Gravitational Positivity and (Beyond) the Standard Model

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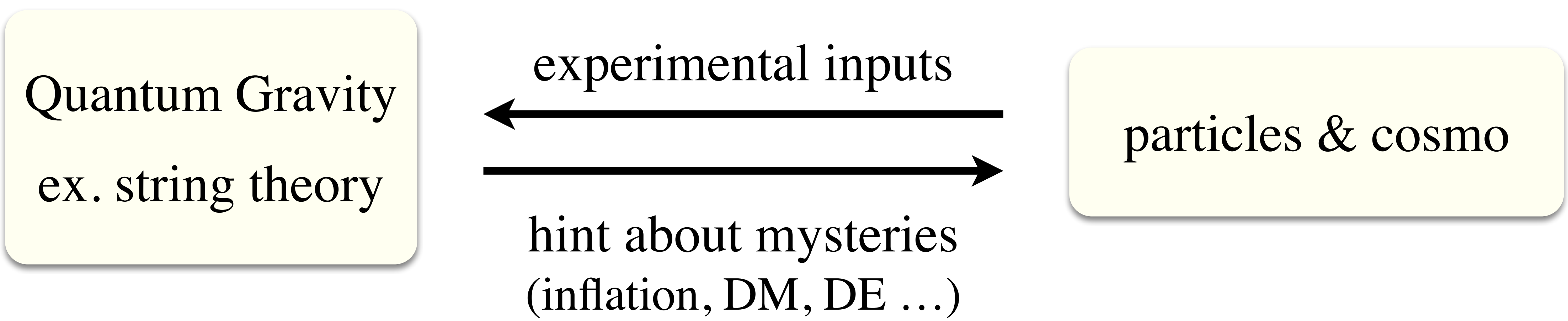
Dec 16th 2022@NCTS ATM

My motivation in this talk:

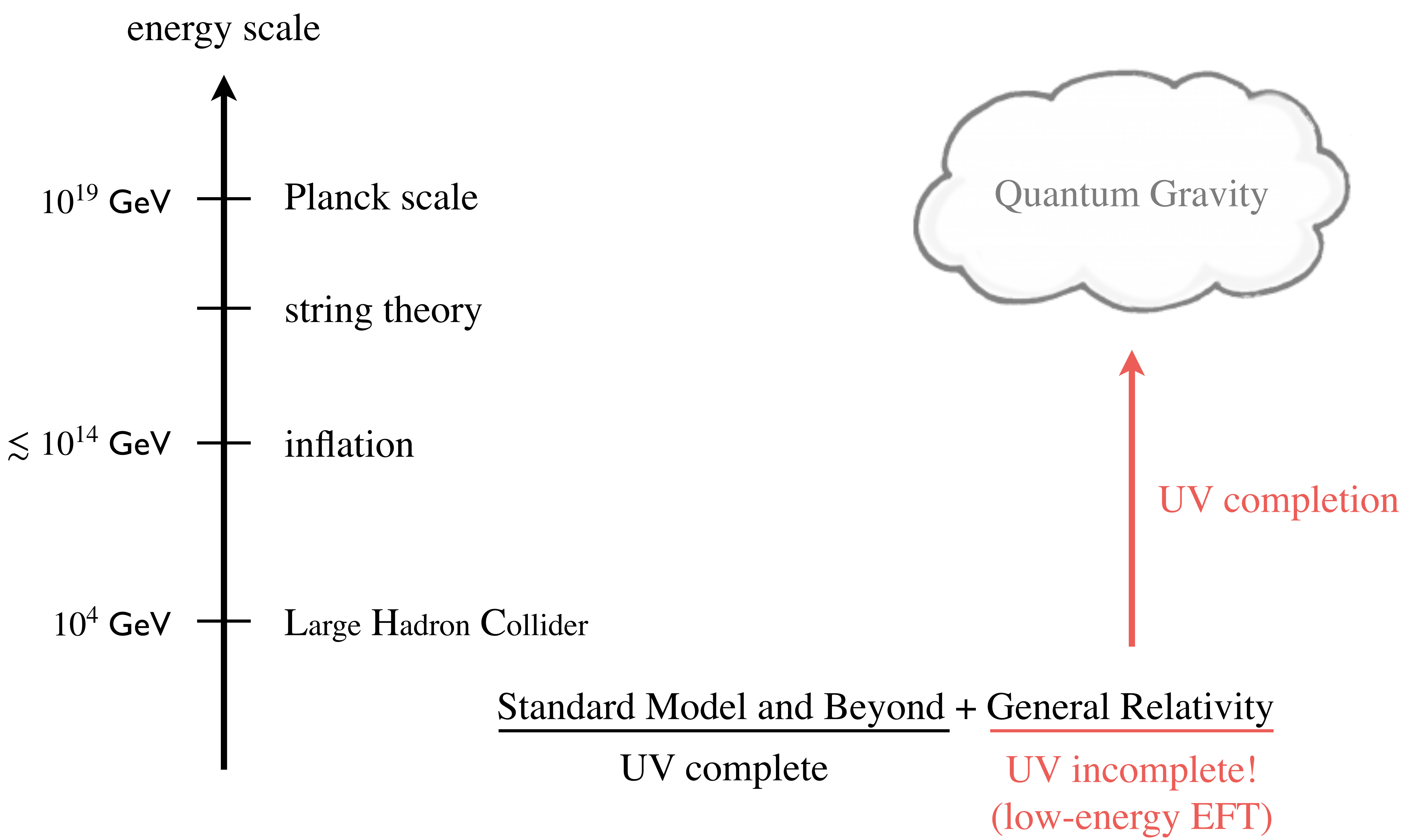
I would like to explore possible interplay

between quantum gravity and pheno (particle physics, cosmology).



Lessons from string theory as a quantum gravity theory!

Particle Physics & Cosmology (QFT + GR)



Particle Physics & Cosmology based on string theory

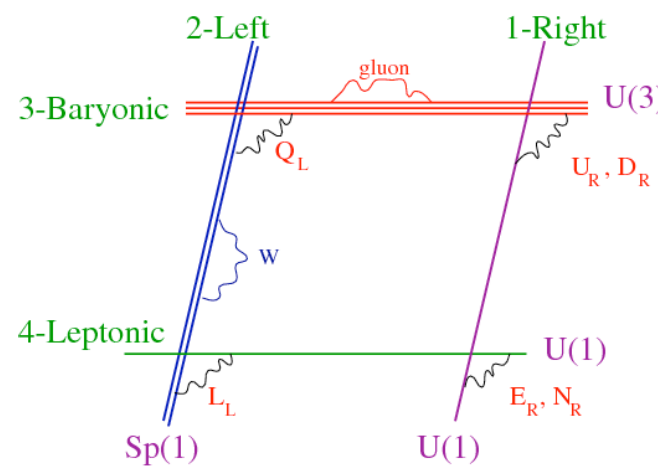
energy scale

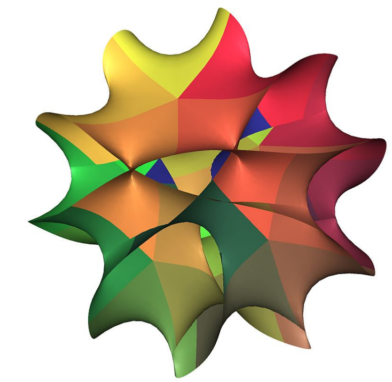
10^{19} GeV — Planck scale

string theory

$\lesssim 10^{14}$ GeV — inflation

10^{4} GeV — Large Hadron Collider





string theory (1+9 dims.)

extra dimensions
brane configurations

UV completion

Standard Model and Beyond + General Relativity

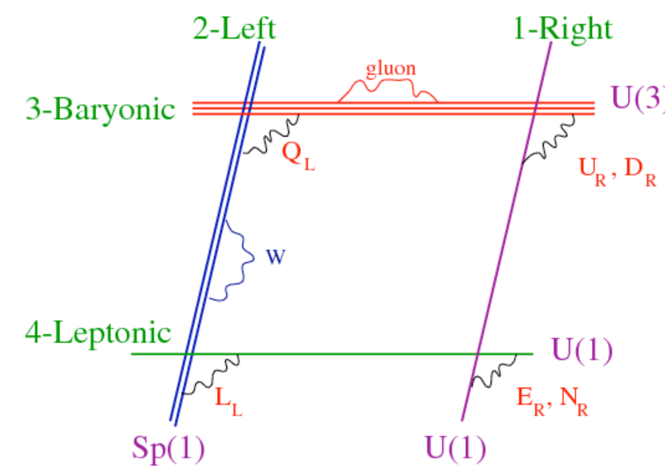
UV complete

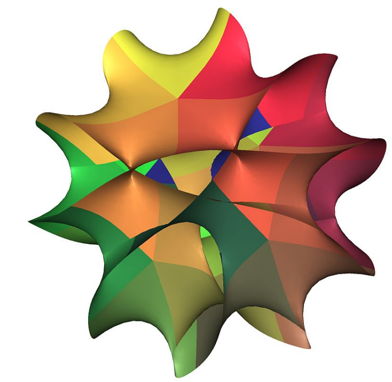
UV incomplete!
(low-energy EFT)

Particle Physics & Cosmology based on string theory

energy scale

10^{19} GeV — Planck scale





Q. What kind of models of particle physics & cosmology are realized in string theory?

→ generic predictions/typicality of string theory, more generally quantum gravity

extra dimensions

brane configurations

UV completion

10^4 GeV — Large Hadron Collider

Standard Model and Beyond + General Relativity

UV complete

UV incomplete!
(low-energy EFT)

An interesting lesson:

There exist non-trivial consistency conditions in QG

that are not present in non-gravitational theories.

- absence of (exact) global symmetries
- subPlanckian axion decay constant,
- weak gravity conjecture, distance conjecture, …

$\longrightarrow$ Various proposals for such Swampland conditions.

The history says that consistency of scattering amplitudes is useful to discuss UV completion of IR EFTs.

- prediction of weak bosons, Higgs boson, …
- string theory emerged in the context of the S-matrix theory.

Is the S-matrix theory useful for the Swampland program?

In this talk, I advertise my works in the past two years

- arXiv:2104.09682 w/Katsuki Aoki (YITP), Tran Quang Loc (Cambridge), **Junsei Tokuda** (Kobe → IBS)
- arXiv:2205.12835 w/**Sota Sato** (Kobe), **Junsei Tokuda** (Kobe → IBS)

See also arXiv:2105.01436 w/**Junsei Tokuda** (Kobe → IBS)

on possible implications of the so-called positivity bounds.

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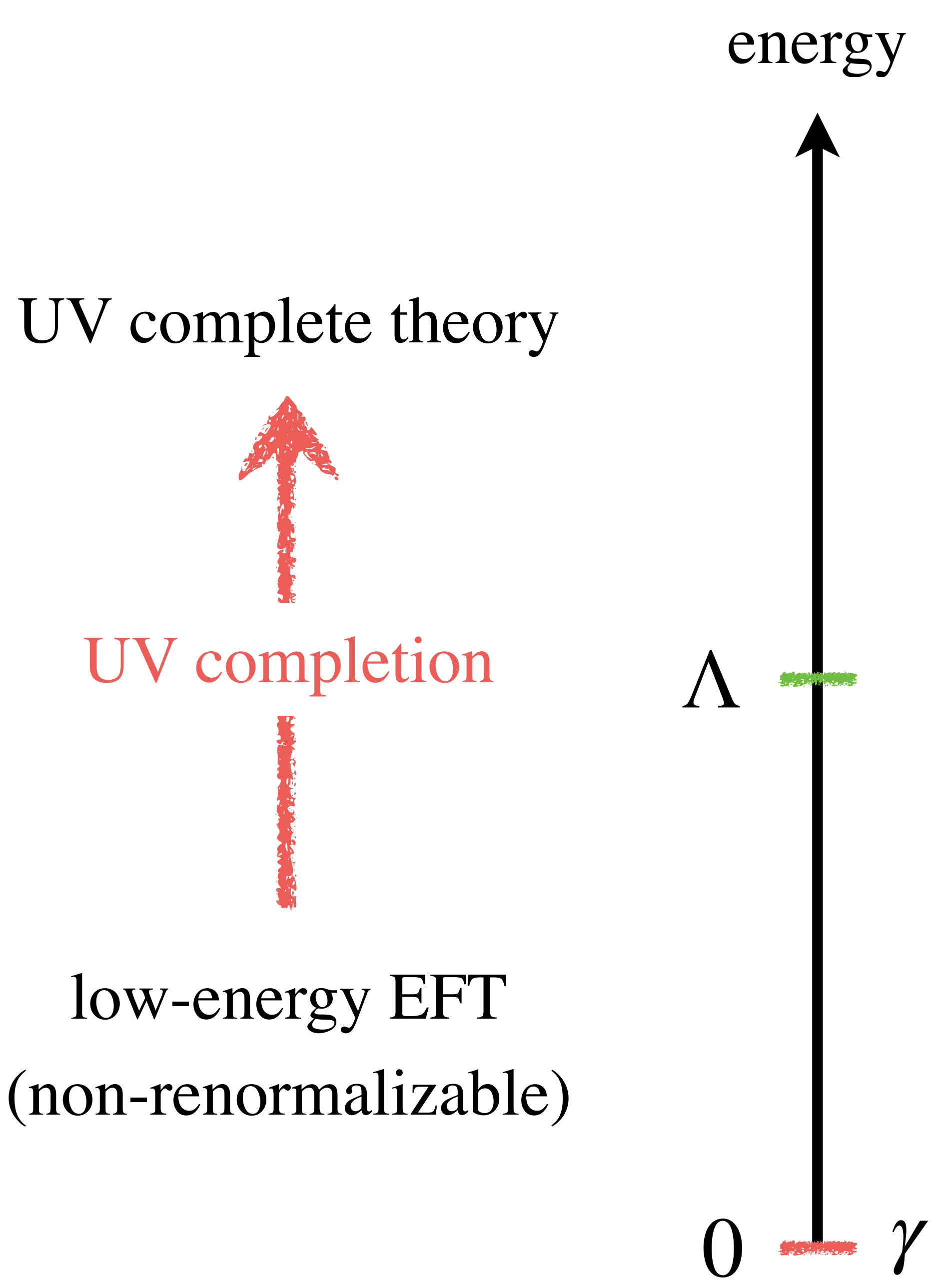
Not every EFT is UV completable even in non-gravitational theories.

A famous criterion is positivity bounds on IR scattering amplitudes.

Positivity Bounds [Adams et al ‘06]



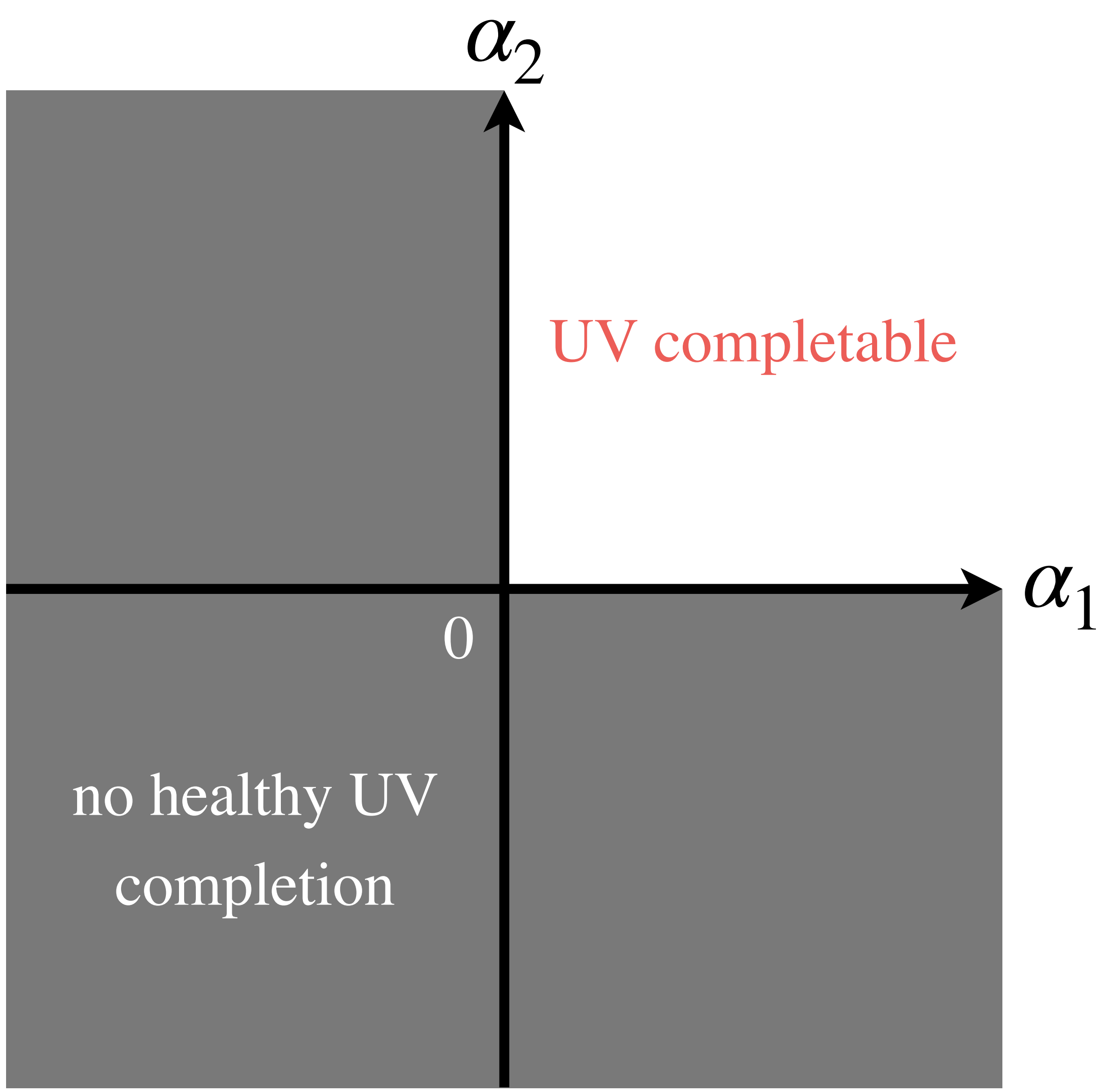
ex. Euler-Heisenberg :

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}^2 + \alpha_1 (F_{\mu\nu}F^{\mu\nu})^2 + \alpha_2 (F_{\mu\nu}\widetilde{F}^{\mu\nu})^2 + \cdots$$

Q. Which parameter region is UV completable?

cf. $\alpha_1 = \dfrac{e^4}{1440\pi^2 m^4}\,, \quad \alpha_2 = \dfrac{7e^4}{5760\pi^2 m^4}$ if the UV theory is QED

Positivity Bounds [Adams et al '06]



Dark region "swampland" cannot be embedded into UV theories with

1. unitary (cross section > 0)
2. analyticity (cf. causality)
3. $|\mathscr{M}(s, t=0)| < s^2$ for $s \to \infty$

※ guaranteed by locality (Froissart bound)

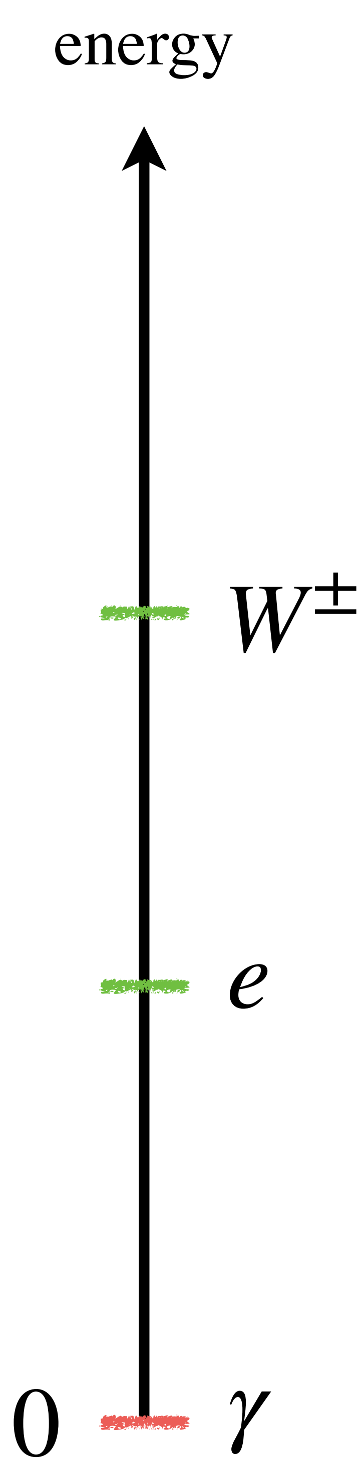
I skip its derivation, but provide an intuitive explanation w/generalization.

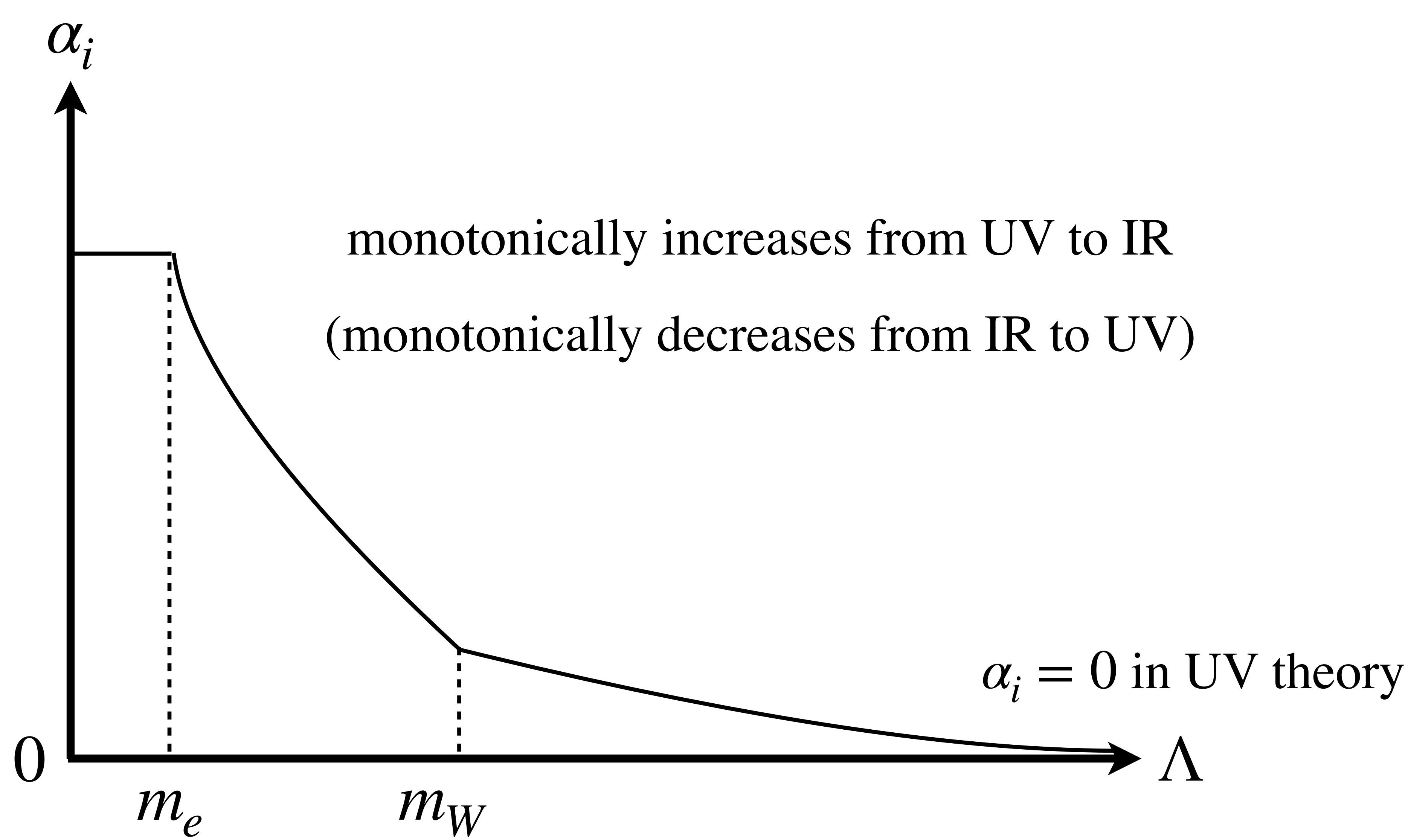
essence of positivity bounds can be captured by the Wilsonian RG picture

Wilsonian RG picture

EFT after integrating out UV modes $E > \Lambda$ (Λ: cutoff):

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}^2 - \mathscr{L}_{\text{charged},E<\Lambda} + \alpha_1(\Lambda)(F_{\mu\nu}F^{\mu\nu})^2 + \alpha_2(\Lambda)(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2 + \cdots$$





We can use dispersion relation to construct a monotonic function of the “cutoff” scale Λ from scattering amplitudes.

Dispersion relation

Consider an s-u crossing amplitude of $\gamma\gamma \to \gamma\gamma$ scattering in the forward limit,

whose low-energy expansion is of the form $\mathscr{M}(s, t=0) = \sum_{n=0}^{\infty} a_{2n} s^{2n}$.

Then, the dispersion relation reads $a_2 = \frac{2}{\pi}\int_{m_{\rm th}^2}^{\infty} ds \frac{{\rm Im}\mathscr{M}(s, t=0)}{s^3}$.

Dispersion relation

Consider an s-u crossing amplitude of $\gamma\gamma \to \gamma\gamma$ scattering in the forward limit, whose low-energy expansion is of the form $\mathcal{M}(s, t=0) = \sum_{n=0}^{\infty} a_{2n} s^{2n}$.

Then, the dispersion relation reads $a_2 = \frac{2}{\pi} \int_{m_{\rm th}^2}^{\infty} ds \frac{\mathrm{Im}\mathcal{M}(s, t=0)}{s^3}$.

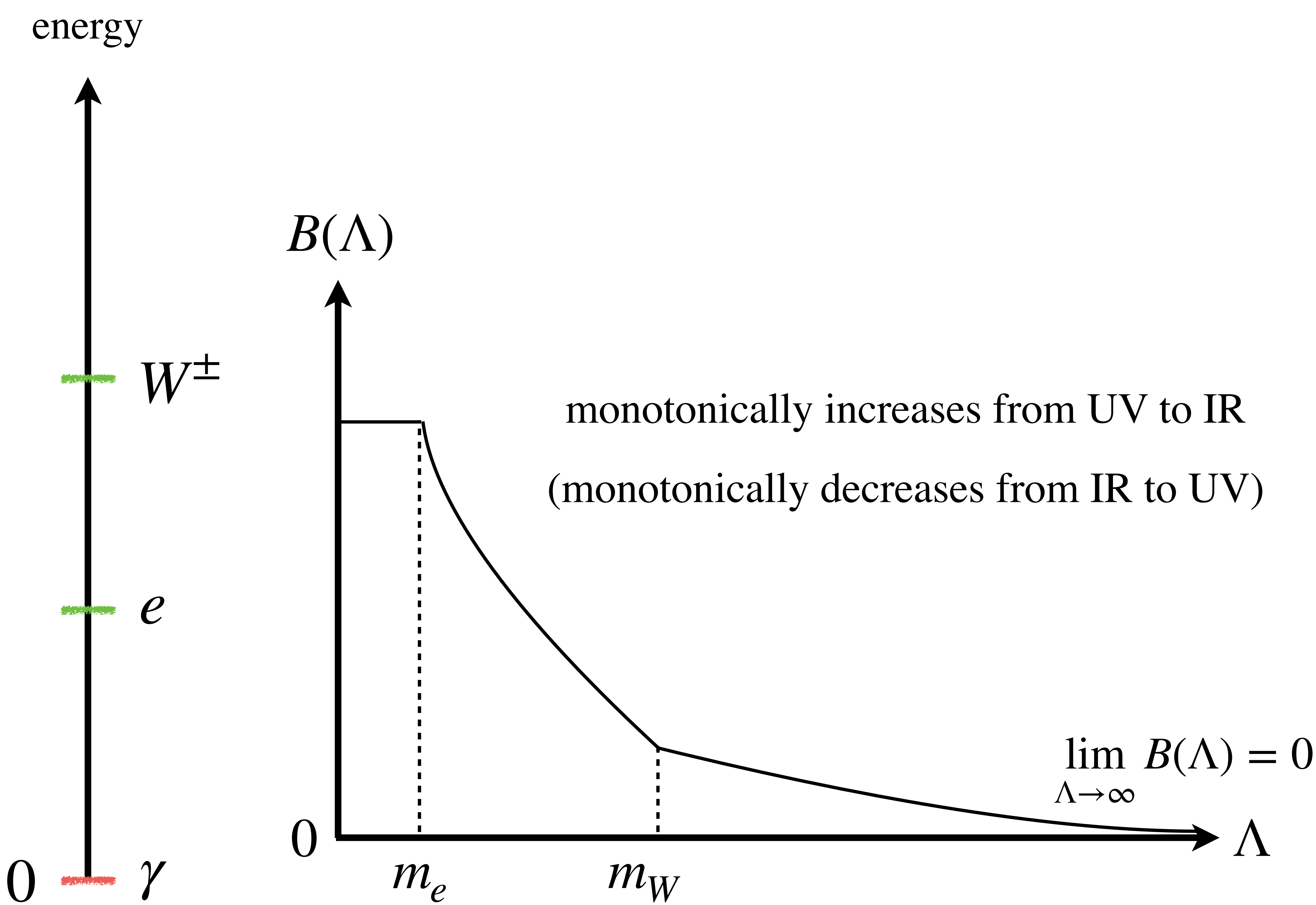
Define a function $B(\Lambda) := a_2 - \frac{2}{\pi} \int_{m_{\rm th}^2}^{\Lambda^2} ds \frac{\mathrm{Im}\mathcal{M}(s, t=0)}{s^3}$.

- $B(\Lambda)$ is defined in terms of scattering amplitudes below the "cutoff" scale Λ.
- $B(\Lambda)$ monotonically decreases as Λ increases, since $\mathrm{Im}\mathcal{M}(s, t=0) \geq 0$.

Then, the dispersion relation implies $B(\Lambda) = \frac{2}{\pi} \int_{\Lambda^2}^{\infty} ds \frac{\mathrm{Im}\mathcal{M}(s, t=0)}{s^3} \geq 0$.

- $\lim_{\Lambda \to \infty} B(\Lambda) = 0$ if the forward amplitude is bounded by s^2.

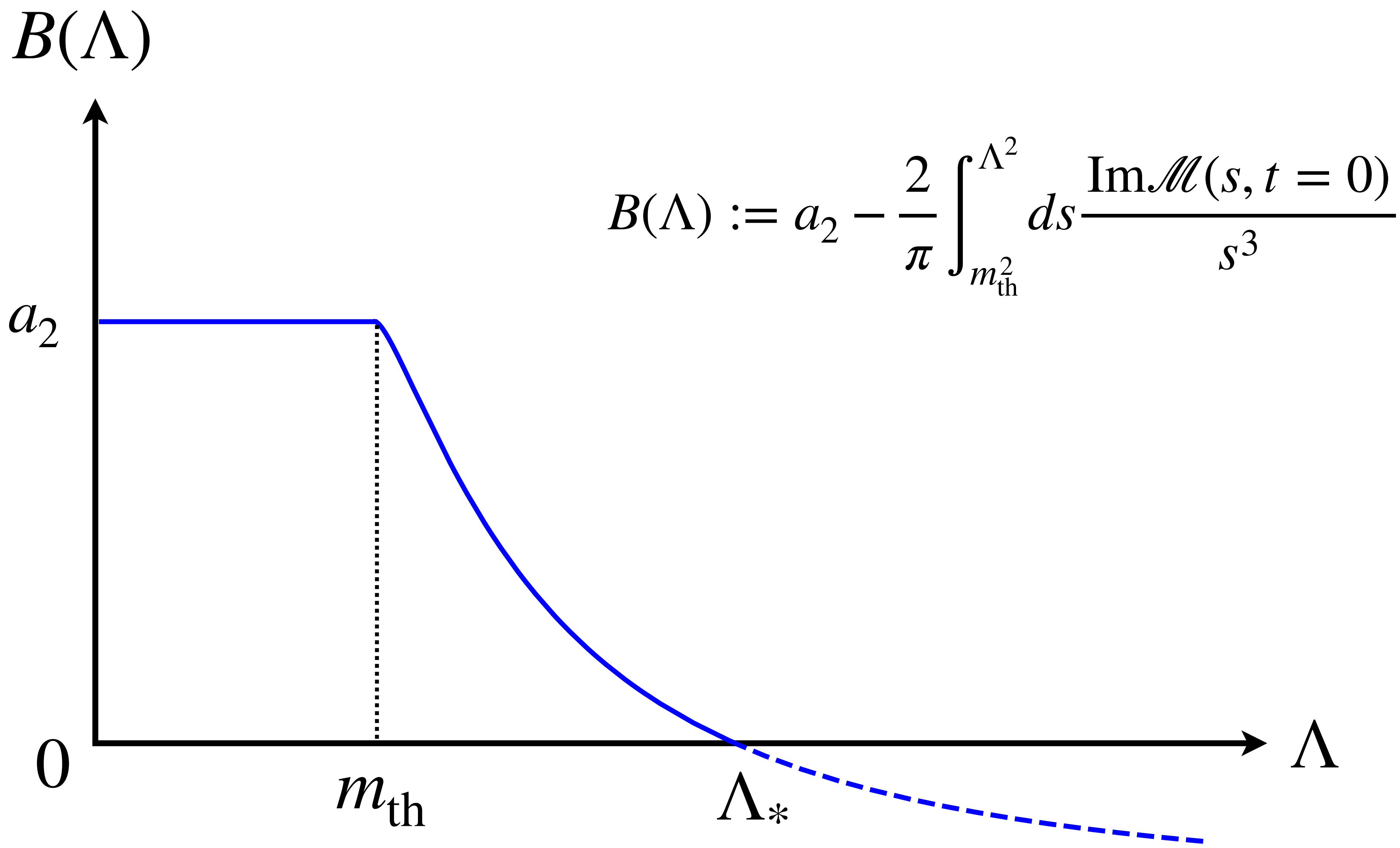
Analogy with Wilsonian RG



Improved positivity bounds [Bellazini '16, de Rham et al '17]:

Identify the EFT cutoff by evaluating $B(\Lambda)$ and extrapolating from IR to UV!

Improved Positivity Bounds

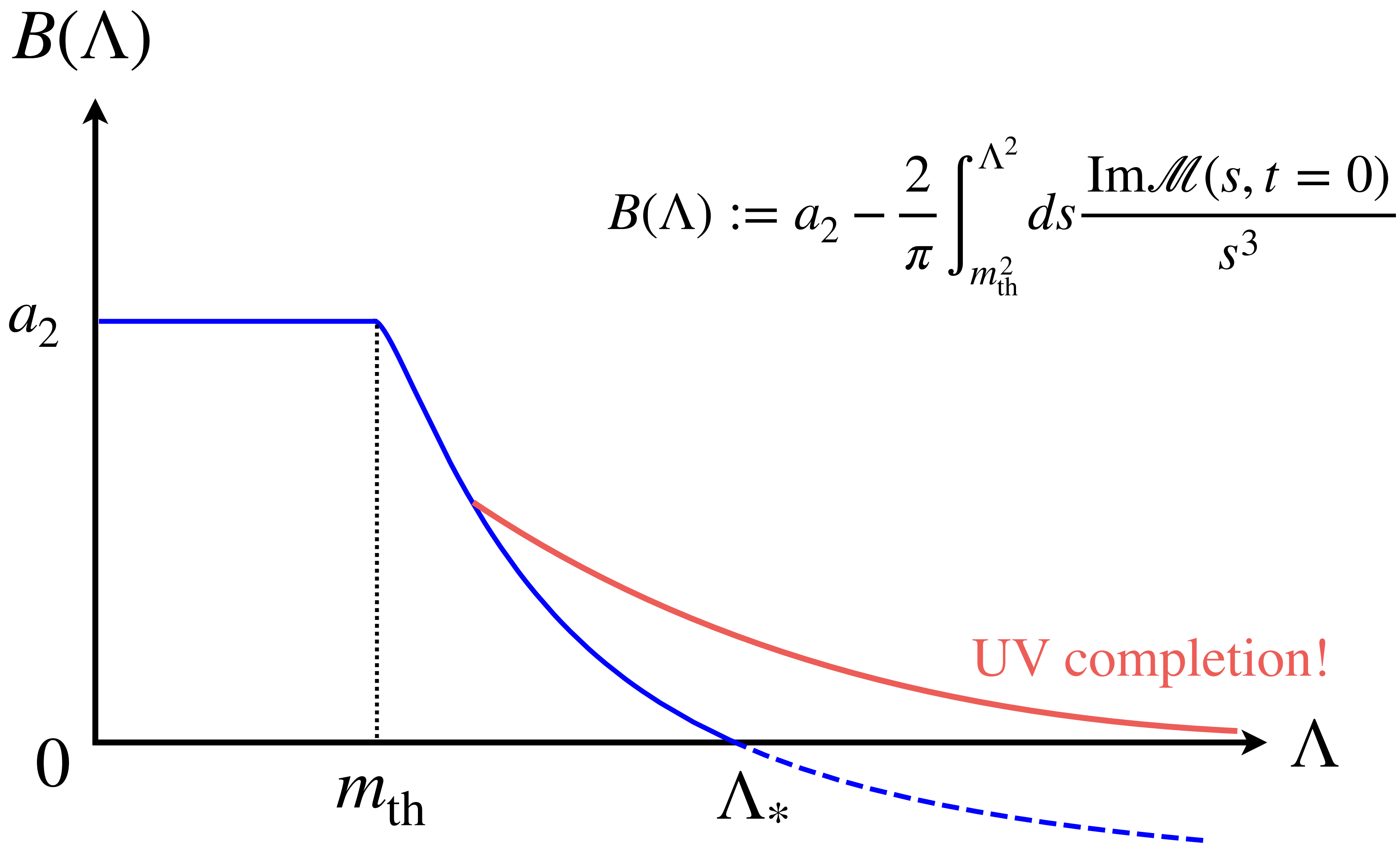


- The EFT breaks down once $B(\Lambda)$ becomes negative.

  $\rightarrow B(\Lambda_*) = 0$ defines the maximum cutoff Λ_*.

- UV completion is required below Λ_*.

Improved Positivity Bounds



- The EFT breaks down once $B(\Lambda)$ becomes negative.

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It would be nice if we can apply those techniques to the Swampland Program.

Recent studies on gravitational EFTs show

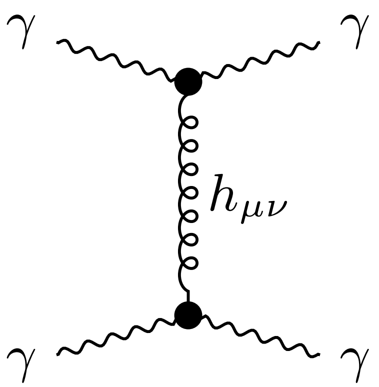
that positivity bounds hold even in gravity theories at least approximately.

See, e.g., Hamada-TN-Shiu '18, Herrero-Valea et al '20, Bellazzini et al'19,
Alberte et al '20, Tokuda-Aoki-Hirano '20, Arkani-Hamed et al '20, Caron-Huot et al '21.

Gravitational effects at IR

For concreteness, let us imagine the graviton-photon EFT:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \alpha_1 (F_{\mu\nu} F^{\mu\nu})^2 + \alpha_2 (F_{\mu\nu} \widetilde{F}^{\mu\nu})^2 + \cdots \right]$$



- the IR expansion includes graviton poles

$$\mathcal{M}(s,t) = \frac{su}{M_{\rm Pl}^2 t} + \frac{tu}{M_{\rm Pl}^2 s} + \frac{ts}{M_{\rm Pl}^2 u} + \sum_{n,m} c_{n,m} s^n t^m .$$

※ I ignore massless loops for simplicity [cf. Herrero-Valea et al '20].

- in the forward limit, the t-channel graviton exchange dominates:

$$\mathcal{M}(s,t) \simeq -\frac{s^2}{M_{\rm Pl}^2 t} + \sum_n c_{n,0} s^n + \mathcal{O}(t).$$

※ The residue of the t-channel pole is s^2 due to the spin 2 nature of graviton.

※ Positivity of the s^2 coefficient does not follow in a straightforward manner.

Gravitational positivity bounds [Tokuda-Aoki-Hirano '20]

Define $B(\Lambda) := c_{2,0} - \frac{2}{\pi}\int_{m_{\rm th}^2}^{\Lambda^2} ds \frac{\mathrm{Im}\mathscr{M}(s,t=0)}{s^3}$ w/monotonic cutoff dependence.

Then, one can show $B(\Lambda) \gtrsim 0$ under the standard assumptions of positivity.

One can quantify "$\gtrsim$" in terms of gravitational Regge amplitudes at UV.

[See Tokuda-Aoki-Hirano '20 for details]

In this talk, I just parameterize it as $B(\Lambda) \geq \pm \frac{1}{M_{\rm Pl}^2 M^2}$.

- In tree-level string theory, we have $M \sim M_{\rm string}$ [cf. Hamada-**TN**-Shiu '18].

  cf. [Caron-Huot et al '21] based on crossing symmetry in 5D and higher

- It is an open problem to identify the scale M for loops, especially in 4D.
- We will find that the scale M is crucial for phenomenological application.

Contents

In [Aoki-Loc-TN-Tokuda ’21],

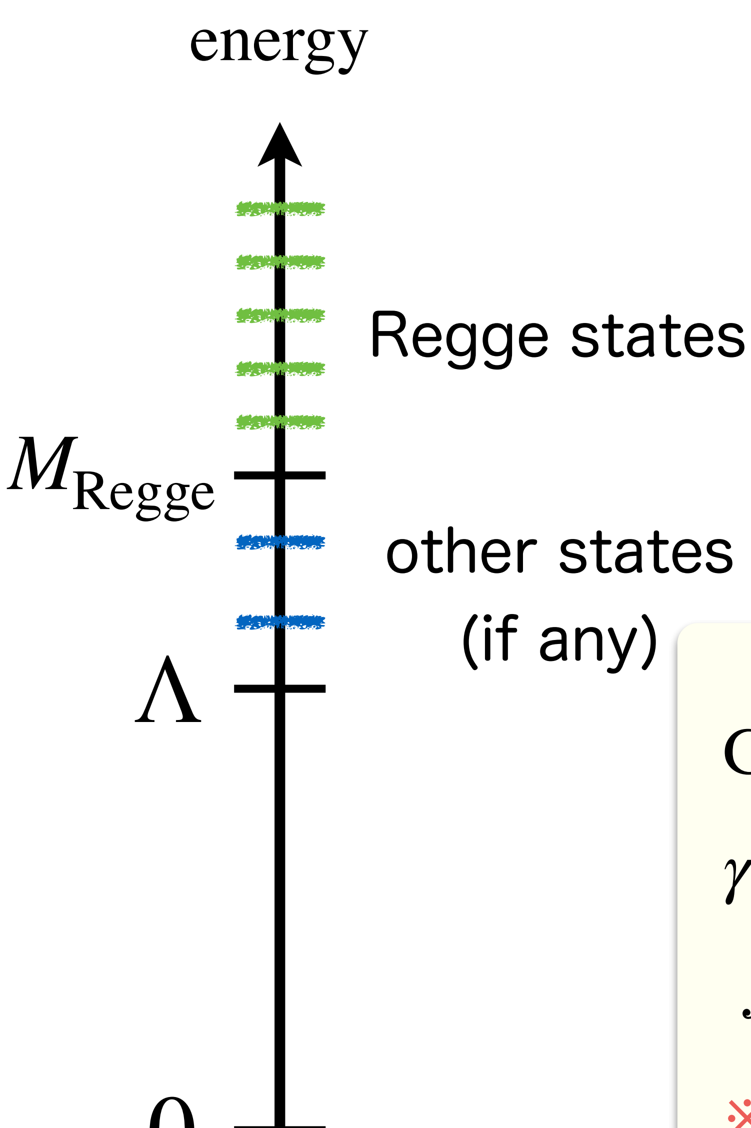
we studied gravitational positivity bounds on the Standard Model,

extending an earlier work [Alberte-de Rham-Jaitly-Tolley ’20] on QED.

cf. earlier works on positivity bounds vs charged particle spectrum

[Cheung-Remmen ’14, Andriolo-Junghans-**TN**-Shiu ’18, Chen-Huang-**TN**-Wen ’19, …]

Gravitational Standard Model



UV completable?

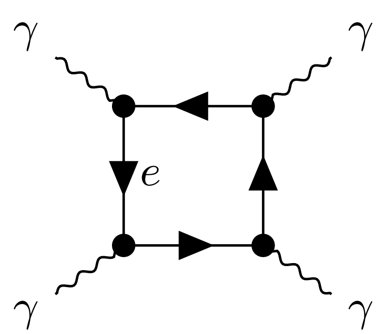
Where is the cutoff?

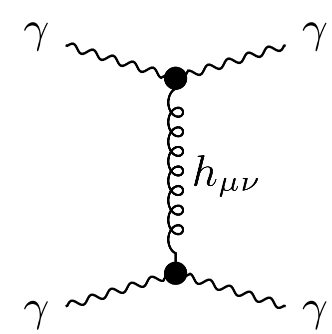
Gravitational Standard Model: $\mathscr{L} = \mathscr{L}_{\rm SM} + \dfrac{M_{\rm Pl}^2}{2} R + \cdots$

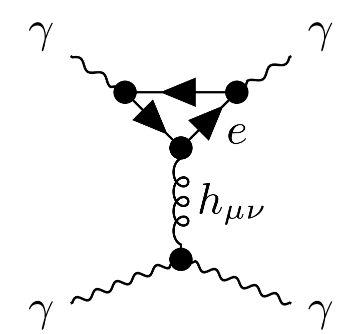
$\gamma\gamma \to \gamma\gamma$ scattering at one-loop:

$$\mathscr{M} = \mathscr{M}_{\rm QED} + \mathscr{M}_{\rm weak} + \mathscr{M}_{\rm QCD} + \mathscr{M}_{\rm GR} + \mathscr{M}_{\rm UV}$$

※ The amplitude grows as s^2, hence it is an IR EFT.

ex. $\mathscr{M}_{\rm QED} =$ 

, $\mathscr{M}_{\rm GR} \simeq$ 

$+$ 

Gravitational electroweak theory (w/o QCD)

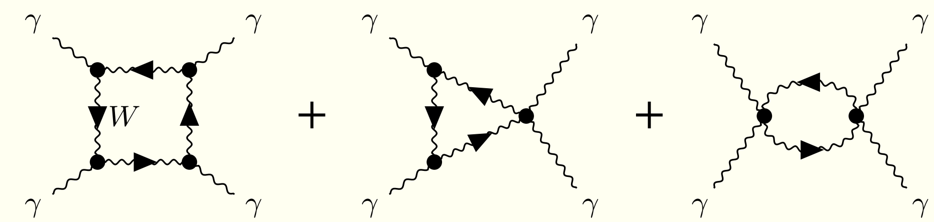
[Aoki-Loc-**TN**-Tokuda '21]

Evaluation of $B(\Lambda)$

1. Non-gravitational contributions to $B(\Lambda)$:

- QED contribution: $B_{\text{QED}}(\Lambda) = \dfrac{2e^4}{\pi^2\Lambda^4}\left(\ln\frac{\Lambda}{m} - \frac{1}{4}\right)$

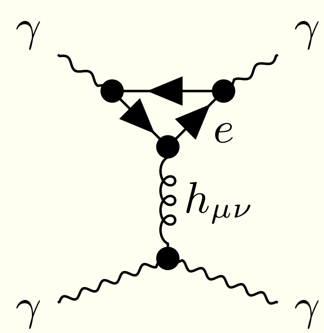
- weak sector: $B_{\text{weak}}(\Lambda) = \dfrac{4e^4}{\pi^2 m_W^2 \Lambda^2}$



※ W boson contributions are dominant because of the spin 1 nature.

2. Gravitational contributions to $B(\Lambda)$:

$$B_{\text{GR}}(\Lambda) \simeq -\frac{11e^2}{180\pi^2 m_e^2 M_{\text{Pl}}^2}$$



※ The electron loop is the dominant contribution.

※ Gravitational contribution is negative!

$$B(\Lambda) := c_{2,0} - \frac{2}{\pi}\int_{s_*}^{\Lambda^2} ds \frac{\text{Im}\mathscr{M}(s,0)}{s^3}$$

Gravitational Positivity

Gravitational positivity $B(\Lambda) > \pm \dfrac{1}{M_{\rm Pl}^2 M^2}$ implies

$$B_{\rm weak}(\Lambda) + B_{\rm GR}(\Lambda) = \frac{4e^4}{\pi^2 m_W^2 \Lambda^2} - \frac{11e^2}{180\pi^2 m_e^2 M_{\rm Pl}^2} > \pm \frac{1}{M_{\rm Pl}^2 M^2}.$$

Consider the following two cases:

1) $M \gg m_e$

RHS is negligible, so that a nontrivial bound appears:

$$B_{\rm weak}(\Lambda) > -B_{\rm GR}(\Lambda) \;\rightleftarrows\; \frac{m_W}{M_{\rm Pl}} < \sqrt{\frac{720}{11}}\, e \frac{m_e}{\Lambda} \;\rightleftarrows\; \Lambda < \sqrt{\frac{1440}{11}} y_e \sin\theta_W M_{\rm Pl}$$

- Explains the hierarchy between the EW scale and the Planck scale??

- A WGC type bound on the Yukawa coupling and the Weinberg angle.

2) $M \sim m_e$ and RHS is negative → Positivity is trivially satisfied

※ This means that Regge amplitudes highly depend on IR physics,

which seems nontrivial ($M \sim M_{\rm string} \gg m_e$ in tree-level string).

[see also Alberte-de Rham-Jaitly-Tolley ’21]

Gravitational Standard Model

[Aoki-Loc-**TN**-Tokuda ’21]

QCD sector analysis

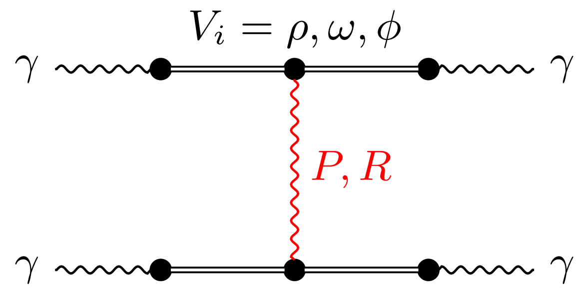
- UV completeness of QCD implies

$$B_{\rm QCD}(\Lambda) = c_{2,0,{\rm QCD}} - \frac{2}{\pi}\int_{m_*^2}^{\Lambda^2} ds \frac{{\rm Im}\mathscr{M}_{\rm QCD}(s,0)}{s^3}$$

$$= \frac{2}{\pi}\left(\int_{m_*^2}^{\infty} - \int_{m_*^2}^{\Lambda^2} 0\right) ds \frac{{\rm Im}\mathscr{M}_{\rm QCD}(s,0)}{s^3} = \frac{2}{\pi}\int_{\Lambda^2}^{\infty} ds \frac{{\rm Im}\mathscr{M}_{\rm QCD}(s,0)}{s^3}$$

- while the amplitude on the r.h.s. is high-energy, the momentum transfer is small

$\rightarrow$ hadron effects in t-channel exchange are relevant

${\rm Im}\mathscr{M}_{\rm QCD} \simeq {\rm Im}$ 

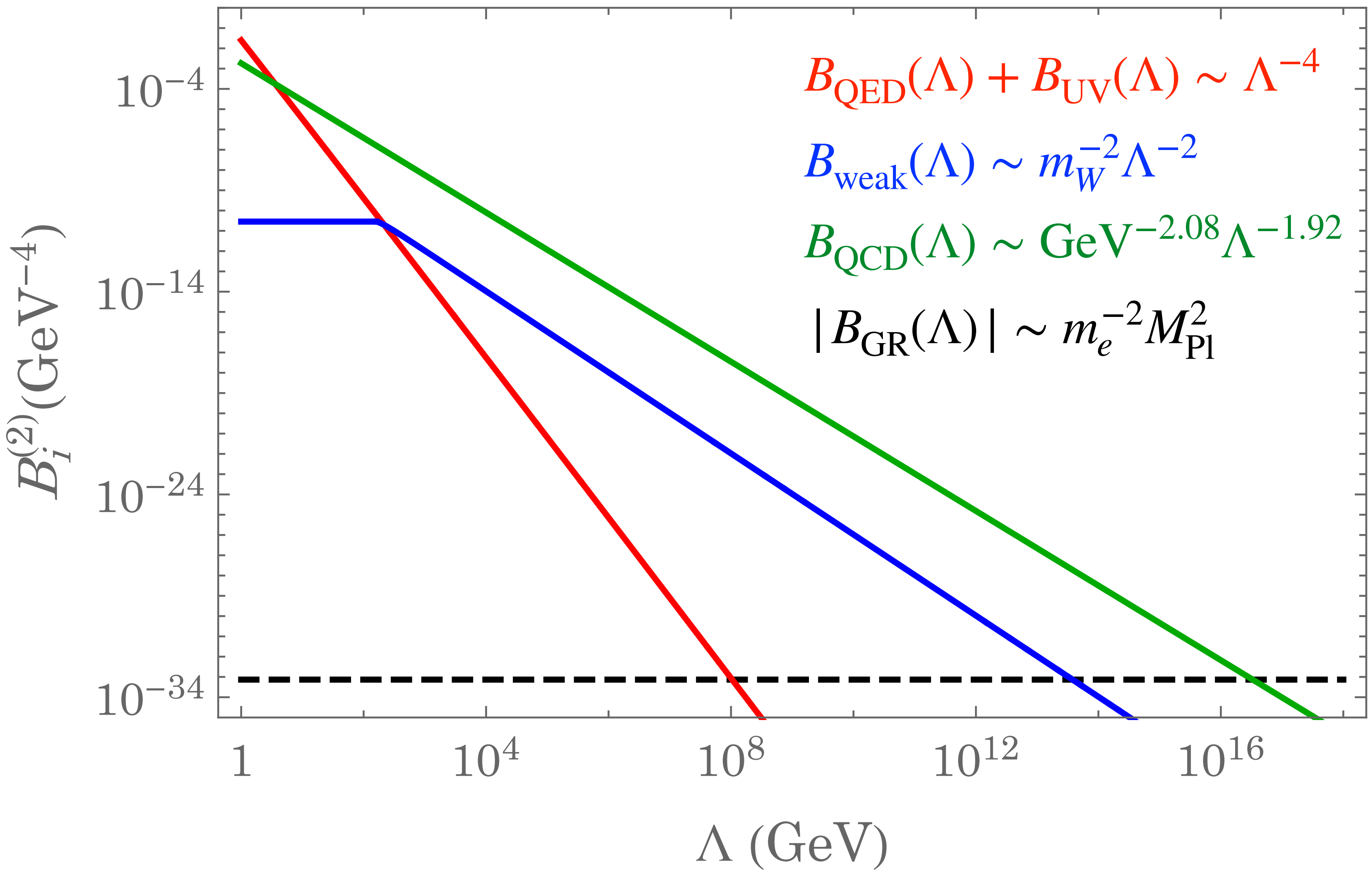
(P: Pomeroon, R: Reggeon)

- extrapolating the Vector Meson Dominance (VDM) model,

$${\rm Im}\mathscr{M}_{\rm QCD} \simeq \frac{25e^4}{16\pi^2}\left(\frac{s}{{\rm GeV}^2}\right)^{1.08}$$

(See our paper for model-(in)sensitivity)

Cutoff scale of gravitational SM



Under the assumption $M \gg m_e$, gravitational positivity implies

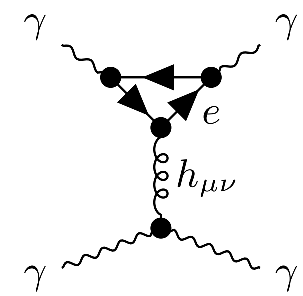
$B_{\rm QED}(\Lambda) + B_{\rm UV}(\Lambda) + B_{\rm weak}(\Lambda) + B_{\rm QCD}(\Lambda) > -B_{\rm GR}(\Lambda)$

$\rightarrow$ this defines the cutoff of the gravitational SM $\Lambda \simeq 3 \times 10^{16}$ GeV.

Summary of the section

We discussed gravitational positivity bounds $B(\Lambda) > \pm \dfrac{1}{M_{\rm Pl}^2 M^2}$ in the SM.

- Negative contributions from GR: $B_{\rm GR}(\Lambda) \simeq -\dfrac{11e^2}{180\pi^2 m_e^2 M_{\rm Pl}^2}$.



- If M is a UV scale, nontrivial constraints on the particle spectrum.
  a) In the EW theory w/o QCD, we found a WGC type bound on Yukawa couplings.
  b) The maximum cutoff is $\Lambda \sim 10^{16}$ GeV, which is reminiscent of grand unification.

- If the sign of RHS is negative and M is an IR scale $M \sim m_e$, no nontrivial constraints,
  but it means the imaginary part of the Regge amplitudes is highly IR-dependent.

[cf. Alberte-de Rham-Jaitly-Tolley '21]

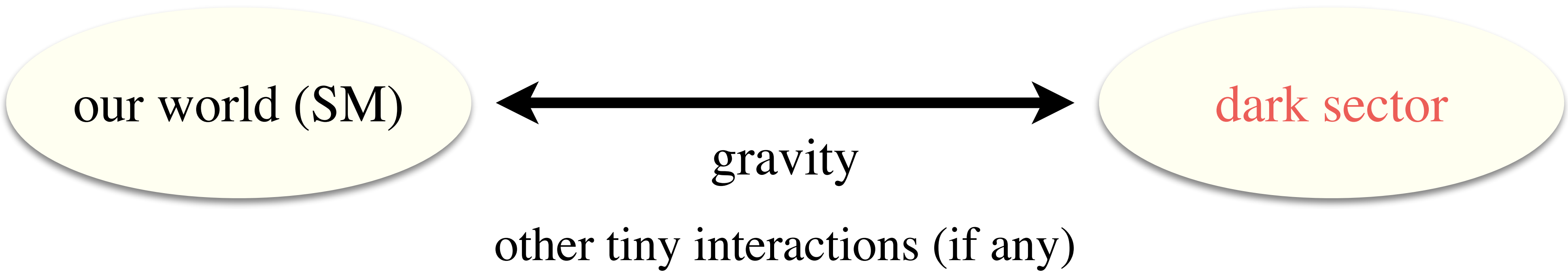
$$B(\Lambda) := c_{2,0} - \frac{2}{\pi}\int_{m_{\rm th}^2}^{\Lambda^2} ds \frac{{\rm Im}\mathcal{M}(s, t=0)}{s^3}$$

Contents

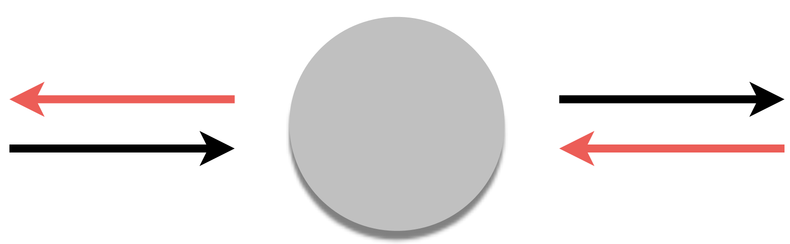
A general consideration about dark sector physics

[Andriolo-Junghans-**TN**-Shiu ’18, **TN**-Sato-Tokuda ’22]

Dark sector cannot be too dark?



- Consider scattering of SM particles and dark sector particles:

 $= \mathscr{M}_{\mathrm{GR}} + \mathscr{M}_{\mathrm{others}}$

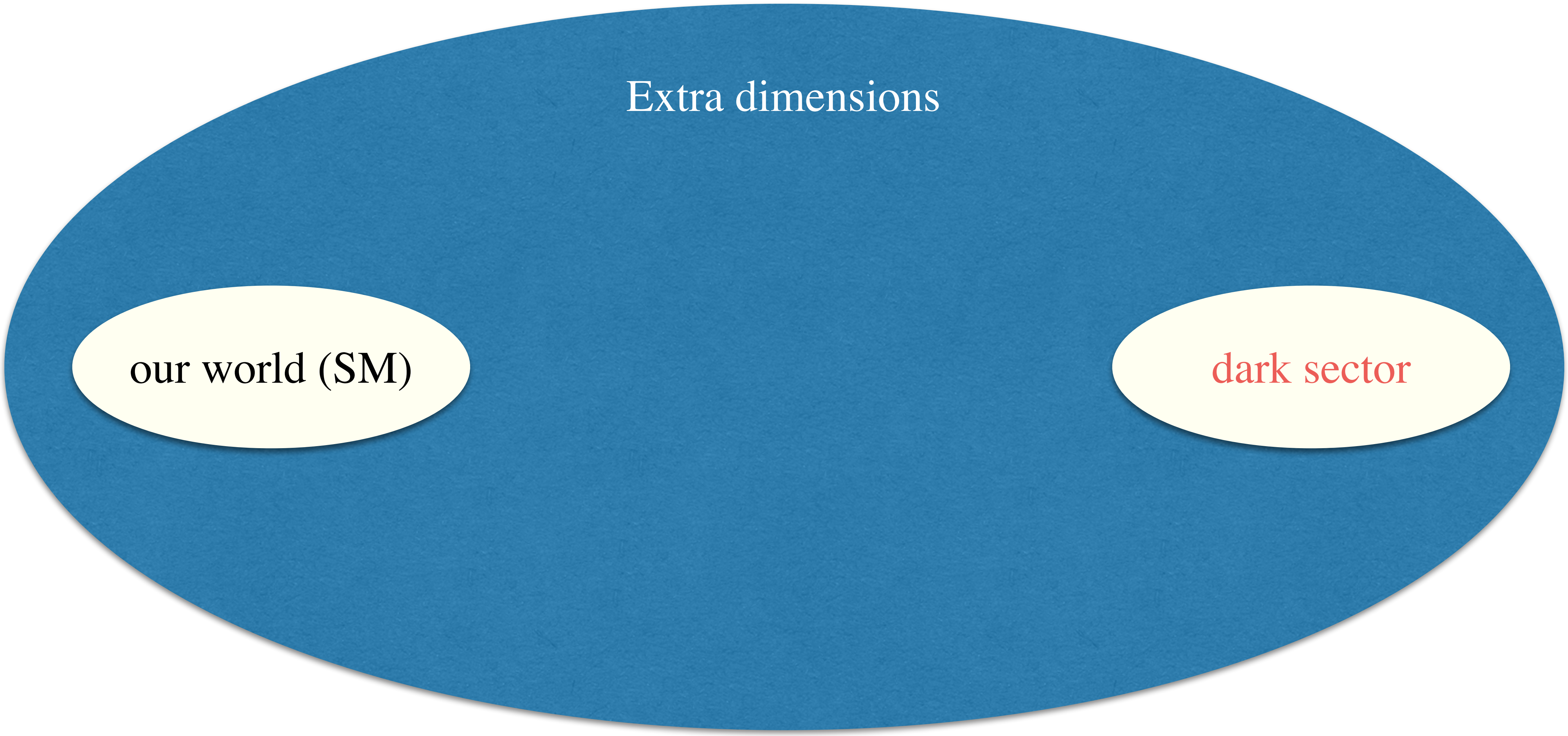
- Positivity implies $B_{\mathrm{others}}(\Lambda) > -B_{\mathrm{GR}}(\Lambda) \pm \dfrac{1}{M_{\mathrm{Pl}}^2 M^2}$

※ To our knowledge, $B_{\mathrm{GR}}(\Lambda) < 0$ is quite universal.

- Under the assumption “$M \gg m_e$,” we have $B_{\mathrm{others}}(\Lambda) > -B_{\mathrm{GR}}(\Lambda)$.

  $\rightarrow$ $B_{\mathrm{others}}(\Lambda)$ cannot be too small, so the dark sector cannot be too dark?

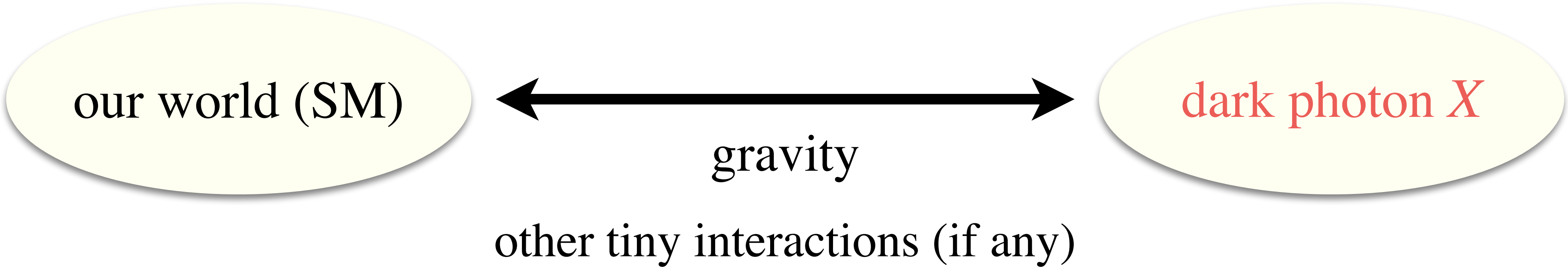
Intuition from extra dimensions

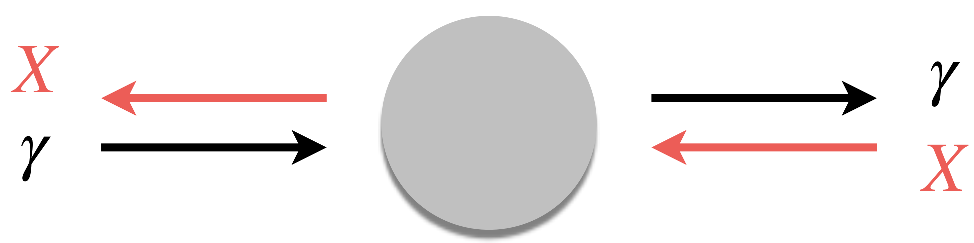


- We need large extra dimensions to separate the dark sector from our world.
- If extra dimensions are too large, gravity becomes weak.
- An upper bound on the distance between our world and dark sector as long as we turn on gravity by keeping extra dimensions finite?

example: dark photons [**TN**-Sato-Tokuda ’22]

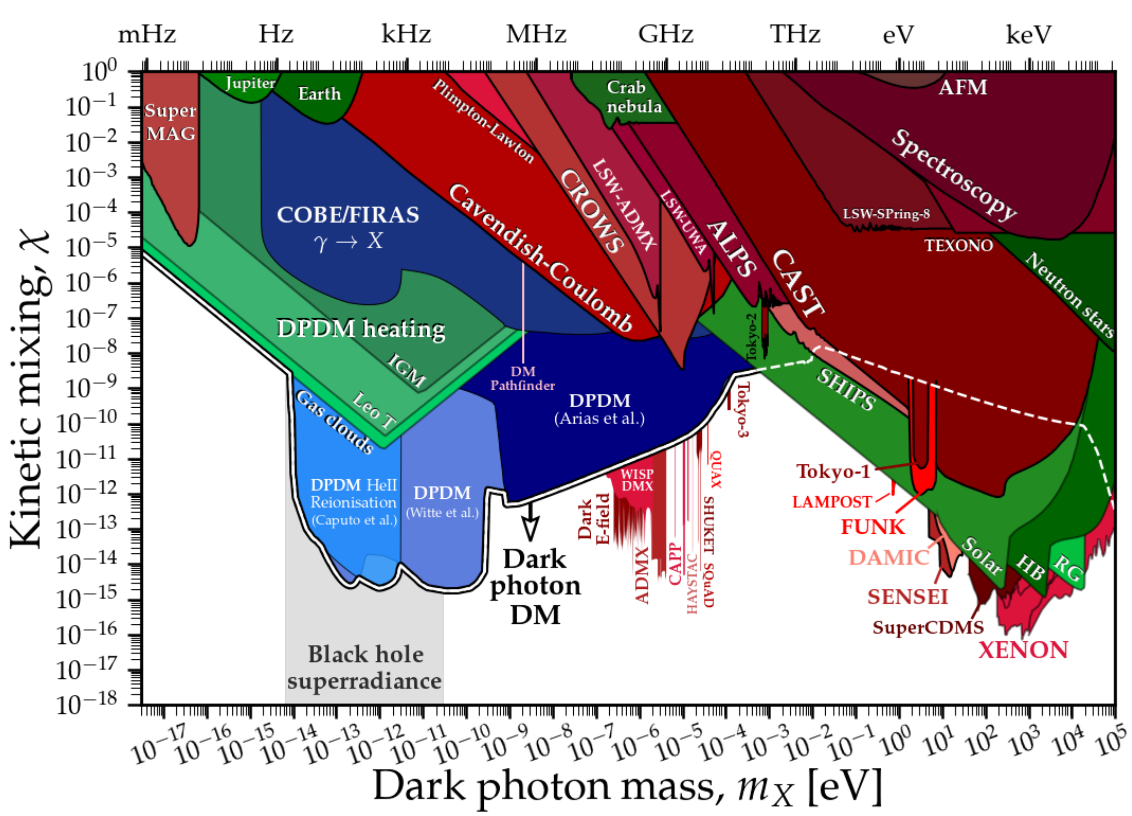
Two scenarios for dark photons





Two types of forward scattering:

1. $\gamma X_T \to \gamma X_T$ (transverse modes)
2. $\gamma X_L \to \gamma X_L$ (longitudinal moes)



How to realize $B_{\rm others}(\Lambda) > -B_{\rm GR}(\Lambda)$?

1. Large enough kinetic mixing χ

$$\mathscr{L} \ni -\frac{1}{4}F_X^2 - \frac{1}{2}m_X^2 X^2 + \chi\, e\, X^\mu J_\mu^{\rm EM}$$

2. Light enough particles charged under both U(1)'s

Scenario 1: large kinetic mixing

Suppose that particles charged under both U(1)'s are too heavy, so that the kinetic mixing χ is the dominant source of $B_{\text{others}}(\Lambda)$.

1. $\gamma X_T \to \gamma X_T$ (transverse modes)

$$B_{\text{others}}(\Lambda) > -B_{\text{GR}}(\Lambda) \iff \frac{2e^4\chi^2}{\pi^2 m_W^2 \Lambda^2} > \frac{11e^2}{720\pi^2 m_e^2 M_{\text{Pl}}^2}$$

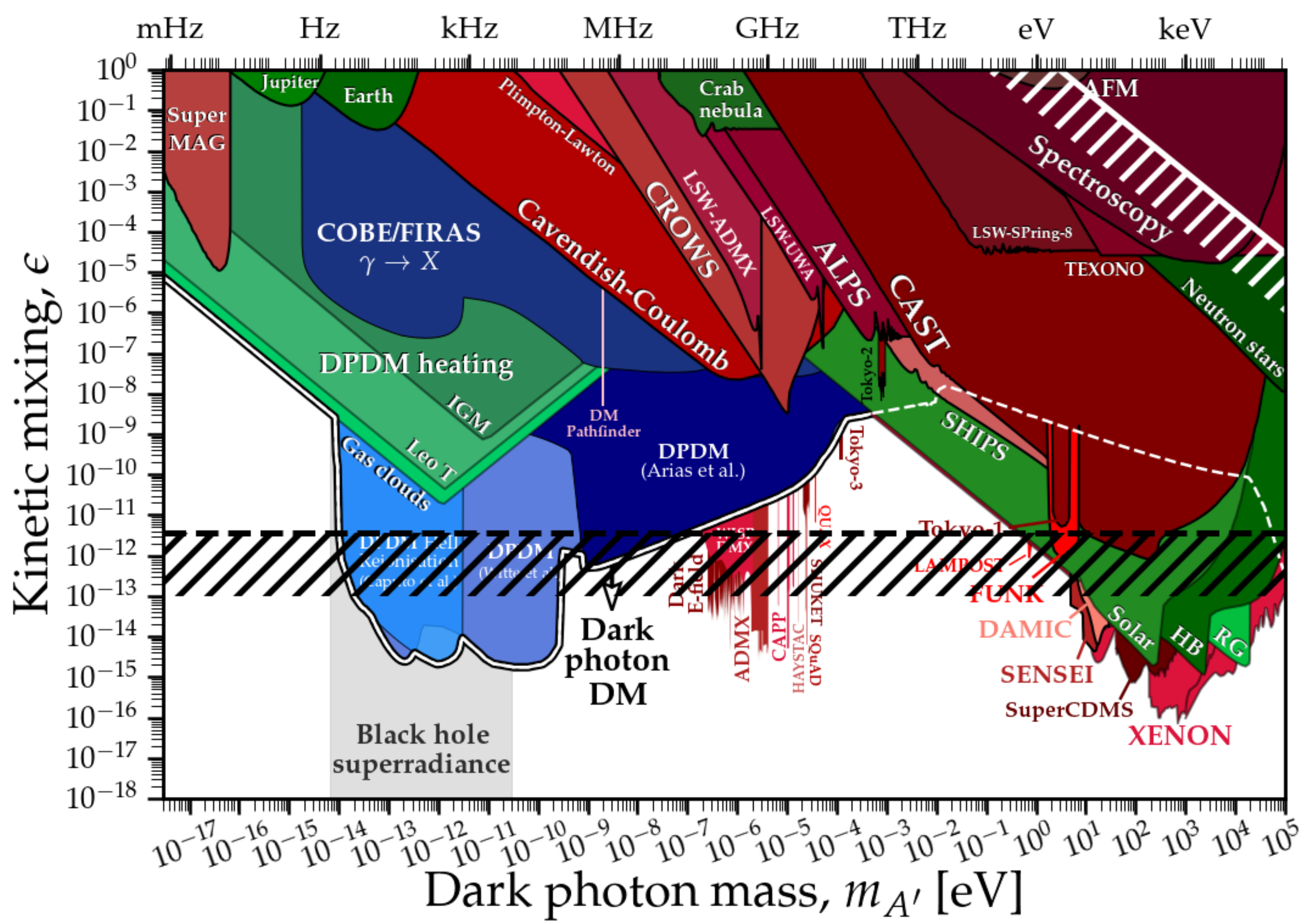
$$\iff \chi > \sqrt{\frac{11}{1440e^2}} \frac{m_W \Lambda}{m_e M_{\text{Pl}}} = 1.9 \times 10^{-11} \frac{\Lambda}{1\text{TeV}}.$$

2. $\gamma X_T \to \gamma X_T$ (longitudinal modes)

$$B_{\text{others}}(\Lambda) > -B_{\text{GR}}(\Lambda) \iff \frac{e^4\chi^2 m_X^2}{2\pi^2 m_W^4 \Lambda^2} > \frac{11e^2}{720\pi^2 m_e^2 M_{\text{Pl}}^2}$$

$$\iff \chi > \sqrt{\frac{11}{360e^2}} \frac{m_W^2 \Lambda}{m_e m_X M_{\text{Pl}}} = 3.0 \frac{\Lambda}{1\text{TeV}} \frac{1\text{eV}}{M_X}.$$

Scenario 1: large kinetic mixing

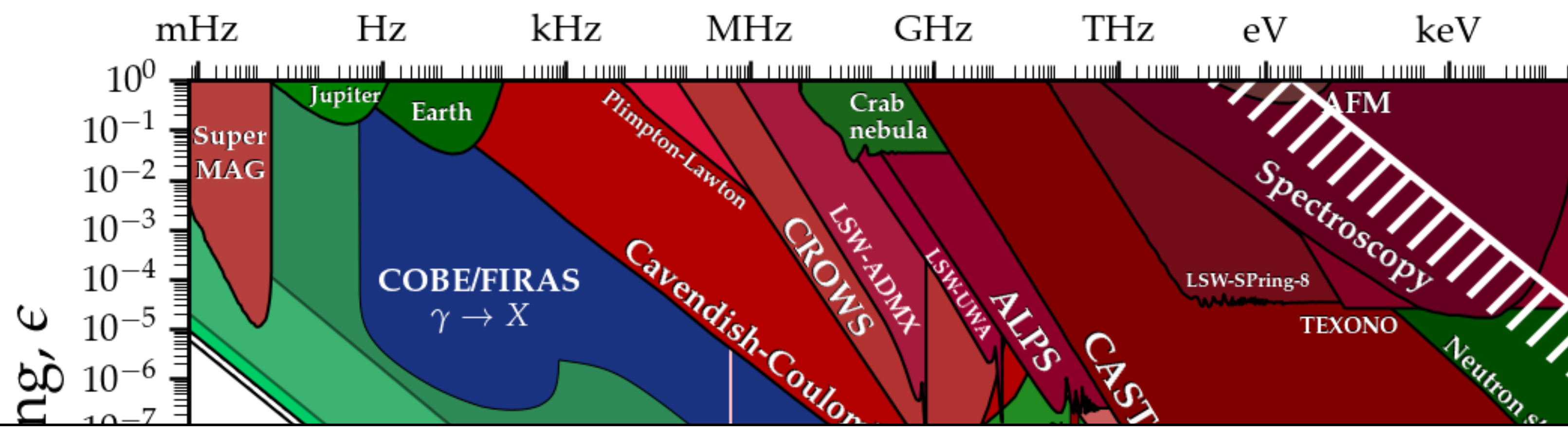


black: transverse, white: longitudinal

This mass range is allowed only when $M \sim m_e$.

(QCD effects will not change the results very much)

Scenario 1: large kinetic mixing



Two lessons:

1. Longitudinal scattering gives a stronger constraint.

2. Scenario 1 seems difficult, so we need light enough bi-charged particles.

black: transverse, white: longitudinal

This mass range is allowed only when $M \sim m_e$.

(QCD effects will not change the results very much)

Scenario 2: bi-charged particles

Suppose that there exists a bi-charged massive vector boson V.

Consider the longitudinal scattering $\gamma X_L \to \gamma X_L$ ($\tilde{e}$: dark photon gauge coupling)

$$B_{\text{others}}(\Lambda) > -B_{\text{GR}}(\Lambda) \quad \rightleftarrows \quad \frac{e^2 \tilde{e}^2 m_X^2}{2\pi^2 m_V^4 \Lambda^2} > \frac{11 e^2}{720 \pi^2 m_e^2 M_{\text{Pl}}^2}$$

$$\rightleftarrows \quad m_V < (m_V^2 \Lambda)^{1/3} < 1.3 \text{ TeV} \left(\frac{\tilde{e}}{e}\right)^{1/3} \left(\frac{m_X}{10^3 \text{ eV}}\right)^{1/3}.$$

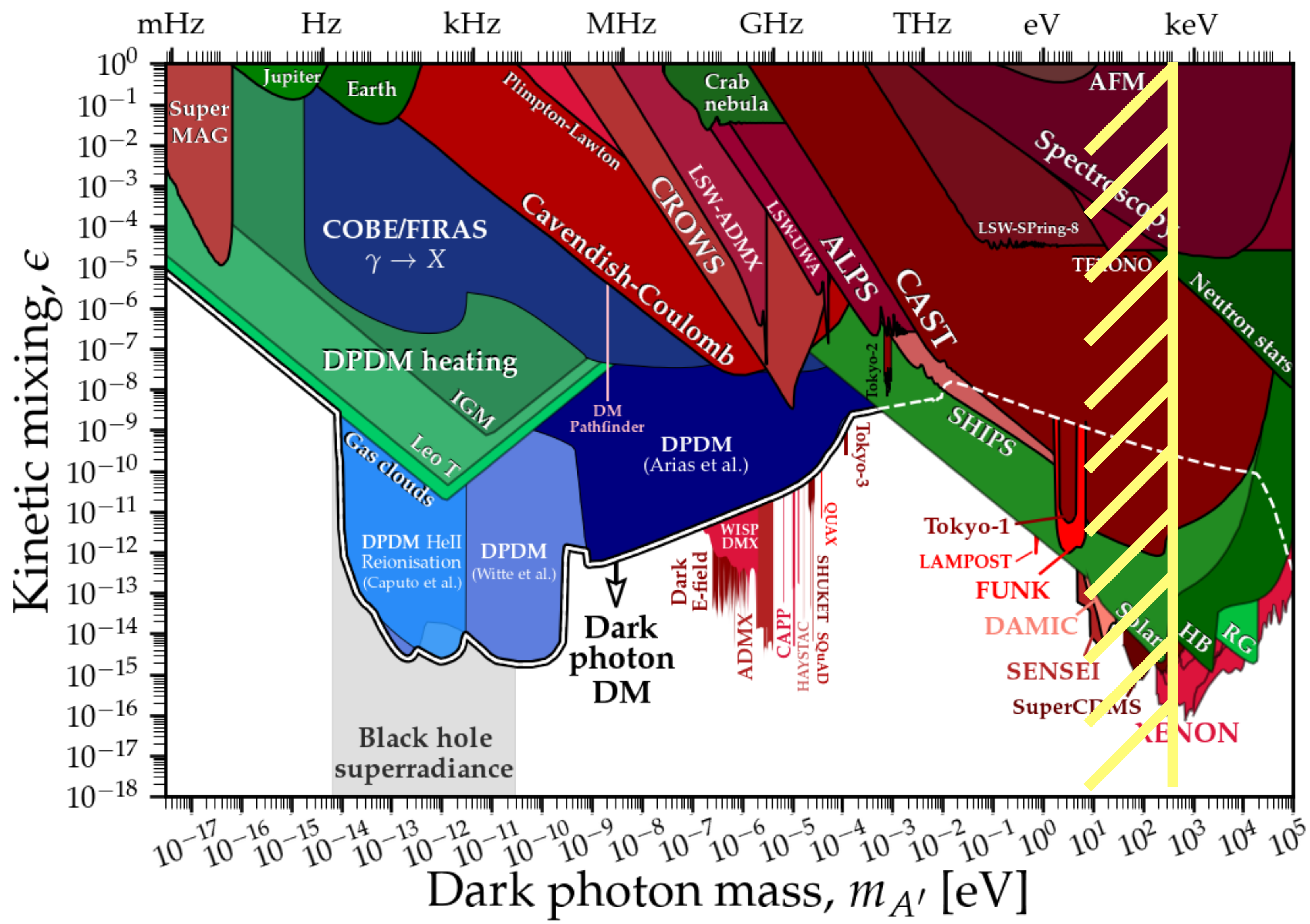
※ dark photon mass cannot be too small, since the vector boson V is coupled to photon.

※ if V were spin 0 or spin 1/2, the situation becomes worse.

We can also think of it as a lower bound on the dark photon mass:

$$B_{\text{others}}(\Lambda) > -B_{\text{GR}}(\Lambda) \rightleftarrows m_X > 4.7 \times 10^2 \text{ eV} \times \frac{e}{\tilde{e}} \left(\frac{M_V}{1 \text{ TeV}}\right)^2 \frac{\Lambda}{1 \text{ TeV}}.$$

bi-charged vector ($M_V = 1$TeV, $\tilde{e} = e$)



lower bound on dark photon mass: $m_{A'} > 500$ eV

(QCD effects will weaken the condition by $\sim 1/10$)

Contents

Summary

1. Positivity bounds on low-energy scattering amplitudes provide

   a criterion for a low-energy EFT to be UV completable in the standard manner

   $\rightarrow$ provides a Swampland condition when applied to gravitational EFTs

2. Positivity in gravitational Standard Model [Aoki-Loc-TN-Tokuda '21]

   Under the assumption "$M \gg m_e$," we found

   - The maximum cutoff scale of gravitational SM is $\Lambda \sim 10^{16}$ GeV

   - A WGC type bound the electron Yukawa coupling and the Weinberg angle.

3. Possible implications for the dark sector [TN-Sato-Tokuda '22]

   The same assumption "$M \gg m_e$" implies that dark sector cannot be too dark.

Future directions

A) sharpen gravitational positivity bounds

cf. [Arkani-Hamed et al ’20, Caron-Huot et al ’21, Alberte et al ’21, …]

- How generic the assumption “$M \gg m_e$” is?
- detailed study of string loop amplitudes in 4D will also be useful.

B) more phenomenological applications (DM, neutrinos, …)

[in progress w/Sato-Tokuda + Aoki-Saito-Shirai-Yamazaki]

C) bootstrap based on other principles

- scattering positivity = positivity of corrections to BH entropy [ex. w/Hamada, Shiu, Loges]

※ BH physics may be useful to sharpen gravitational positivity???

- recent developments on BH evaporation vs unitary time-evolution

※ Is symmetry-resolved entropy useful? [Milekhin-Tajdini ’21, Lau-TN-Tamaoka-Takii ’22]

D) cosmological bootstrap: bootstrapping dS correlators

- useful for the dark energy problem??? (IR completion)

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Thank you!

backup

Positivity bounds w/o gravity

[Adams et al ’06]

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha\left(\partial_\mu\phi\partial^\mu\phi\right)^2 + \cdots$$

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_1\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \alpha_2\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2 + \ldots$$

$$\alpha, \alpha_1, \alpha_2 \geq 0$$

A key idea of positivity bounds:

connect UV and IR using analyticity of scattering amplitudes

Positivity bounds w/o gravity

Consider an s-u symmetric scattering amplitude $\mathscr{M}(s,t)$ in the forward limit.

IR expansion in the forward limit:

$$\mathscr{M}(s,t=0)=\sum_n a_{2n}\,s^{2n},$$

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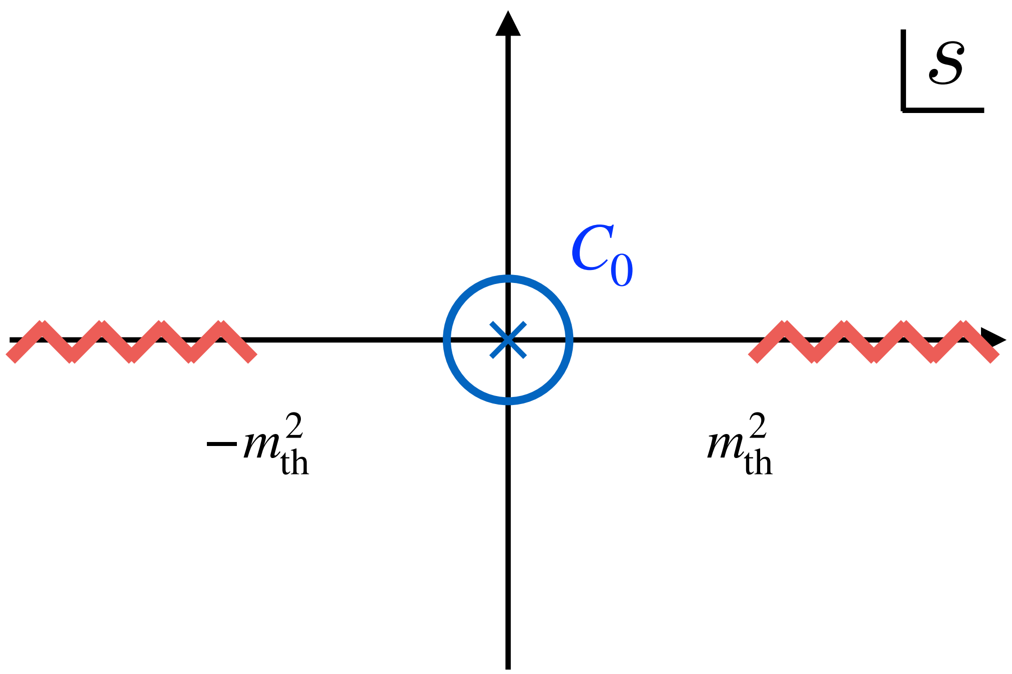

analytic structure of $\mathscr{M}(s,t=0)$

IR expansion in the forward limit:

$$\mathscr{M}(s,t=0)=\sum_n a_{2n}\,s^{2n},$$

$$a_{2n}=\oint_{C_0}\frac{ds}{2\pi i}\frac{\mathscr{M}(s,t=0)}{s^{2n+1}}.$$

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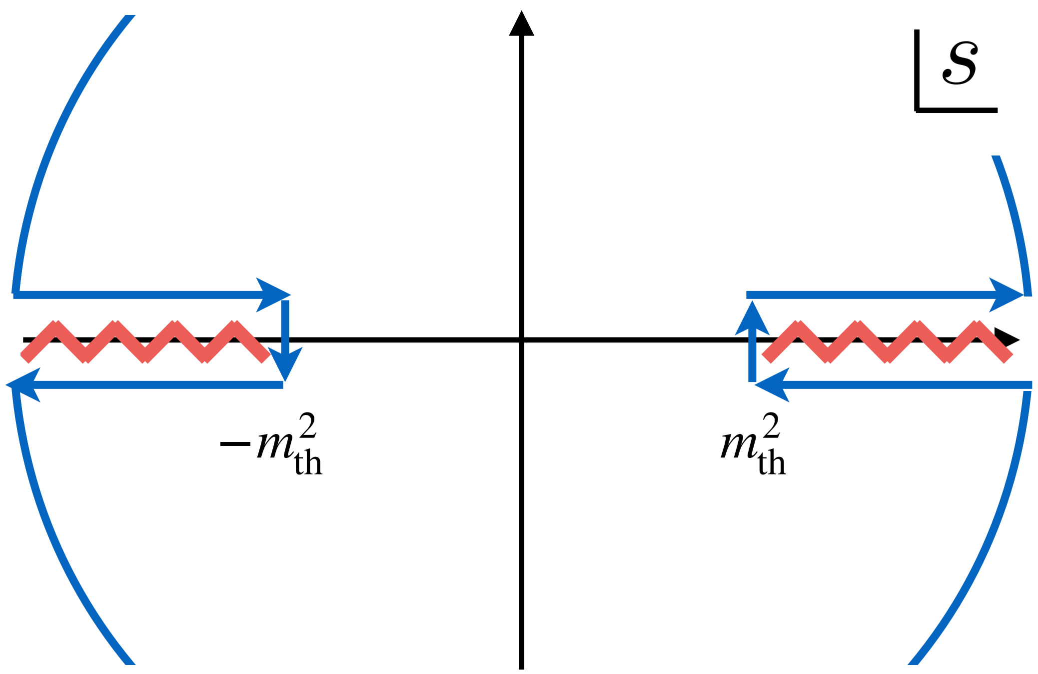

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Deform the integration contour to rewrite it in the UV language:

$$a_{2n}=\frac{2}{\pi}\int_{m_{\rm th}^2}^{\infty}ds\frac{\mathrm{Im}\mathcal{M}(s,t=0)}{s^{2n+1}}+\oint_{C_\infty}\frac{ds}{2\pi i}\frac{\mathcal{M}(s,t=0)}{s^{2n+1}}.$$

※ used the s-u symmetry and $\mathrm{Disc}\,\mathcal{M}(s,t=0)=2i\,\mathrm{Im}\,\mathcal{M}(s,t=0)$

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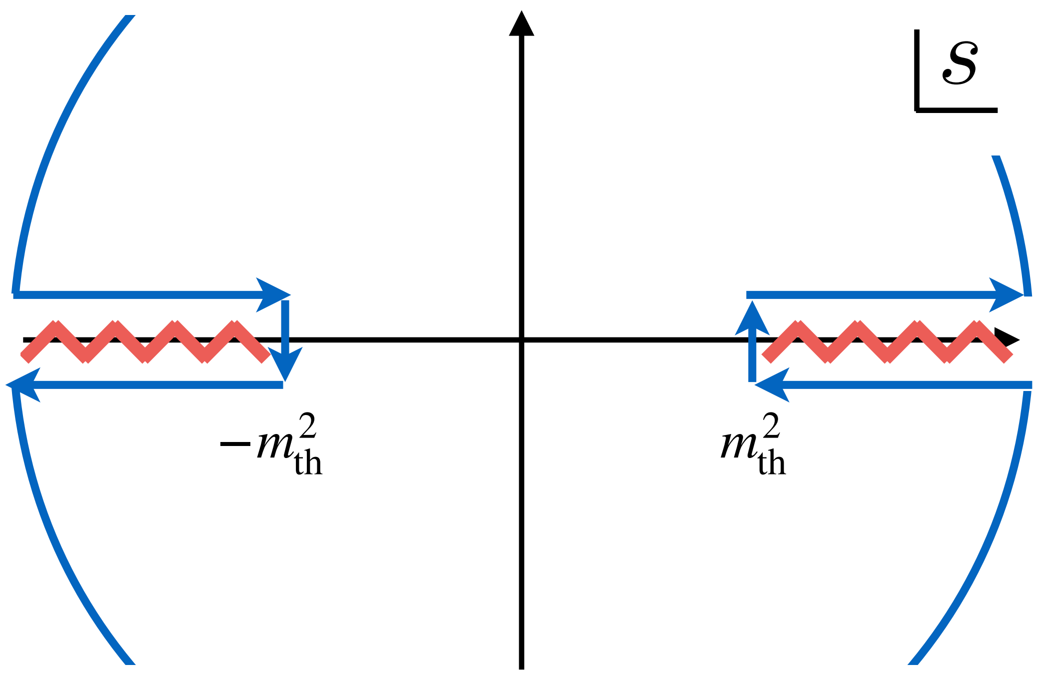

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$$\mathscr{M}(s,t=0) = \sum_n a_{2n}\, s^{2n},$$

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Deform the integration contour to rewrite it in the UV language:

If $|\mathscr{M}(s,t=0)| < |s|^{2n}$

$$a_{2n} = \frac{2}{\pi}\int_{m_{\rm th}^2}^{\infty} ds \frac{{\rm Im}\mathscr{M}(s,t=0)}{s^{2n+1}} + \cancel{\oint_{C_\infty} \frac{ds}{2\pi i} \frac{\mathscr{M}(s,t=0)}{s^{2n+1}}}.$$

※ used the s-u symmetry and Disc $\mathscr{M}(s,t=0) = 2i\,{\rm Im}\,\mathscr{M}(s,t=0)$

In local gapped theories, unitarity implies $|\mathscr{M}(s,t=0)| < s\ln^2 s \quad (s\to\infty)$.

(Froissart bound)

This leads to the following dispersion relation and the positivity:

$$a_{2n} = \frac{2}{\pi}\int_{m_{\text{th}}^2}^{\infty} ds \frac{\mathrm{Im}\mathscr{M}(s,t=0)}{s^{2n+1}} \geq 0 \quad \text{for} \quad 2n = 2,4,\ldots$$

Positive because of unitarity!

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \alpha\left(\partial_\mu\phi\partial^\mu\phi\right)^2 + \cdots$$

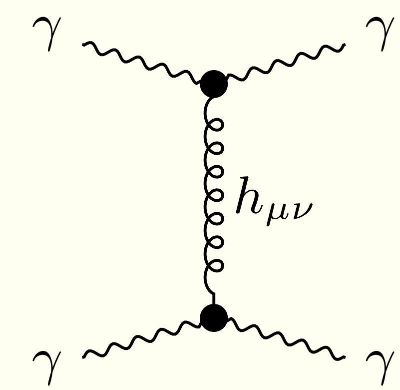
$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_1\left(F_{\mu\nu}F^{\mu\nu}\right)^2 + \alpha_2\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right)^2 + \ldots$$

$\alpha, \alpha_1, \alpha_2 \geq 0$

How the story changes in the presence of gravity?

Gravitational Positivity Bounds [Tokuda-Aoki-Hirano '20]

In the presence of gravity, IR scattering amplitudes behave as



$$\mathscr{M}(s,t) = \frac{su}{M_{\rm Pl}^2 t} + \frac{tu}{M_{\rm Pl}^2 s} + \frac{ts}{M_{\rm Pl}^2 u} + \sum_{n,m} c_{n,m}\, s^n t^m.$$

- In the forward limit, the t-channel graviton exchange dominates:

$$\mathscr{M}(s,t) \simeq -\frac{s^2}{M_{\rm Pl}^2 t} + \sum_n c_{n,0}\, s^n + \mathscr{O}(t).$$

※ Positivity of the s^2 coefficient does not follow in a straightforward manner.

If we assume the s^2 boundedness for $t < 0$,

the dispersion relation reads $c_{2,0} = \lim_{t\to 0}\left[\frac{1}{M_{\rm Pl}^2 t} + \frac{2}{\pi}\int_{m_{\rm th}^2}^{\infty} ds \frac{{\rm Im}\mathscr{M}(s,t)}{s^3}\right].$

※ Cancelation of the t-pole implies Reggeization for $s > M_{\rm Regge}^2$:

$${\rm Im}\mathscr{M}(s,t<0) \simeq f(t)\left(\frac{s}{M_{\rm Regge}^2}\right)^{2+\alpha' t+\alpha'' t^2+\cdots} \text{ w/ } f(0) = \frac{\pi}{2\alpha' M_{\rm Pl}^2}.$$

Gravitational Positivity Bounds [Tokuda-Aoki-Hirano '20]

Approximate positivity holds even in gravity theories:

$$c_{2,0} = \lim_{t\to 0}\left[\frac{4}{M_{\rm Pl}^2 t} + \frac{2}{\pi}\int_{M_{\rm Regge}^2}^{\infty} ds \frac{{\rm Im}\mathcal{M}(s,t)}{s^3}\right] + \frac{2}{\pi}\int_{m_{\rm th}^2}^{M_{\rm Regge}^2} ds \frac{{\rm Im}\mathcal{M}(s,0)}{s^3}$$

$$\geq -\frac{1}{M_{\rm Pl}^2}\left(\frac{f'(0)}{f(0)} - \frac{\alpha''}{\alpha'}\right) := \pm\frac{1}{M_{\rm Pl}^2 M^2}$$

※ The energy scale M in the approximate positivity $c_{2,0} > \pm\dfrac{1}{M_{\rm Pl}^2 M^2}$

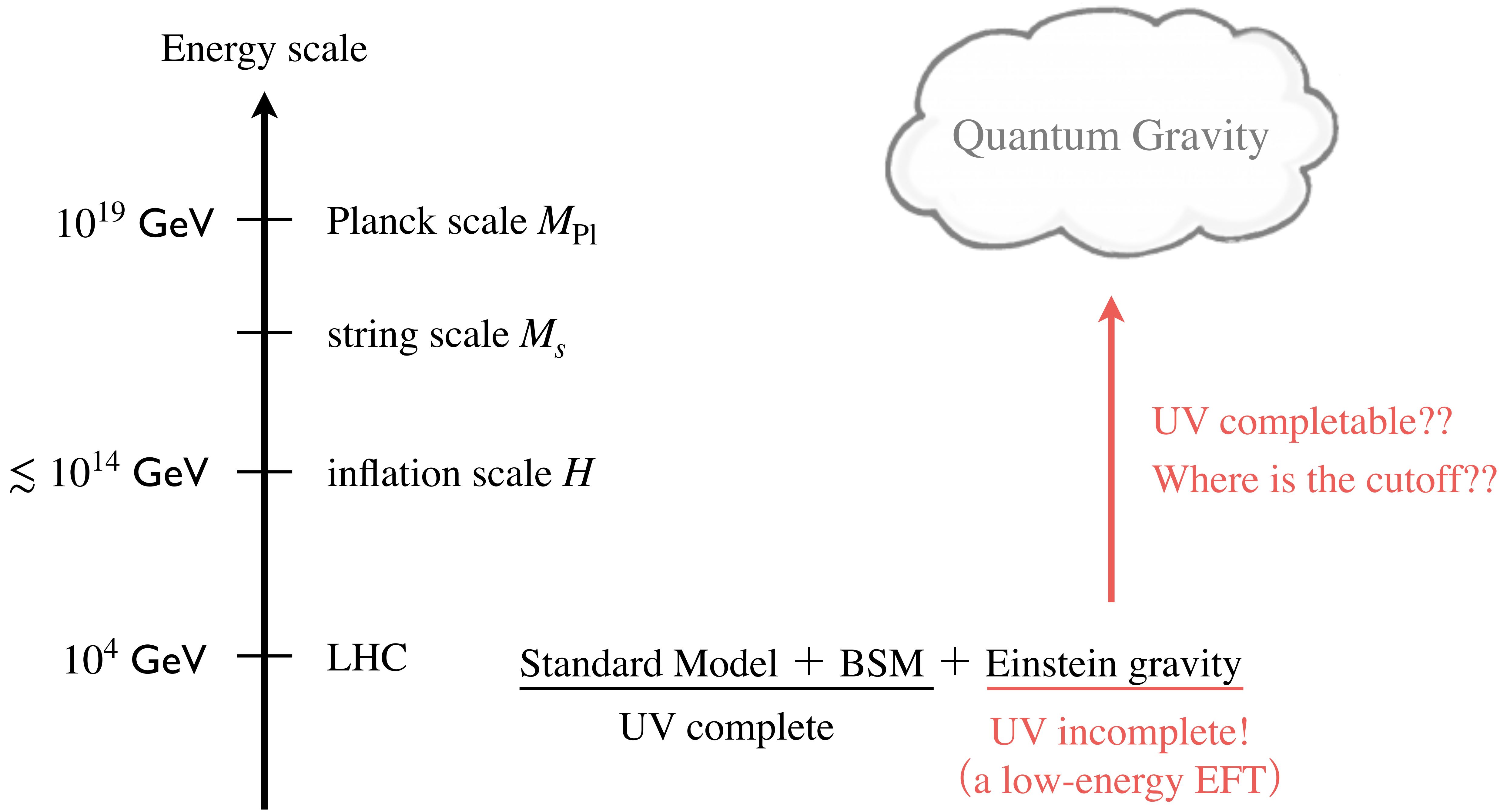
depends on details of Regge states required for UV completion of gravity.

- In tree-level string theory, we have $M \sim M_{\rm string}$ [cf. Hamada-TN-Shiu '18].

  cf. [Caron-Huot et al '21] based on crossing symmetry in 5D and higher

- It is an open problem to identify the scale M for loops, especially in 4D.

- We will find that the scale M is crucial for phenomenological application.



Apply (improved) positivity bounds for this problem!

Improved Positivity Bounds

Improved Positivity Bounds

In the previous section, we derived the dispersion relation

$$c_{2,0} = \frac{2}{\pi}\int_{m_{\mathrm{th}}^2}^{M_{\mathrm{Regge}}^2} ds \frac{\mathrm{Im}\mathscr{M}(s,0)}{s^3} - \frac{1}{M_{\mathrm{Pl}}^2}\left(\frac{f'(0)}{f(0)} - \frac{\alpha''}{\alpha'}\right).$$

When the threshold energy m_{th} is below the cutoff scale Λ,

it is convenient to define [Bellazini '16, de Rham et al '17]:

$$B(\Lambda) := c_{2,0} - \frac{2}{\pi}\int_{m_{\mathrm{th}}^2}^{\Lambda^2} ds \frac{\mathrm{Im}\mathscr{M}(s,0)}{s^3},$$ which is calculable within the EFT.

※ $B(\Lambda)$ monotonically decreases as Λ increases

Then, the dispersion relation implies

$$B(\Lambda) := \frac{2}{\pi}\int_{\Lambda^2}^{M_{\mathrm{Regge}}^2} ds \frac{\mathrm{Im}\mathscr{M}(s,0)}{s^3} - \frac{1}{M_{\mathrm{Pl}}^2}\left(\frac{f'(0)}{f(0)} - \frac{\alpha''}{\alpha'}\right) \geq \pm \frac{1}{M_{\mathrm{Pl}}^2 M^2}.$$

※ If this improved positivity bound is violated at some UV scale Λ,

the EFT breaks down and UV completion is required below Λ.