case well-known
  first-order PT at finite β and λ
  crossover at small β and large λ
  confinement and Higgs phases analytically connected
- $N_h = 2$ symmetric Higgs case showed first-order surface covers the entire phase diagram
- Important to look for a critical point where continuum limit may be taken

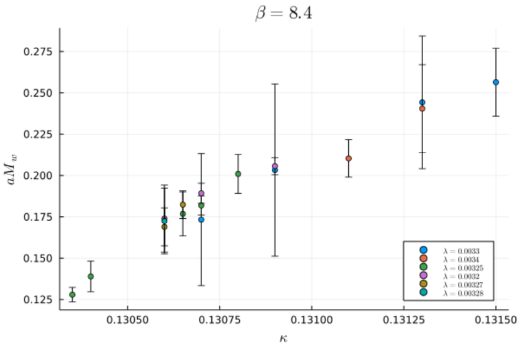
Preliminary phase diagram

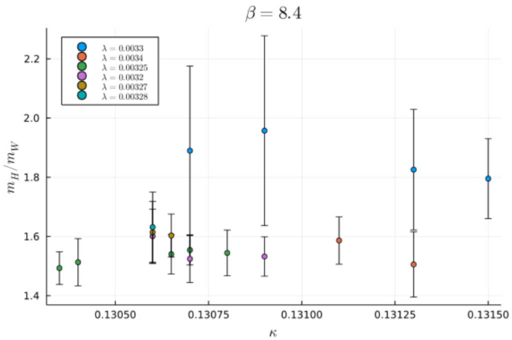




Evidence of first-order transition which gradually weakens with increasing gauge coupling β

Spectrum dependence on couplings in single Higgs model [Preliminary]



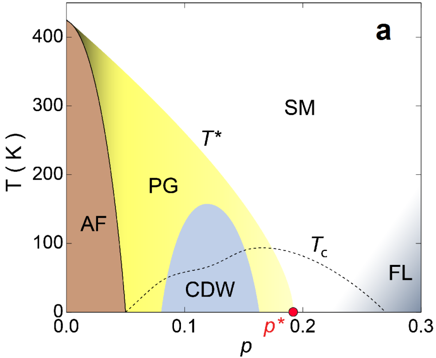


- smearing of scalar fields and gradient flow of gauge links for improving signal
- lattice spacing increases with κ at fixed β in Higgs phase,
- for $aM_W \sim 0.2$, the cutoff scale $\Lambda = 1/a \sim 400$ GeV

Ongoing and future work

- Spectrum of exotic scalar particles in the Higgs phase at the scale of physical W and SM Higgs
- Dependence of the spectrum on the couplings
- Searching the large phase diagram for critical points
- Nature of the electroweak phase transition at finite temperature

Motivation



Doping density

[C. Proust and L. Taillefer, Annu. Rev. Condens. Matter Phys. 2019. 10:409–29]

- Cuprates show superconductivity exceeding 150 K $\sim$ half of room temperature
- Large magnetic field suppresses superconductivity
- Underlying states still not well understood
- Attempts to explain the critical point and surrounding phases as fluctuations of spin density wave order (SDW)

Proposal

Electron spin magnetic moment in spacetime dependent rotating reference frame $\boldsymbol{\sigma} \cdot \boldsymbol{S}_i = R_i \boldsymbol{\sigma} R_i^\dagger \cdot \boldsymbol{H}_i$

$\boldsymbol{H}_i$ transform under adjoint representation of $SU(2)$

Parametrize : $\boldsymbol{H}_i = \mathrm{Re}\left[\boldsymbol{\mathcal{H}}_x e^{i\boldsymbol{K}_x \cdot \boldsymbol{r}_i} + \boldsymbol{\mathcal{H}}_y e^{i\boldsymbol{K}_y \cdot \boldsymbol{r}_i}\right]$

[S. Sachdev, H. D. Scammell, M. S. Scheurer, G. Tarnopolsky, Phy Rev B 99, 054516 (2019)]

$\boldsymbol{\mathcal{H}}_x, \boldsymbol{\mathcal{H}}_y$: complex fields
$\rightarrow$ 4 adjoint Higgs

$$\mathcal{L}_{\mathcal{H}} = \frac{1}{4g^2}\boldsymbol{F}_{\mu\nu} \cdot \boldsymbol{F}_{\mu\nu} + |\partial_\mu \boldsymbol{\mathcal{H}}_x - \boldsymbol{A}_\mu \times \boldsymbol{\mathcal{H}}_x|^2 + |\partial_\mu \boldsymbol{\mathcal{H}}_y - \boldsymbol{A}_\mu \times \boldsymbol{\mathcal{H}}_y|^2 + V(\boldsymbol{\mathcal{H}}_{x,y})$$

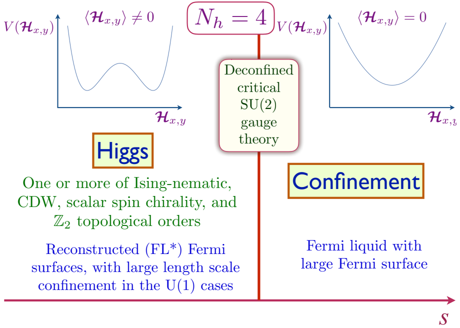
$$V(\boldsymbol{\mathcal{H}}_{x,y}) = s\left(\boldsymbol{\mathcal{H}}_x^* \cdot \boldsymbol{\mathcal{H}}_x + \boldsymbol{\mathcal{H}}_y^* \cdot \boldsymbol{\mathcal{H}}_y\right) + u_0\left(\boldsymbol{\mathcal{H}}_x^* \cdot \boldsymbol{\mathcal{H}}_x + \boldsymbol{\mathcal{H}}_y^* \cdot \boldsymbol{\mathcal{H}}_y\right)^2 + \frac{u_1}{4}\phi^2 + \frac{u_2}{2}\left(|\Phi_x|^2 + |\Phi_y|^2\right) + u_3\left(|\Phi_+|^2 + |\Phi_-|^2\right)$$

($u_1 = u_2 = u_3 \rightarrow$ global $O(4)$ symmetry)

Gauge-invariant bilinears

$$\phi = |\boldsymbol{\mathcal{H}}_x|^2 - |\boldsymbol{\mathcal{H}}_y|^2\,,$$
$$\Phi_x = \boldsymbol{\mathcal{H}}_x \cdot \boldsymbol{\mathcal{H}}_x\,, \Phi_y = \boldsymbol{\mathcal{H}}_y \cdot \boldsymbol{\mathcal{H}}_y\,,$$
$$\Phi_+ = \boldsymbol{\mathcal{H}}_x \cdot \boldsymbol{\mathcal{H}}_y\,, \Phi_- = \boldsymbol{\mathcal{H}}_x \cdot \boldsymbol{\mathcal{H}}_y^*$$

Predictions and Previous work

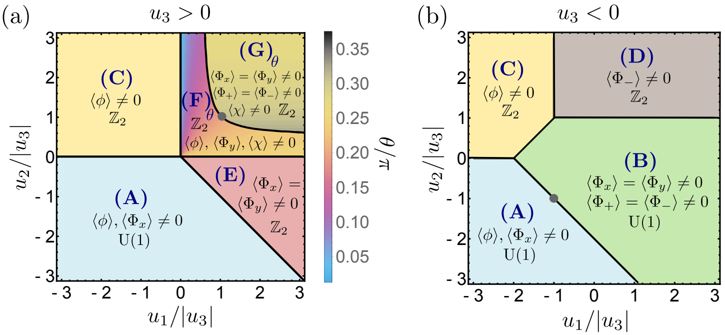


[S. Sachdev, H. D. Scammell, M. S. Scheurer, G. Tarnopolsky, Phy. Rev. B 99, 054516 (2019)]

- Higgs phase hosts different broken symmetries which resemble hole-doped cuprates
- $SU(2)$ broken to either $U(1)$ or $Z(2)$
- Recent numerical studies with the $O(4)$ symmetric potential finds the two patterns of symmetry breaking

  [H. D. Scammell, K. Patekar, M. S. Scheurer, S. Sachdev, Phy. Rev. B 101, 205124 (2020)]

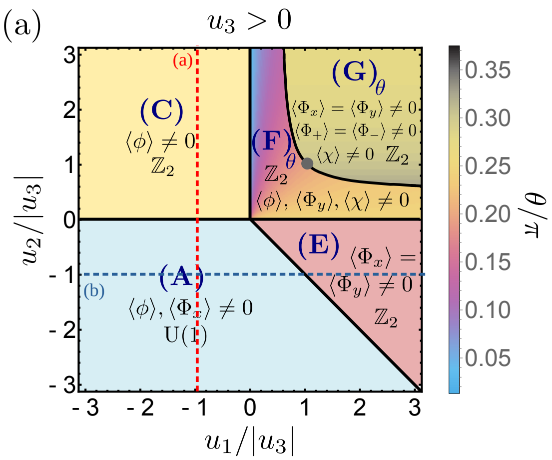
  [C. Bonati, A. Franchi, A. Pelissetto, E. Vicari, Phy. Rev. B 104, 115166 (2021)]

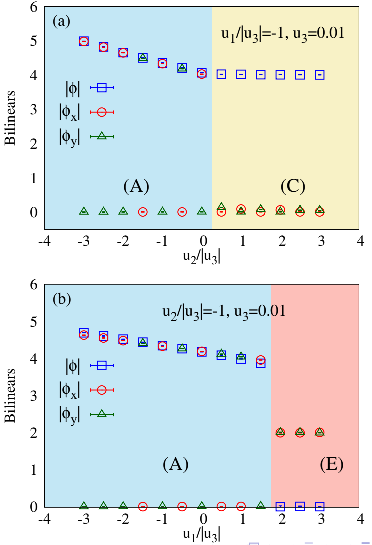


[S. Sachdev, H. D. Scammell, M. S. Scheurer, G. Tarnopolsky, Phy. Rev. B 99, 054516 (2019)]

- first-time numerical study of complete phase diagram
- using Hybrid Monte Carlo Algorithm
- CPU code with MPI parallelization
- preliminary study on 12^3 lattices, s chosen negative and u_0 positive enough to be in the broken phase

Preliminary results $u_3 > 0$





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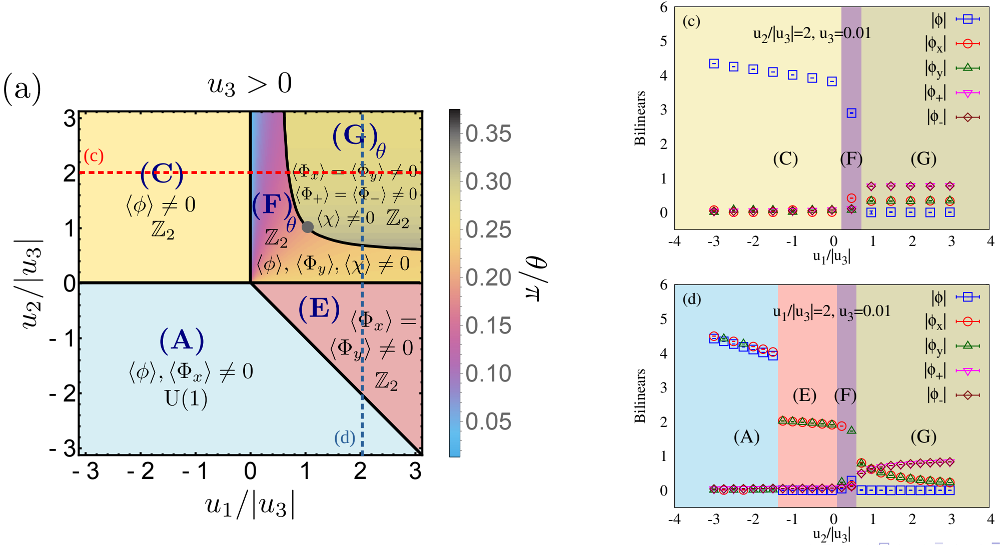

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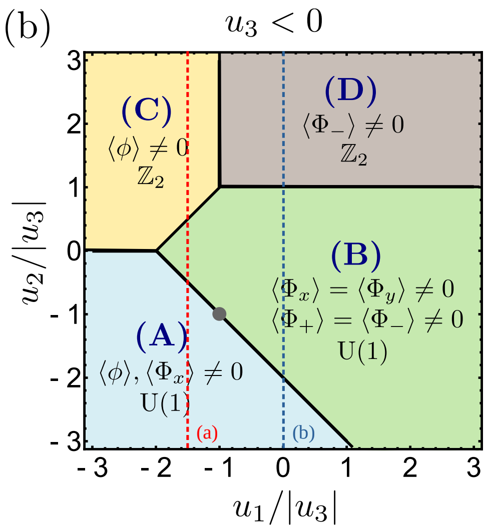

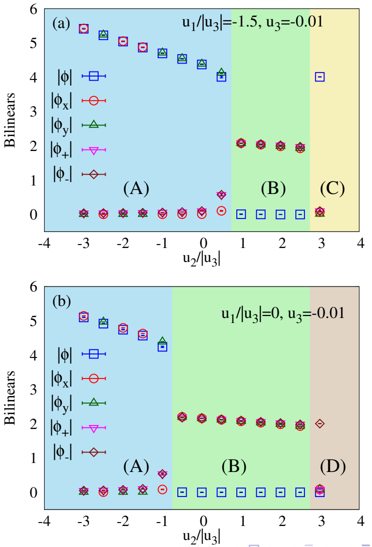

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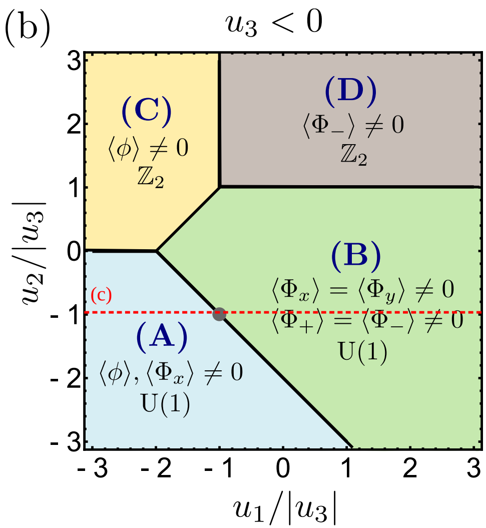

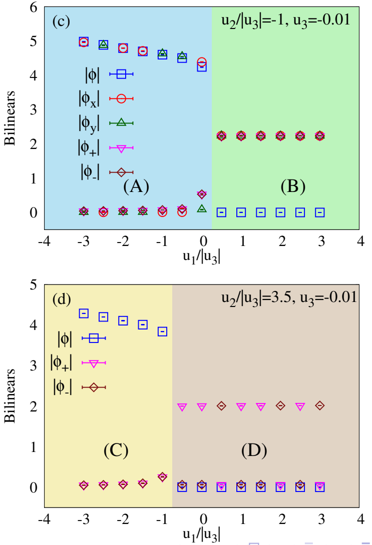

Conclusions and Outlook

$4d$ SU(2) gauge theory with 2 fundamental Higgs doublet

- First-time study of the complete potential with real couplings in 4d
- Preliminary study of the phase diagram and scale setting
- Study of the exotic spectrum currently ongoing

$3d$ SU(2) gauge theory with 4 adjoint Higgs

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- Preliminary numerical results qualitatively confirm the mean-field expectations
- Future investigation of the purported deconfined quantum critical point and topological orders

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Thank you for your attention!