Stacked Hilbert-Huang Transform and Its Applications

From High-Energy Astrophysics to Gravitational-Wave Astrophysics

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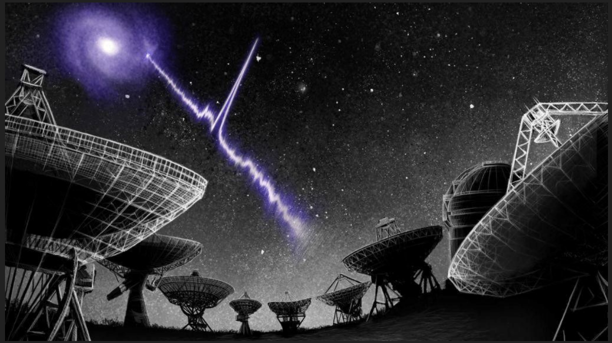
2022 NCTS Annual Theory Meeting

Time-Domain Astronomy

- The Universe is dynamic – time-domain astronomy is more and more important
- Many transient, non-stationary, and quasi-periodic phenomena are dicovered
- To describe the detailed frequency evolution and investigate underlying physics, advanced time-frequency analysis is needed
  - Fourier (with priori): Spectrogram, dynamic power spectrum, wavelet analysis
    - Trade-off between time resolution and frequency resolution
  - Adaptive (without priori): Hilbert-Huang transform



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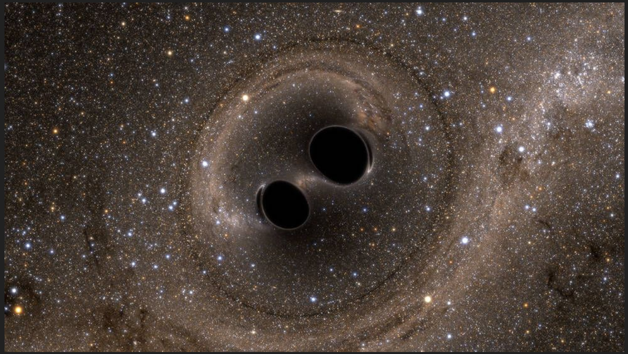

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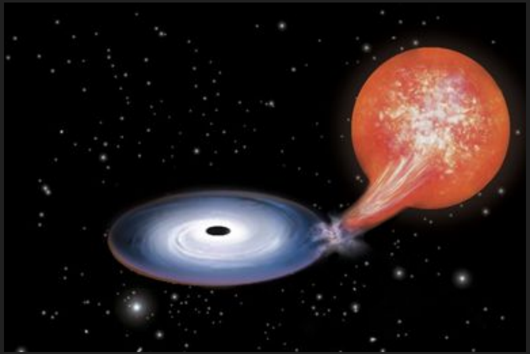

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Frequency: Definition

- Given the period of a modulation as P, the frequnecy can be represented as

$$f = \frac{1}{P}$$

- This definition is too crude
- Does not apply to non-stationary processes
- Adding a moving window is able to trace the evolution of a non-stationary process in limited time/frequency resolution.

Instantaneous Frequency

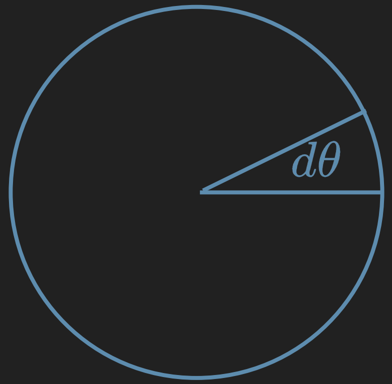
Analog to the linear motion:

The velocity can be defined as $v = \frac{\Delta x}{\Delta t}$ … constant speed, or mean velocity

The instantaneous velocity can be defined as: $v = \frac{dx}{dt}$

Angular frequency in Fourier analysis: $\omega = \frac{2\pi}{P}$

The instantaneous angular frequency: $\omega = \frac{d\theta}{dt}$



θ is the (angular) phase function – how to define?

Instantaneous Frequency and Hilbert Transform

- For an oscillation signal x(t), the instantaneous frequency can be obtained by calculating the time derivative of the phase function.
- The phase function can be defined as

$$y(t) = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty} \frac{x(t)}{t - t'} dt'$$

Hilbert transform, normalization is not needed but recommended to overcome the difficulties in Bedrosian theory

$$y(t) = \sqrt{1 - [x(t)]^2}$$

Direct quadrature, normalization is needed

- Some other approches are also available. Then we can calculate

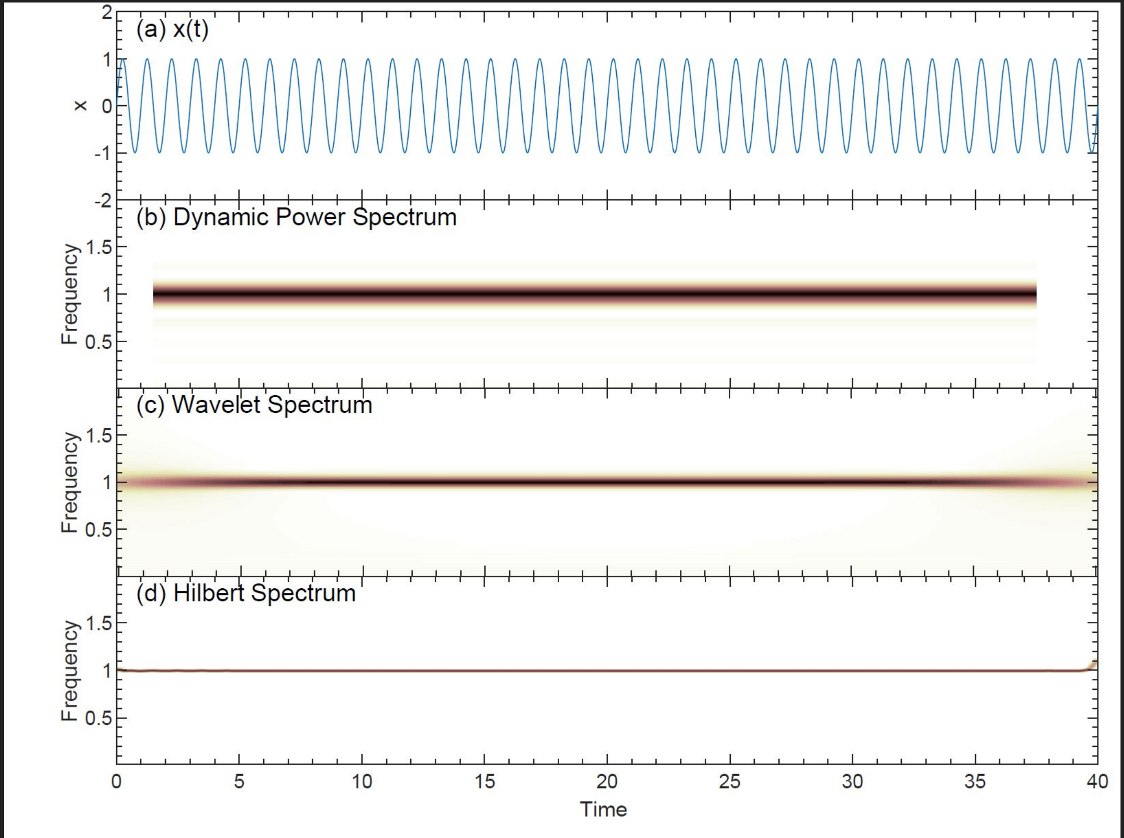
$$\theta(t) = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$

phase function

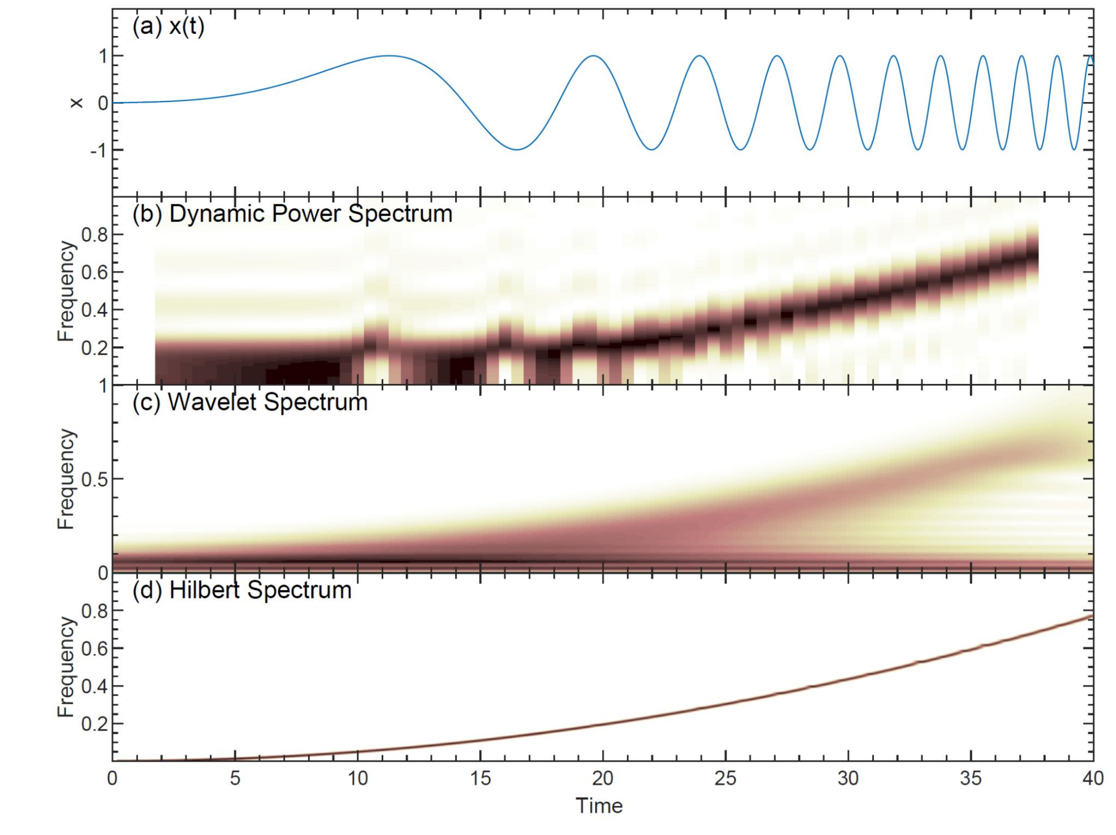
$$\omega(t) = \frac{d\theta(t)}{dt}$$

instantaneous (angular) frequency

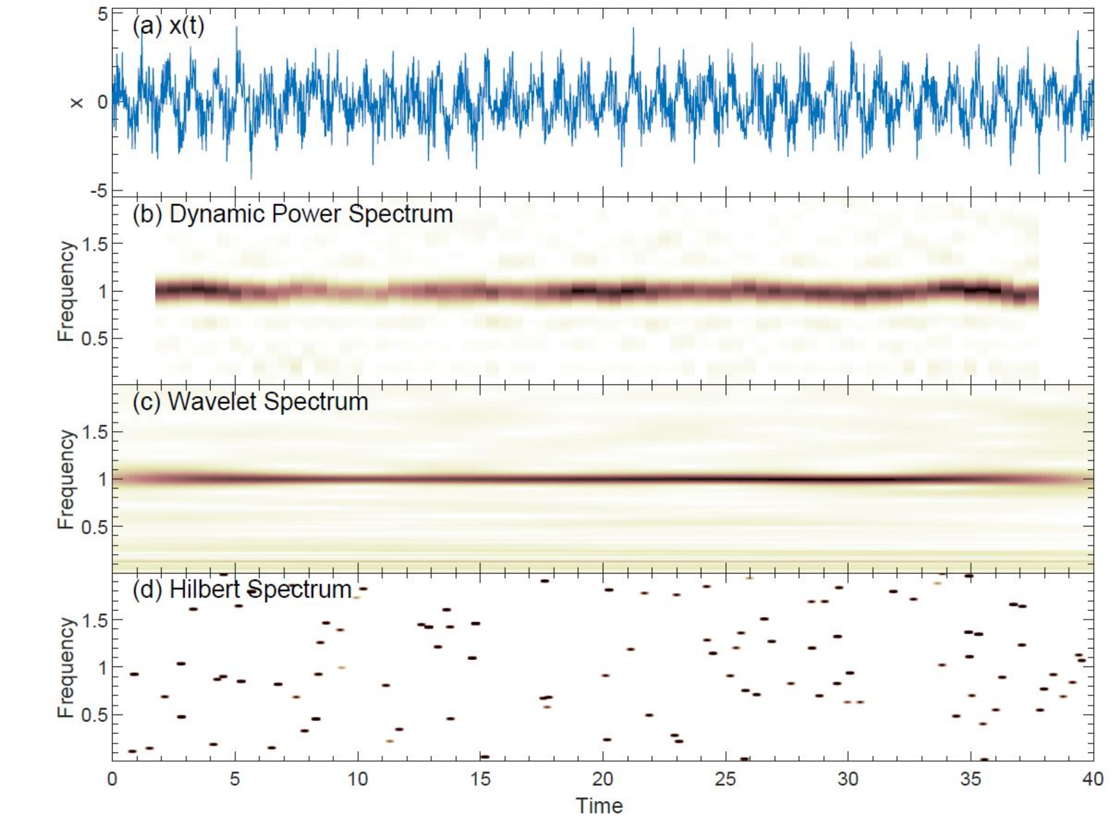
Hilbert Transform: Pure Sinusoidal Function



Hilbert Transform: Chirp Signal



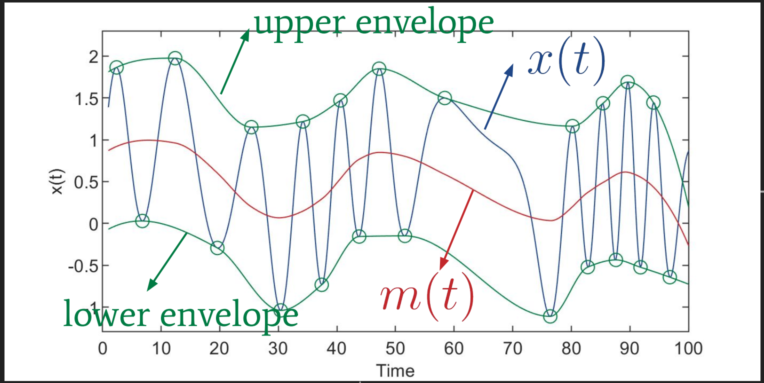
It Looks Great, BUT…

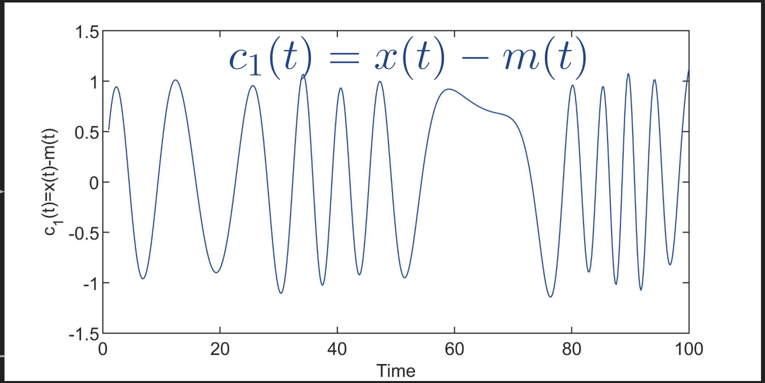


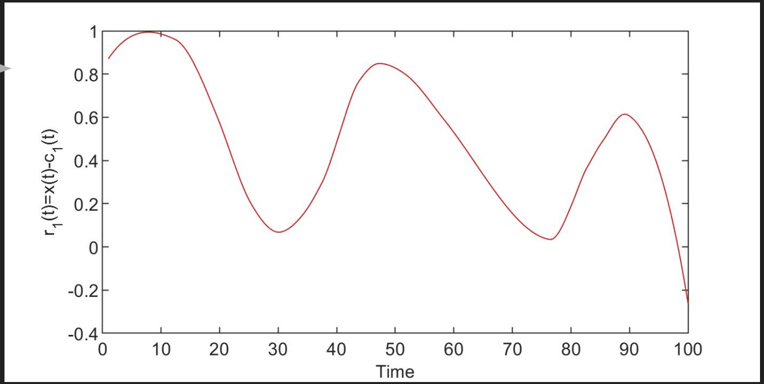
Intrinsic Mode Function

- The instantaneous frequency is physically meaningful only if the input signal is an intrinsic mode function (IMF, Huang et al. 1998)
  - The number of extrema and the number of zero crossings must be identical or differ by one
  - At any points in the series, the mean value of the upper envelope (dened by the local maxima) and the lower envelope (dened by the local minima) is zero.
- Thus, the empirical mode decomposition (EMD, Huang et al. 1998) was proposed to decompose the signal x(t) into finite number of IMFs.
  - An iteration process to calculate the local mean curve of the upper/lower envelopes, and subtract the curve from the signal.
  - Working on the same process until the residual (trend) can no longer be decomposed.

Empirical Mode Decomposition



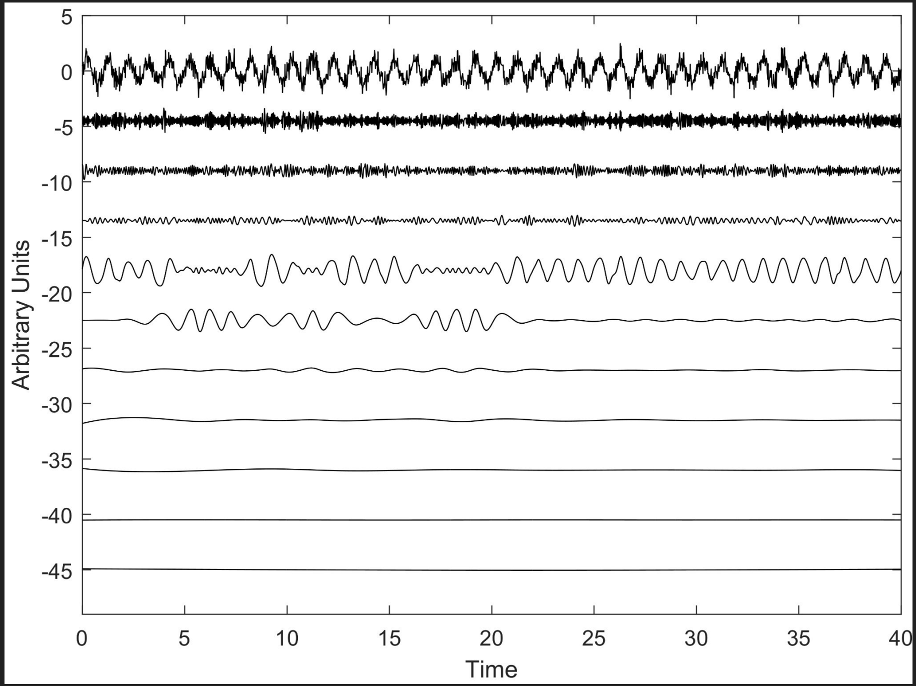


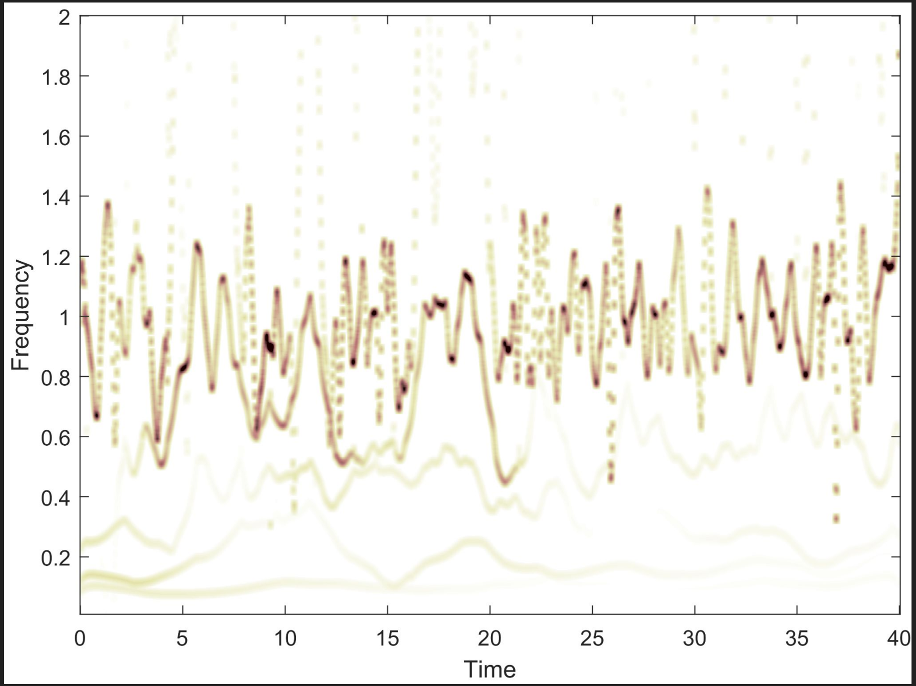


If no, work on the same process on $c_1(t)$ until it fits the IMF criteria.

If $c_1(t)$ fit the IMF criteria, we treat $c_1(t)$ as the first IMF, and subtract it from the original signal. Then, we do the same operation to the residual to obtain the next IMF.

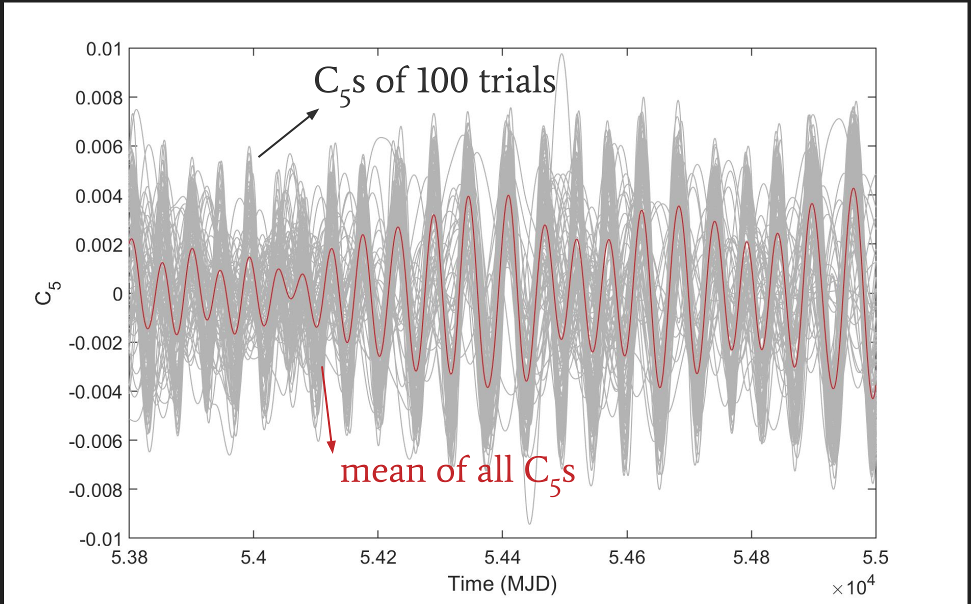
Empirical Mode Decomposition



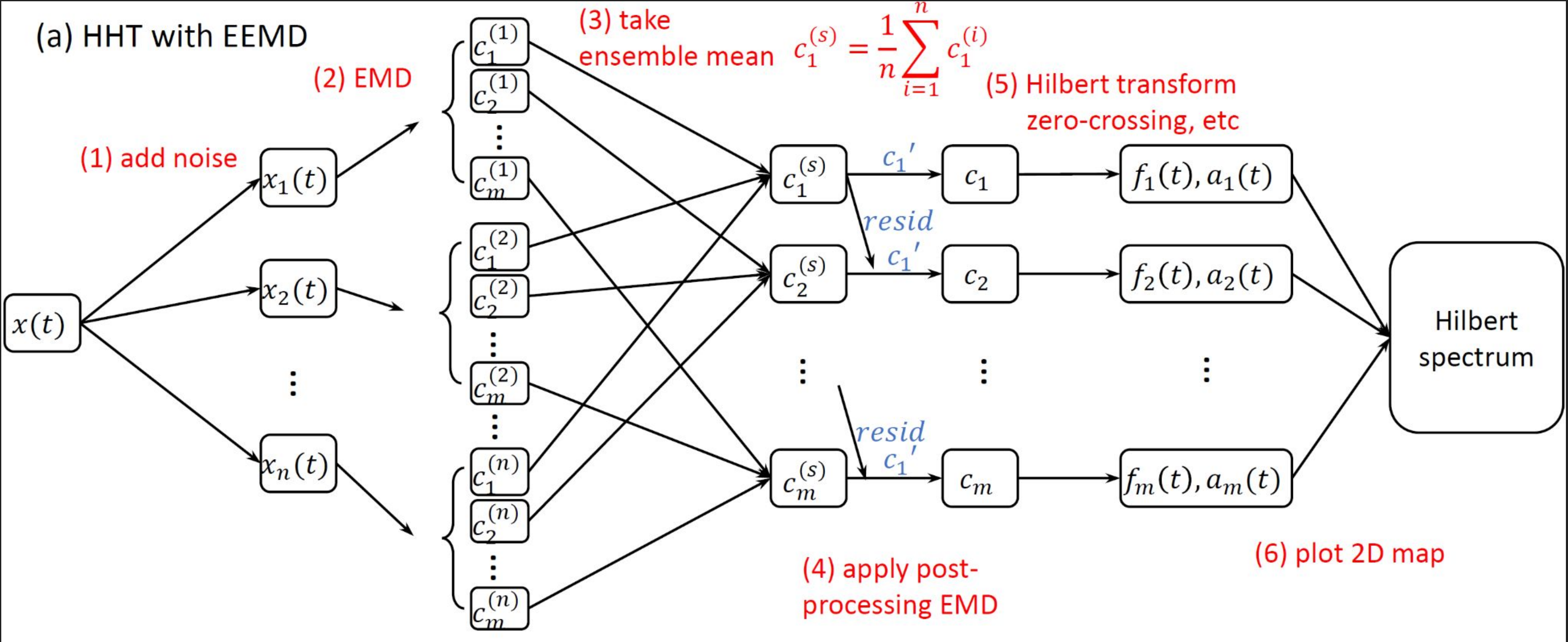


Ensemble EMD (EEMD)

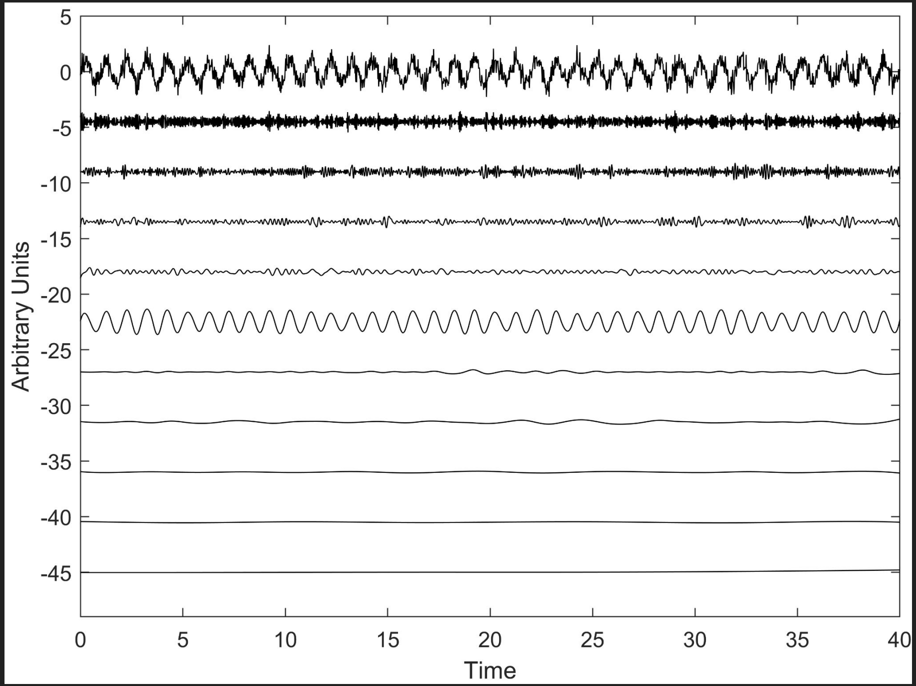
- By adding white noise and take the ensemble mean of IMFs, the mode mixing problem could be solved (Wu et al. 2009).
  - The noise level should be chosen carefully – could be optimized by minimizing the orthogonality index
  - The summation of IMFs is no longer an IMF – a post-processing EMD is needed



EEMD


(a) HHT with EEMD
(1) add noise
(2) EMD
(3) take ensemble mean $c_1^{(s)} = \frac{1}{n}\sum_{i=1}^{n} c_1^{(i)}$
(4) apply post-processing EMD
(5) Hilbert transform zero-crossing, etc
(6) plot 2D map
$x(t)$
$x_1(t)$
$x_2(t)$
$x_n(t)$
$c_1^{(1)}$
$c_2^{(1)}$
$c_m^{(1)}$
$c_1^{(2)}$
$c_2^{(2)}$
$c_m^{(2)}$
$c_1^{(n)}$
$c_2^{(n)}$
$c_m^{(n)}$
$c_1^{(s)}$
$c_2^{(s)}$
$c_m^{(s)}$
c_1'
resid
c_1
c_2
c_m
$f_1(t), a_1(t)$
$f_2(t), a_2(t)$
$f_m(t), a_m(t)$
Hilbert spectrum

EEMD Result



5
0
-5
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-15
-20
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-40
-45
Arbitrary Units
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20
25
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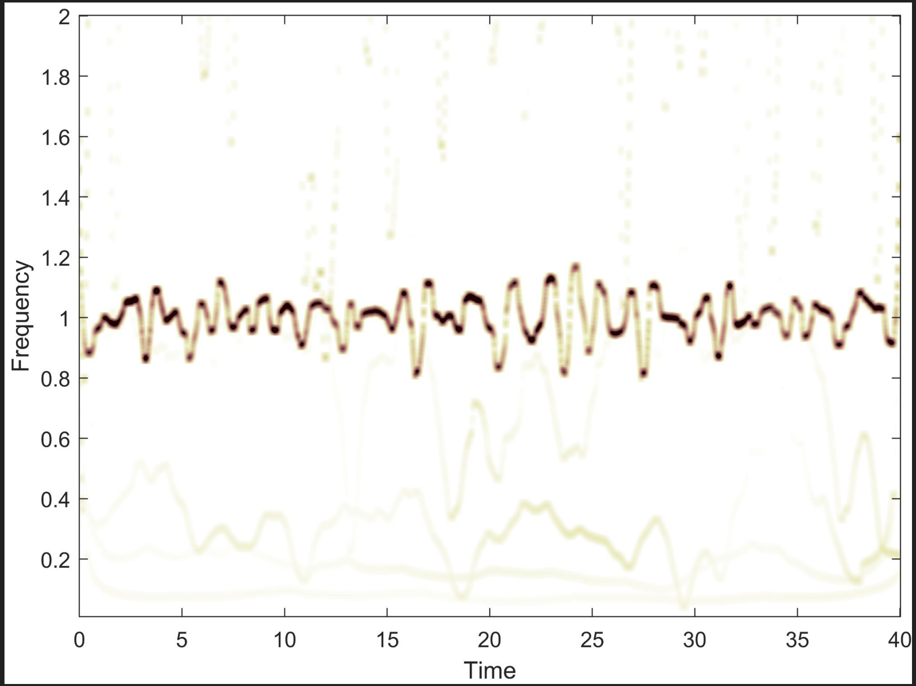
35
40
Time



2
1.8
1.6
1.4
1.2
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0.8
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Frequency
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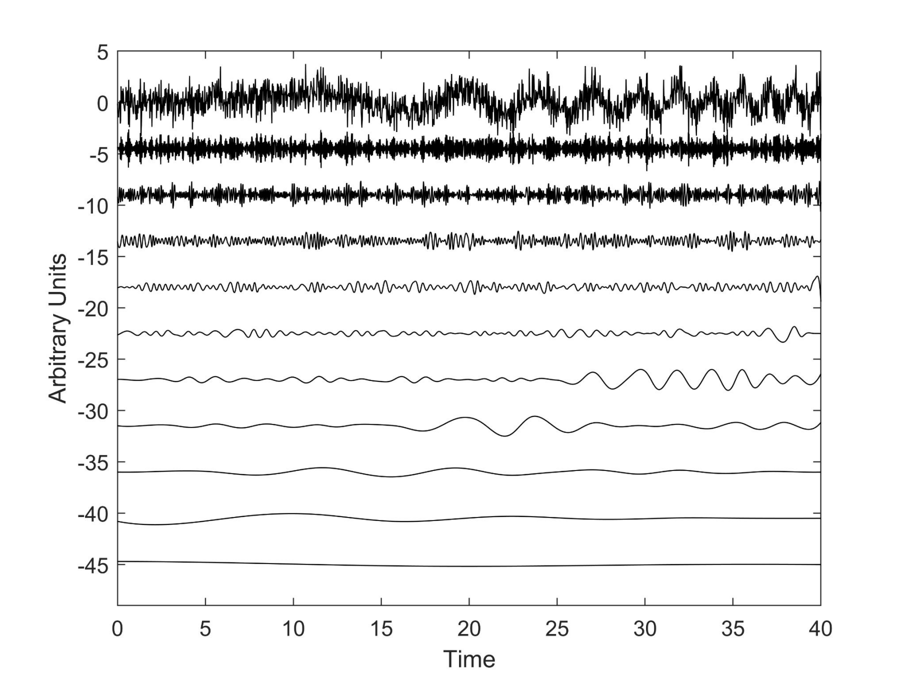
35
40
Time

However



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Arbitrary Units
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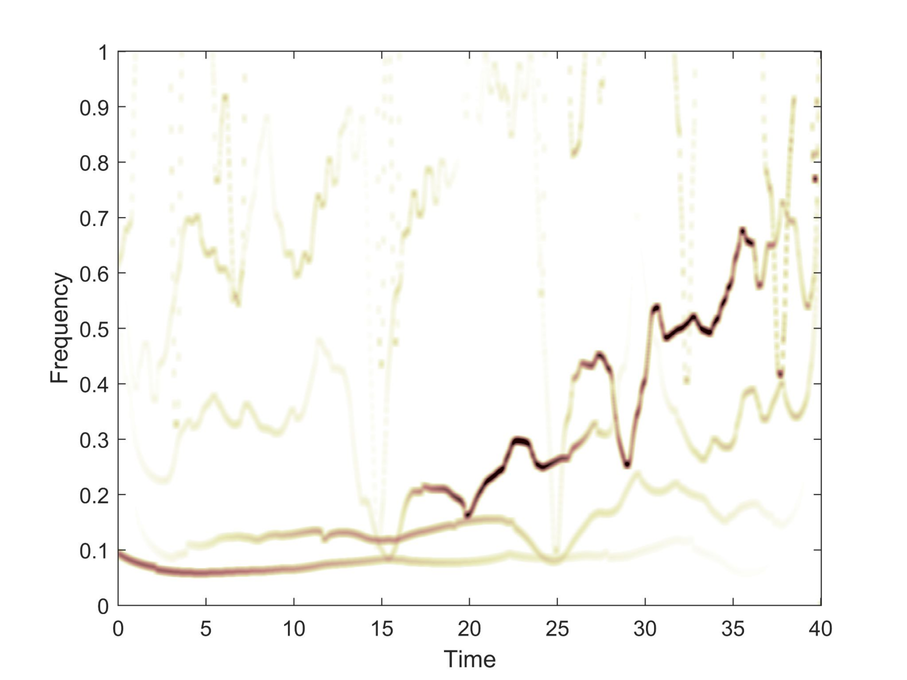
Time



1
0.9
0.8
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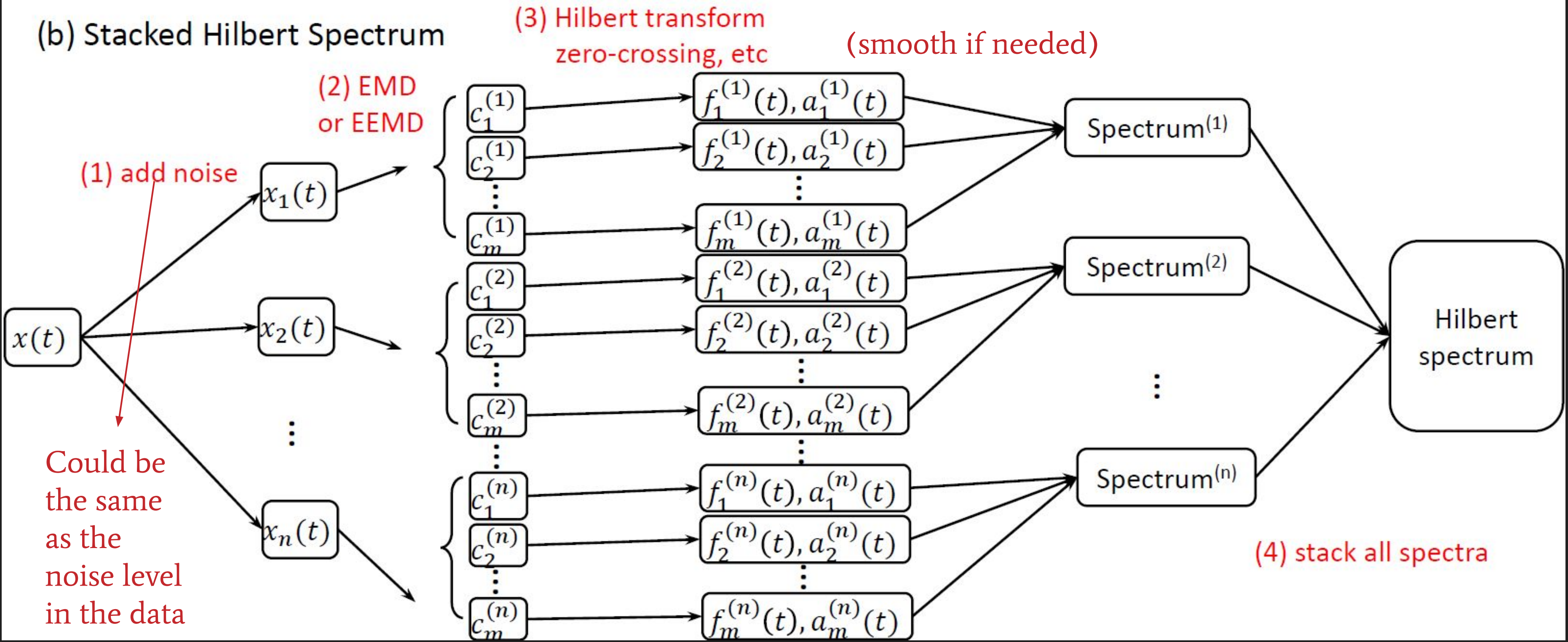
0.6
0.5
0.4
0.3
0.2
0.1
0
Frequency
0
5
10
15
20
25
30
35
40
Time

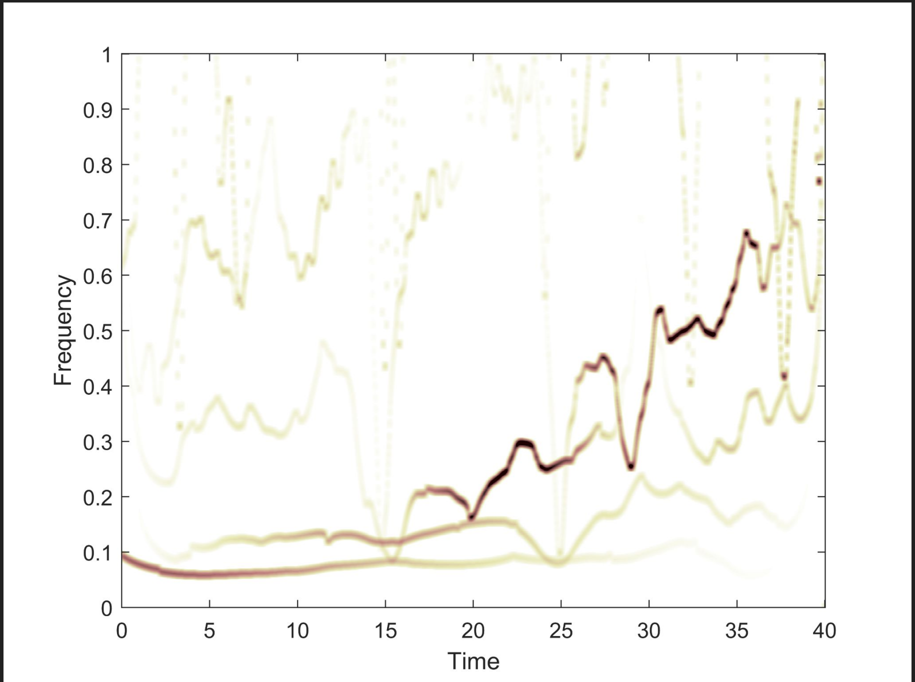
Problems of EEMD

- If the frequency range of a signal is larger than a factor of ~2, mode-mixing (or mode-splitting) is unavoidable.
    - The instantaneous frequency near the boundary that the signall across from one IMF to another would be unphysically modulated
    - Some (probably) unreal signals are introduced by the added/intrinsic noise.

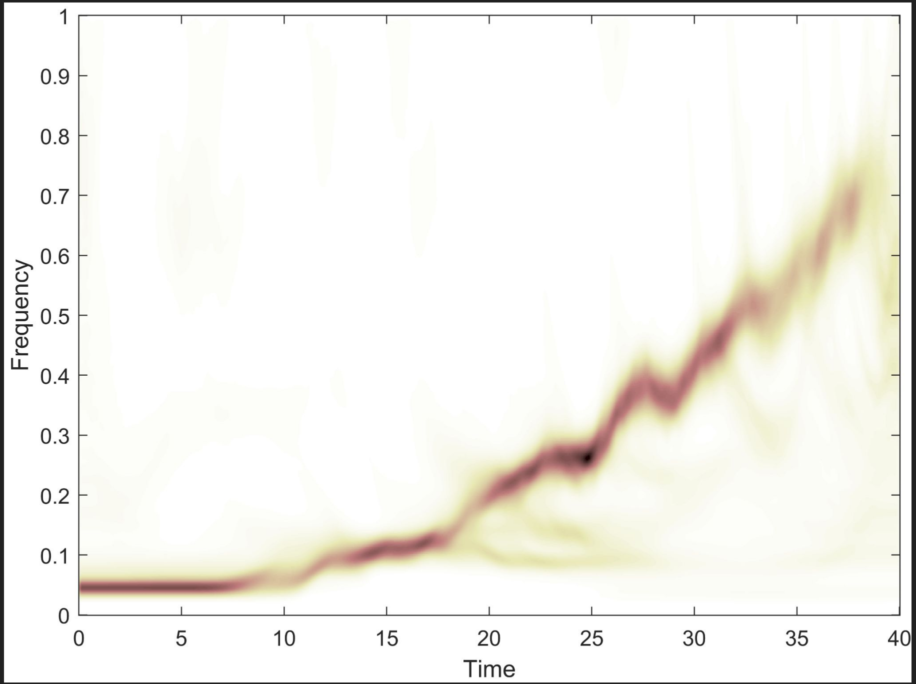
New Approach: Stacking on the Time-Frequency Map



SHHT Result



Frequency
Time
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35
40

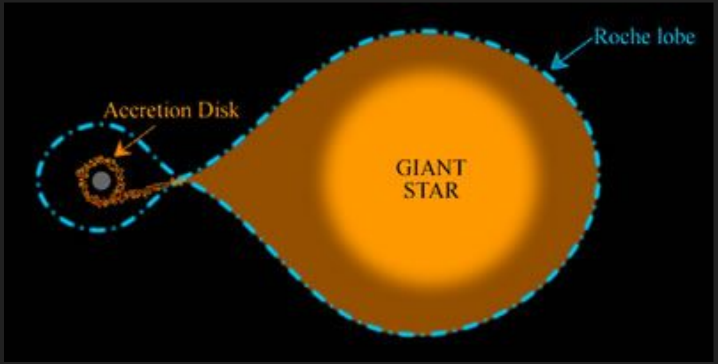


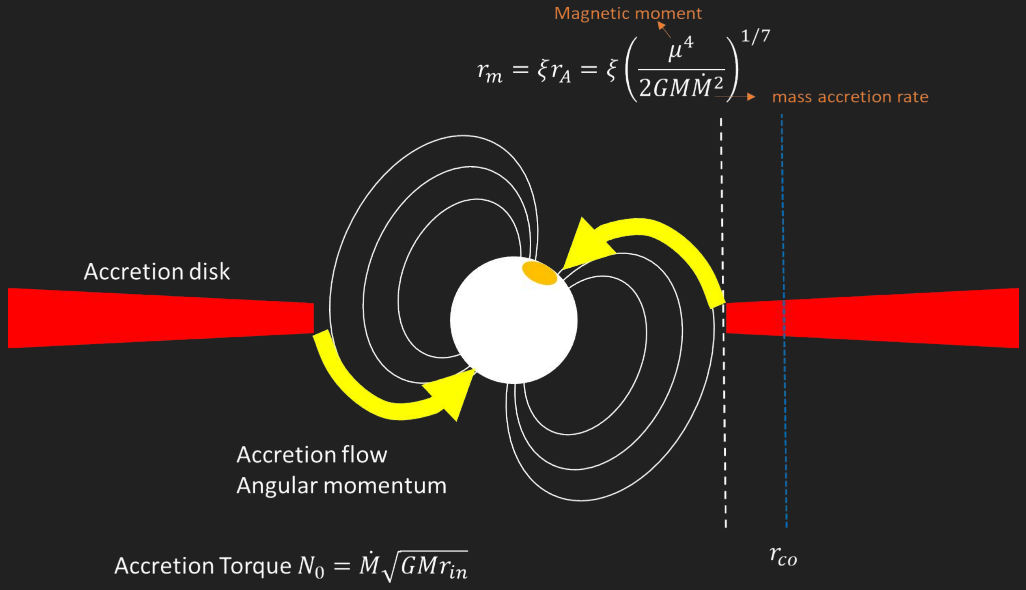
Frequency
Time
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35
40

Application 1: Superorbital Modulation of SMC X-1

SMC X-1

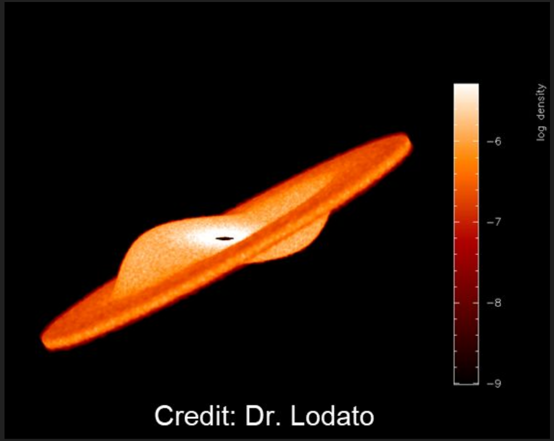
- High-mass X-ray binary with a supergiant companion
  - Roche-lobe overflow
  - X-ray pulsar with P = 0.7 s
    - Spin-up owing to accretion

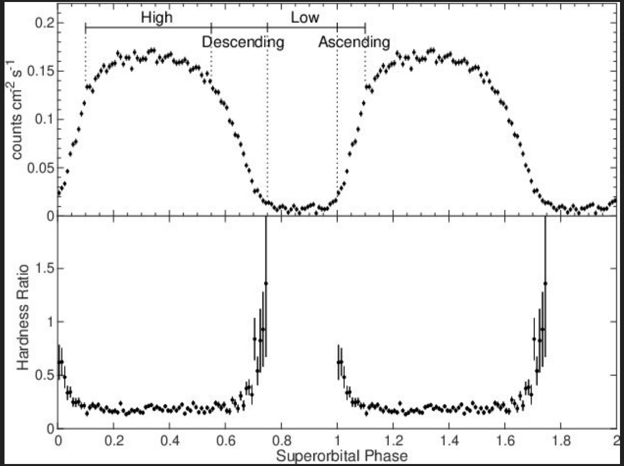




SMC X-1

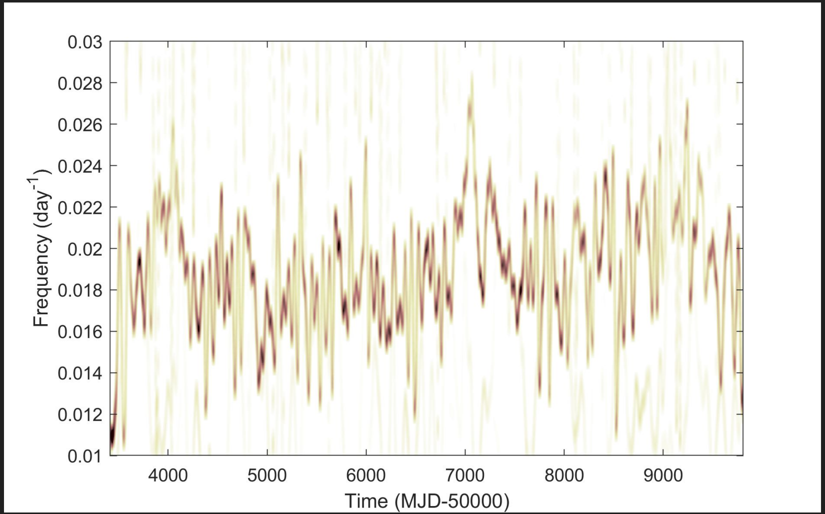
- Orbital period = 3.9 days
  - Eclipse: inclination ~ 60 – 70 degree
- Superorbital period varies between ~35 – ~65 days
  - by precession of warped accretion disk
  - Radiation-driven warp – spin-up rate and superorbital period should have connection to X-ray luminosity.
- X-ray luminosity ~ 5E38 erg/s – super-Eddington
  - Local analog of pulsating ULXs



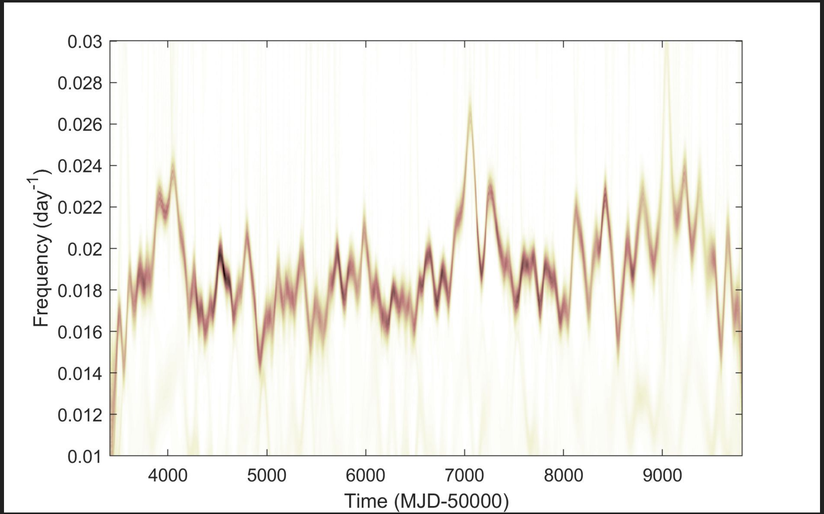


SMC X-1: Superorbital Excursion

HHT

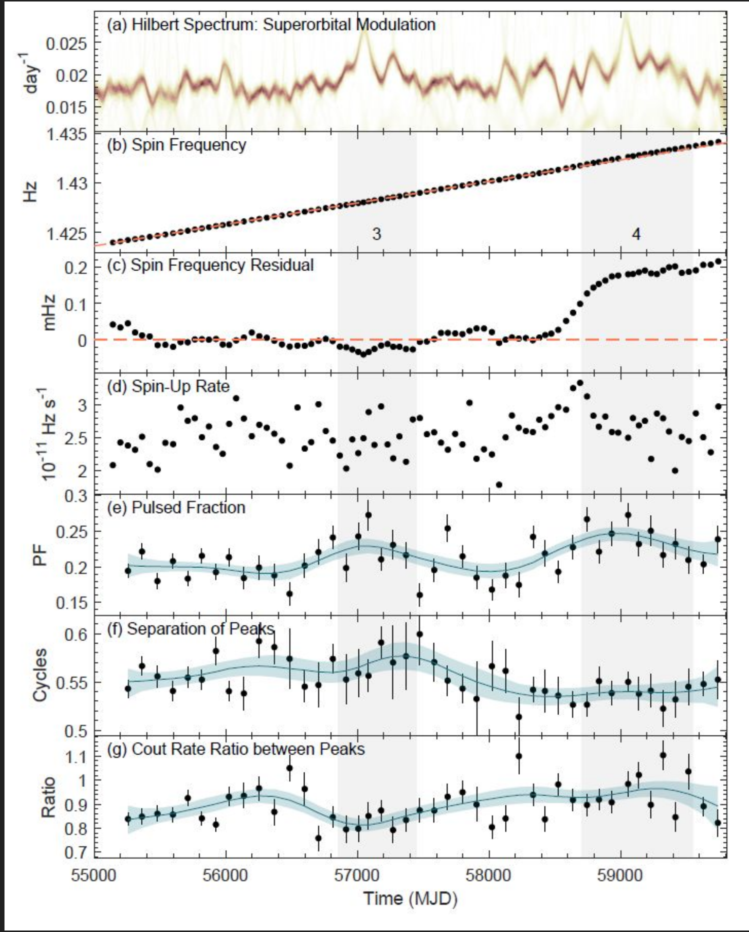


Stacked HHT



SMC X-1: Spin Evolution

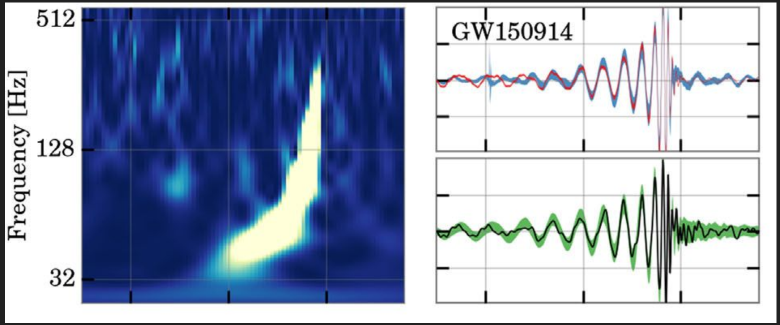
- Spin-up rate changes randomly with time
  - The spin-up accelerates monotonically 2 year before the onset of the 4th excursion, and decay back in ~1 year
    - A significant increase in the residual
    - An inside-out process connecting spin-up rate change and superorbital excursion?
    - Similar event is NOT seen in the 3rd excursion
    - A threshold? A coincidence?
  - The pulsed fraction seems increase during the superorbital excursion
    - Consistent with radiation driven model
    - Future detailed observation with phase-connected analysis is essential



Application 2: Gravitational Wave Signal From Coalscence and Core-Collapse Supernovae

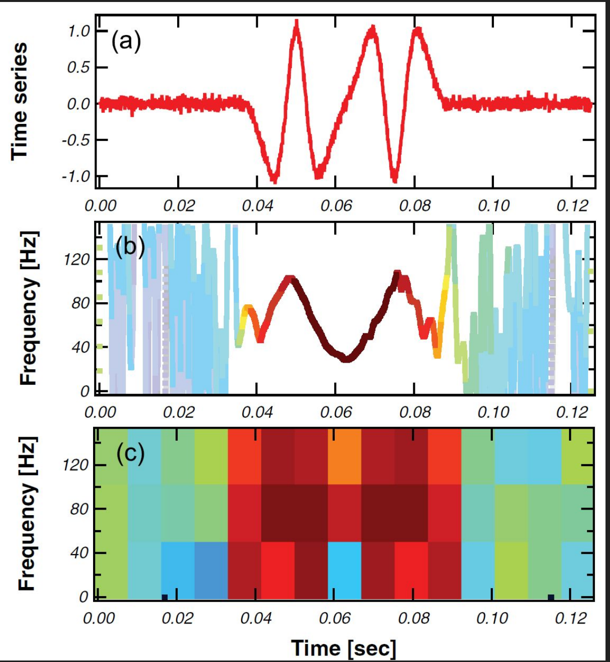
Gravitaional Wave Signal: Binary Coalescence Events

- Gravitational wave signals from the coalescence of binary black holes are transient chirp signals
  - The frequency changes dramatically within a few cycles (e.g., from ~30 Hz to ~300 Hz with 6-7 cycles)
    - A factor of ~10
  - The amplitude also changes quickly.
  - They are difficult to be decomposed into a single IMF.
    - As long as EMD/EEMD is a dyadic filter.
    - Large noise could result in spurious frequency modulation

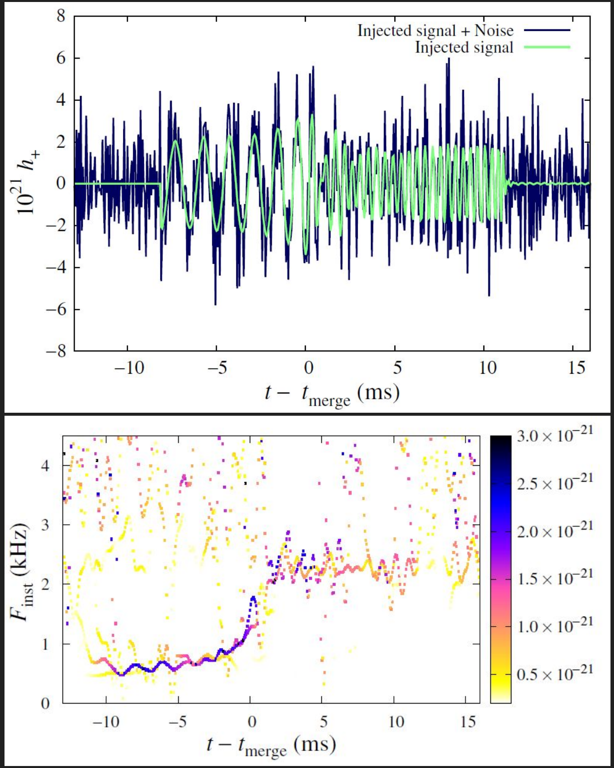


From GWTC-1 (Abbott+2019)

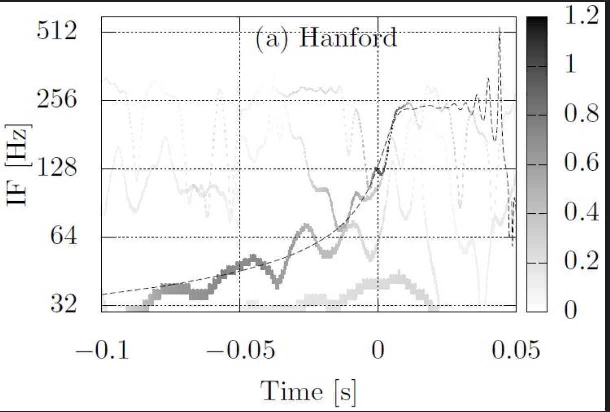
HHT analysis to Gravitational Wave data



Camp+2007

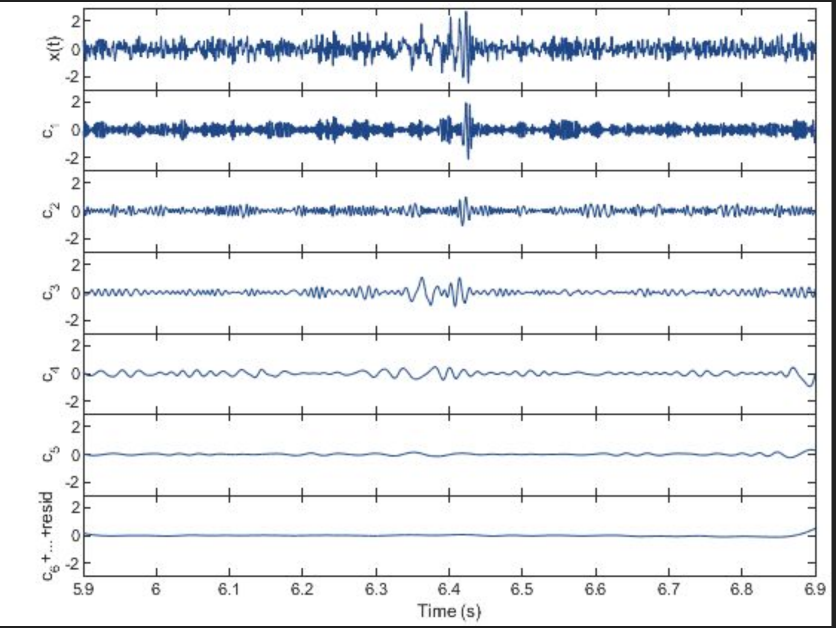


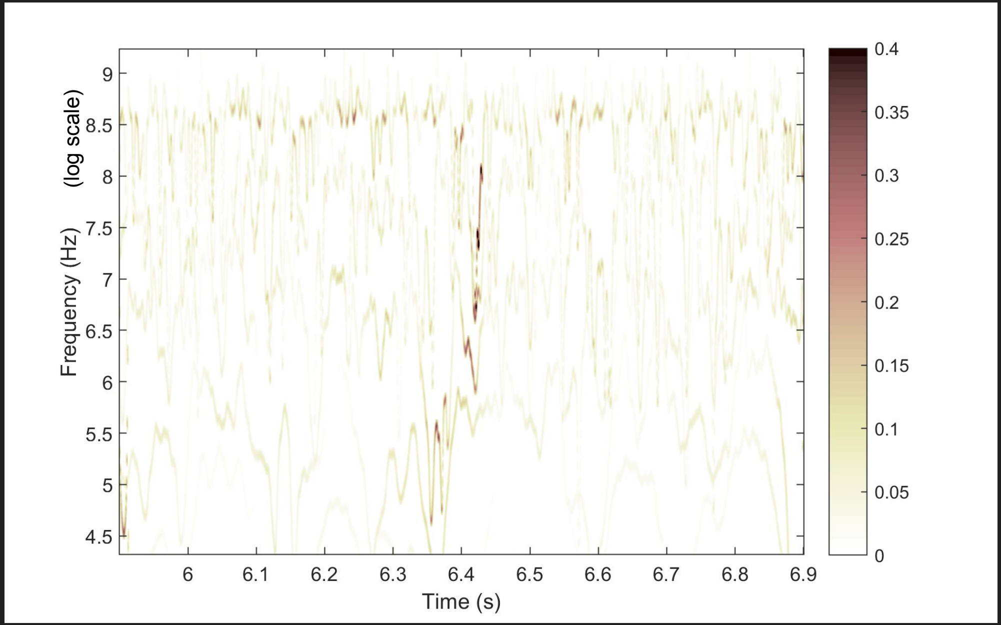
Kaneyama+2016



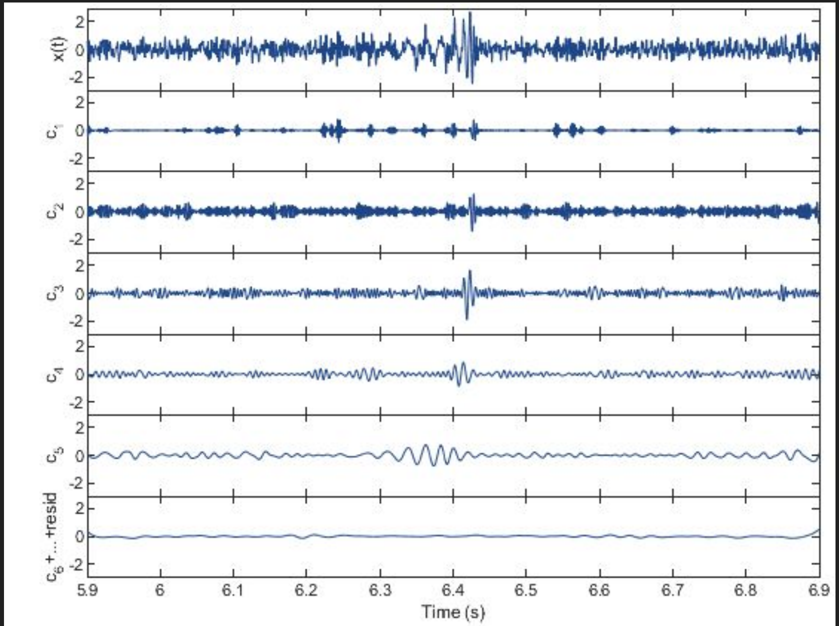
Sakai+2017

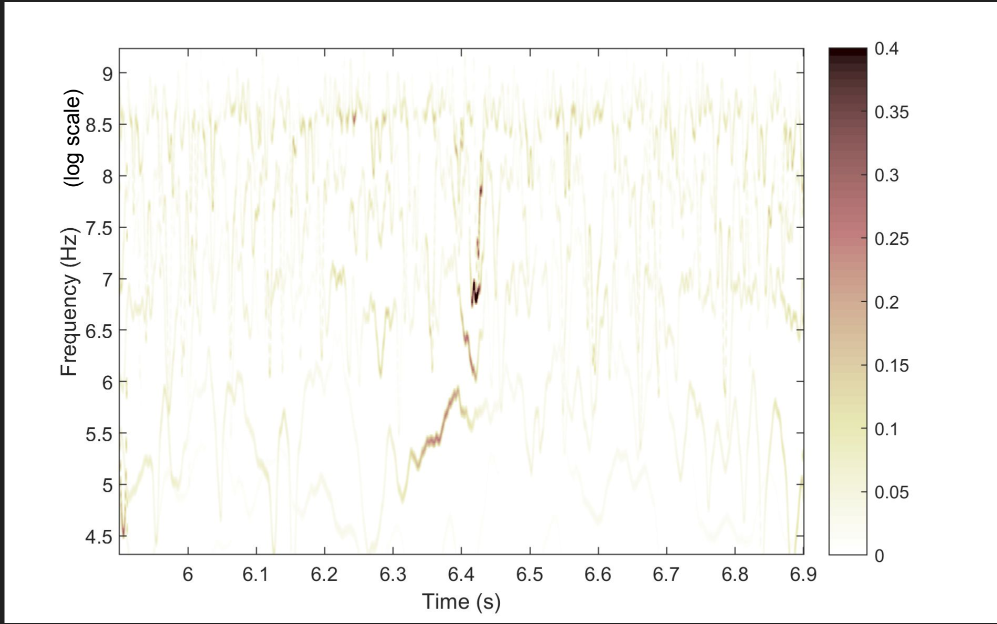
Result: GW150914 (H) with EMD



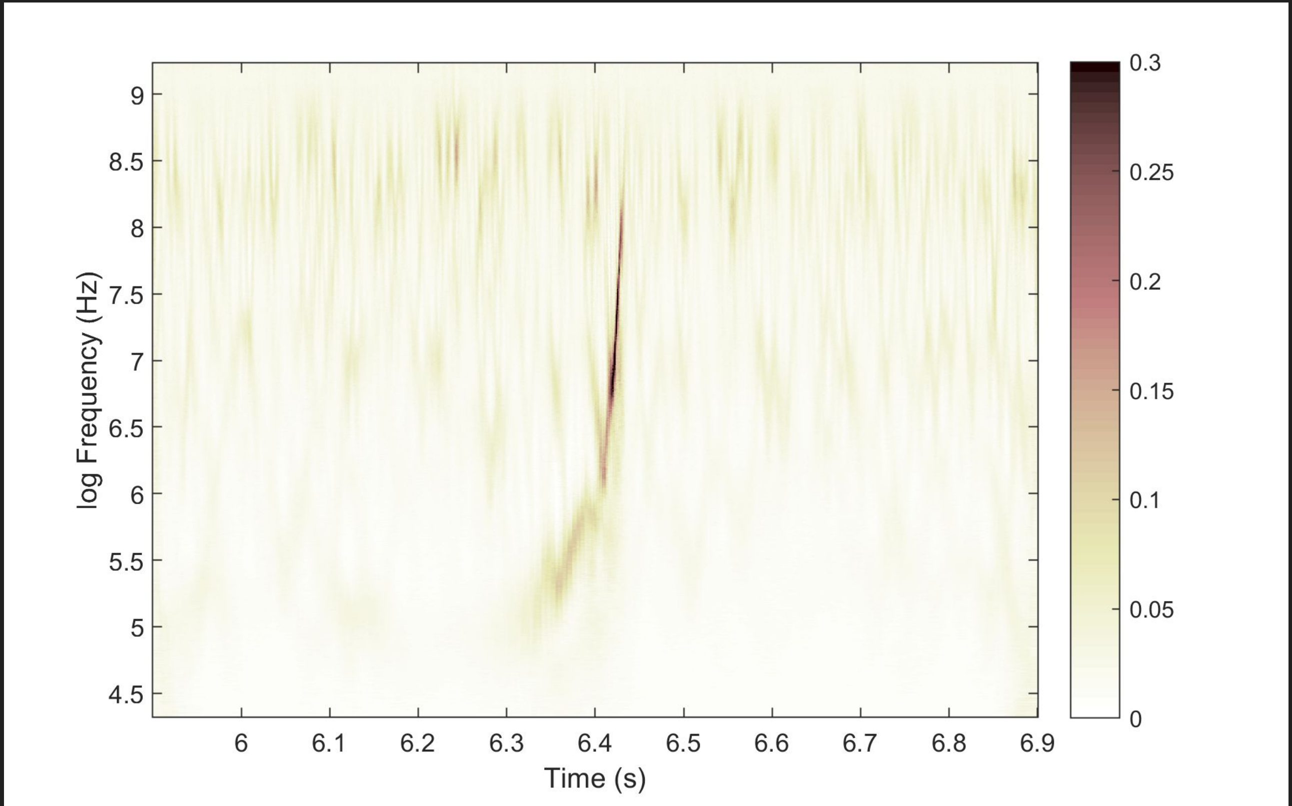


Result: GW150914 with EEMD (noise level = 0.2)





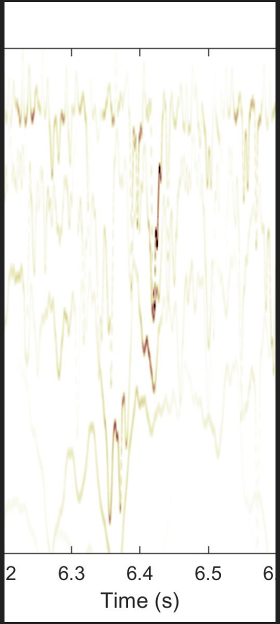
Result: GW150914 with Stacked HHT



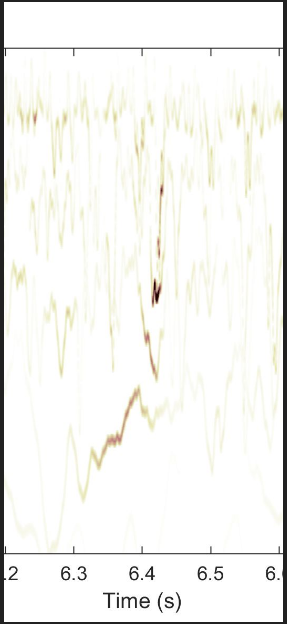
Comparison

HHT

EMD



EEMD



Stacked HHT

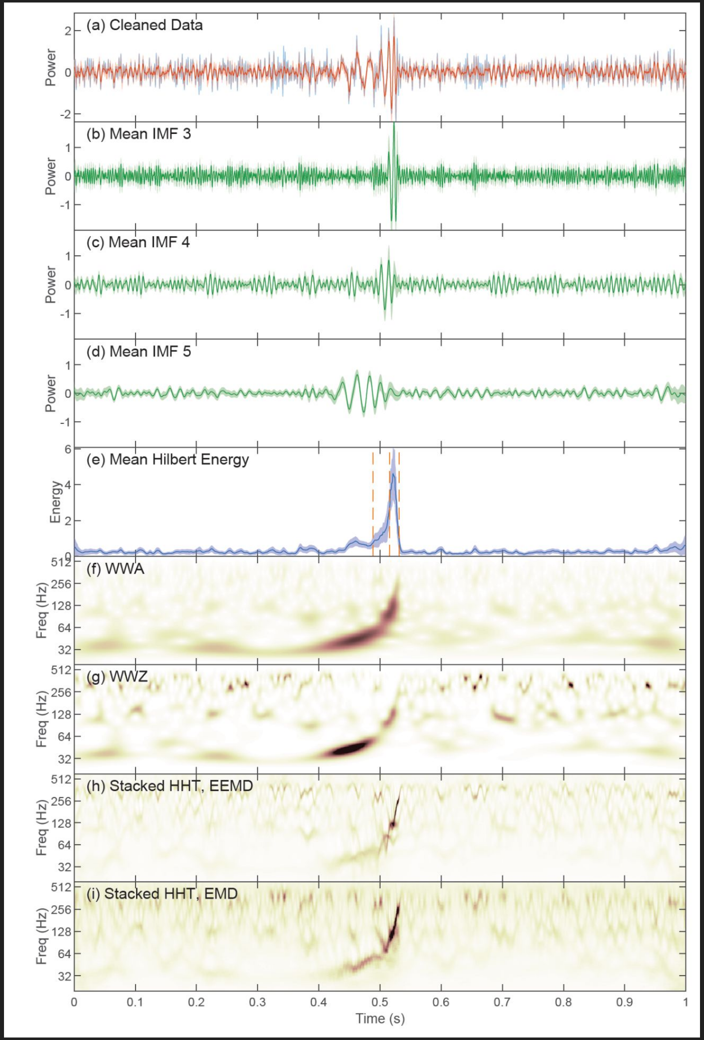
EMD

EEMD

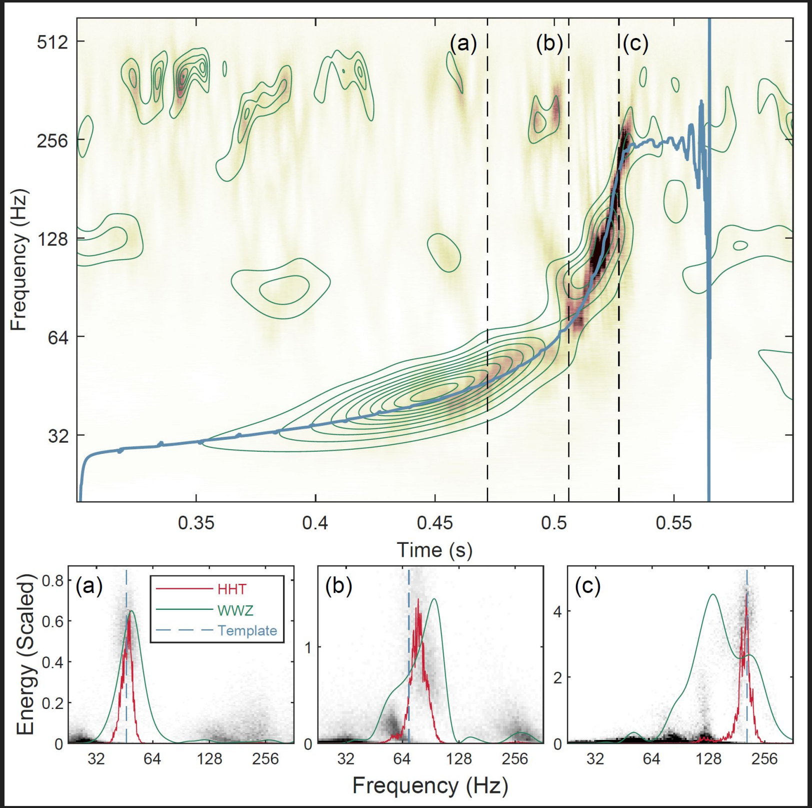
Smoothed using Gaussian Filter

Result: GW150914

- GW150914: A strongest gravitational signal.
    - The signal can be seen by eyes in the cleaned data.
    - The signal is decomposed into at least three IMFs
    - The frequency evolution can be traced very well with both the WWZ and the SHHT

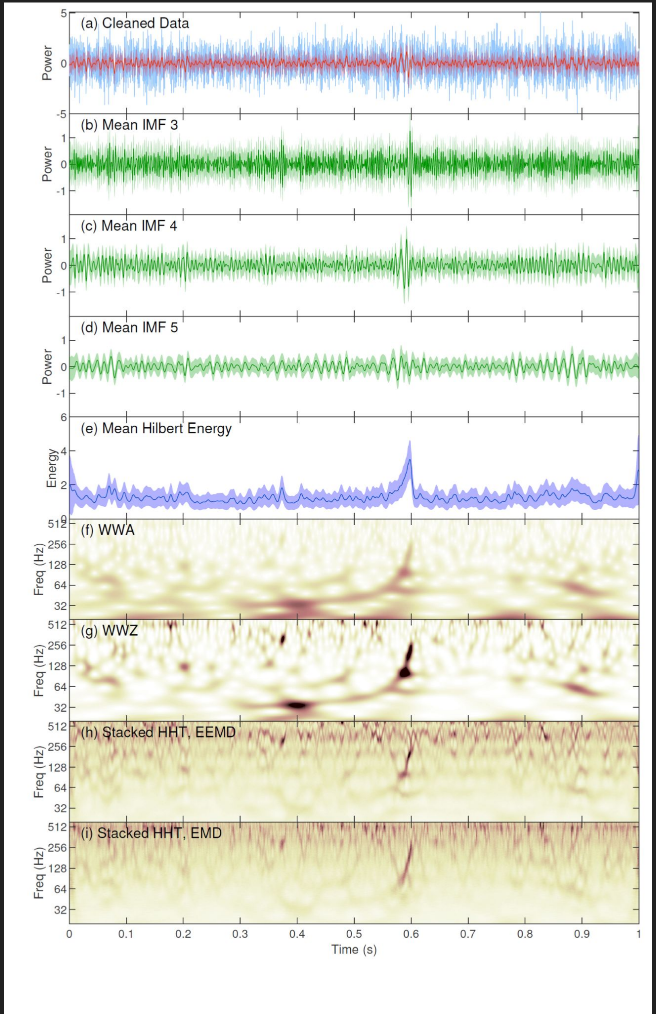


GW 150914: Detailed Comparison



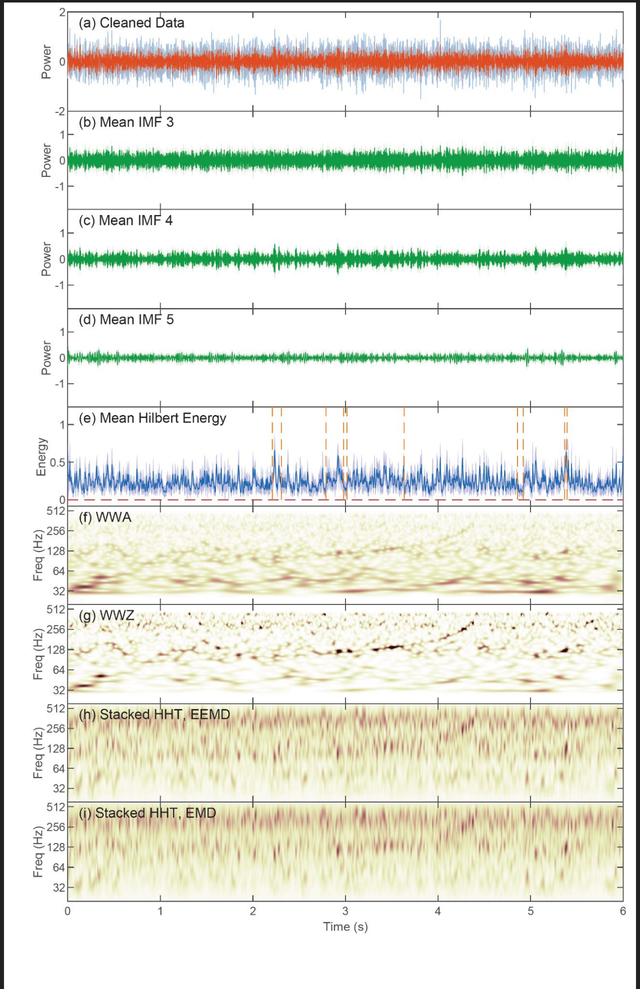
Result: GW170104 (L)

- GW170104: A relatively weak signal
  - The signal can still be seen marginally in decomposed IMFs
  - The time-frequency pattern can still be seen in the stacked Hilbert spectrum.



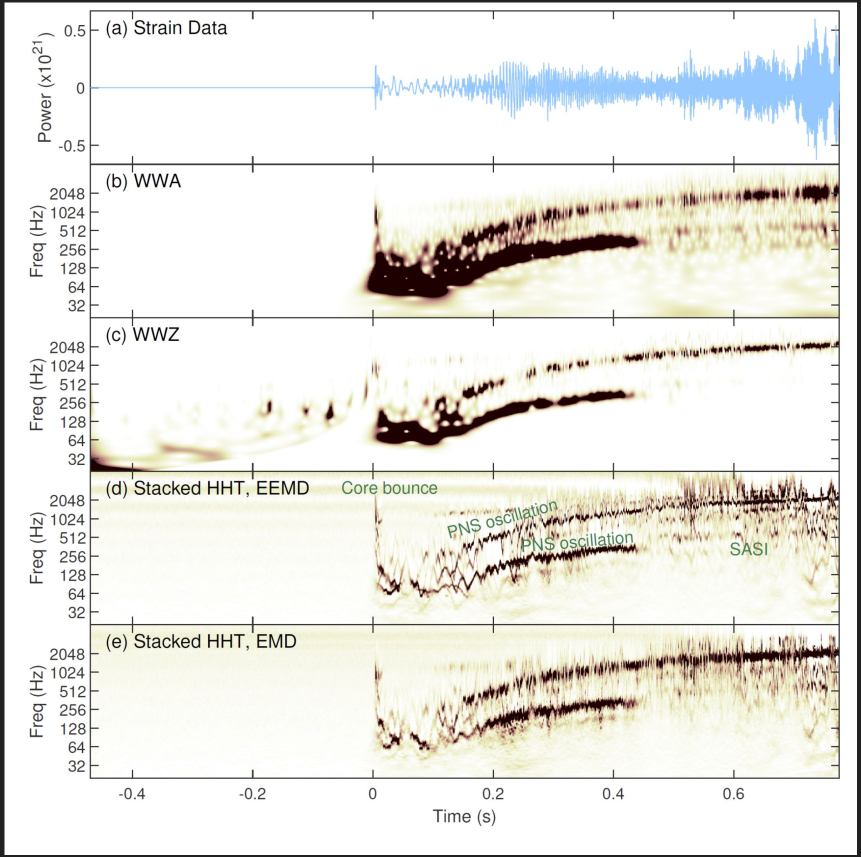
Result: GW170817 (H)

- However, the signal of GW170817 cannot be observed with this algorithm
  - Probably the instantaneous signal to noise ratio is too low?
  - Detailed mathematical proof and statistical tests are under developing (Yen et al. in prep)

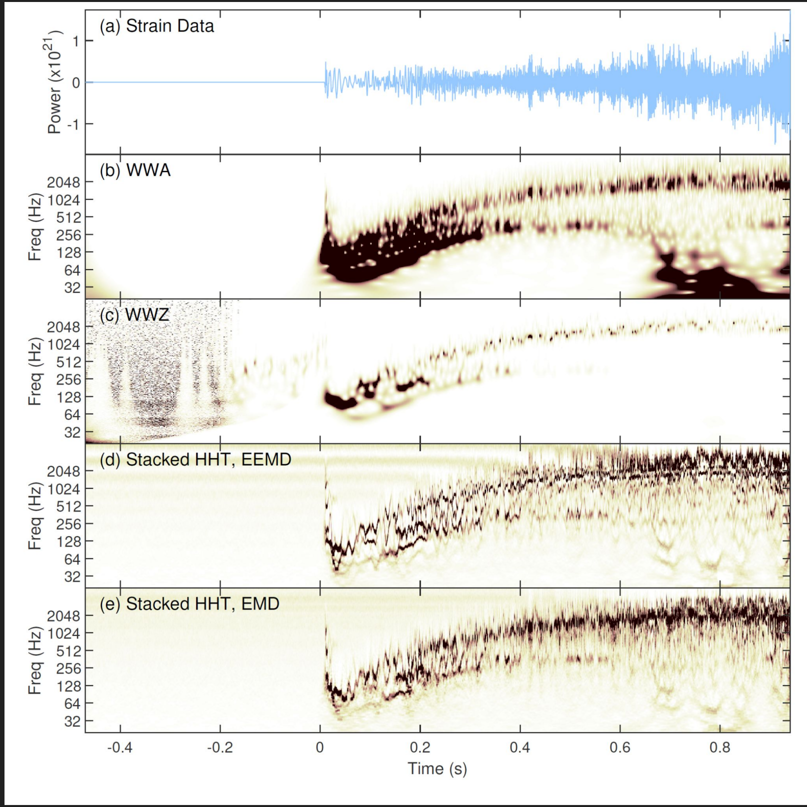


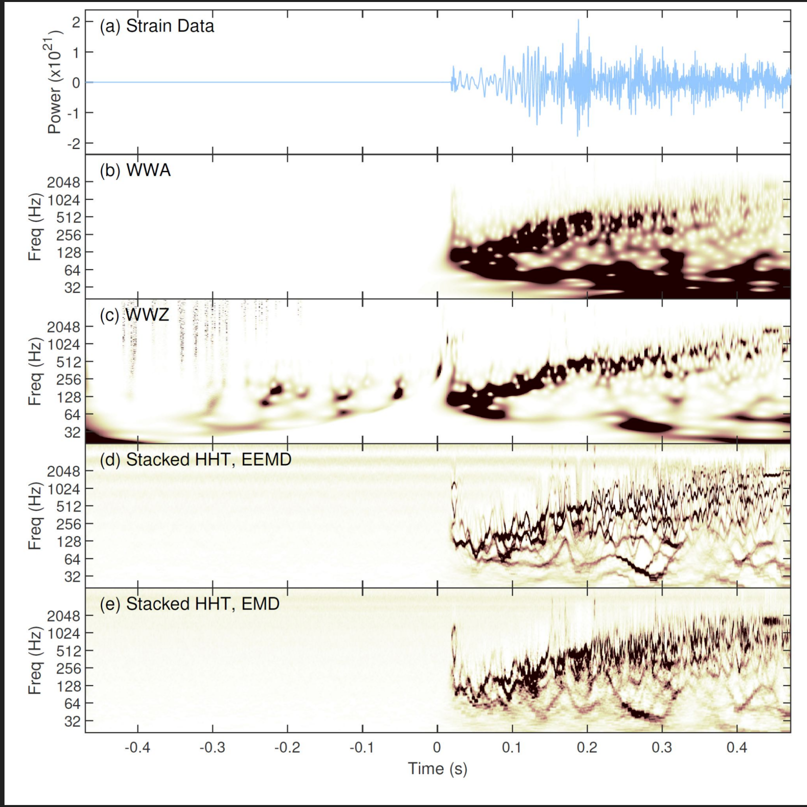
Result: Core-Collapse Supernovae

- Detection of GW signals from a nearby CCSN will be the milestone of next-generation GW observatory
  - The cross component seen from the equatorial plane with 40 solar mass progenitor without rotation
  - The signal could be much more complicated
  - The stacked HHT would be helpfin in recognizing different components of the GW signal from a CCSN.

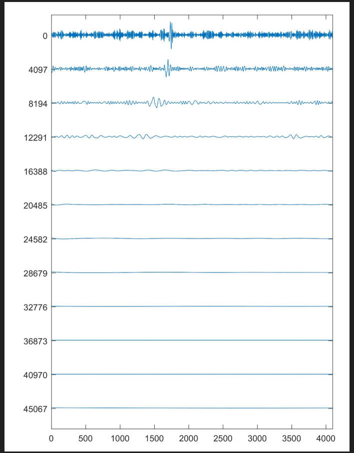


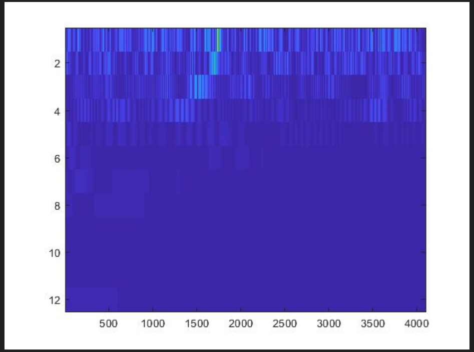
Result: Slow Rotators and Fast Rotators

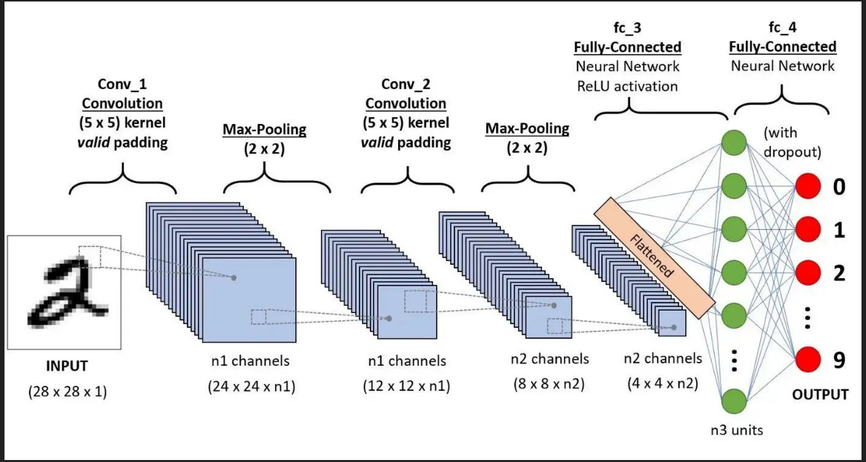




Future Work: EMD-CNN Model for GW Detection







- The IMFs effectively forms a 2-D map, and the GW signal draws a special pattern on the map
  - Good for convolutional neural network: image classification
  - Precise time-frequency information is not required

Summary

- The HHT is a novel technique for time-frequency analysis although the mode-mixing problem is severe for signals with huge frequency variability.
  - We propose a method that could minimize the spurious signals by stacking the time-frequency map instead of taking the ensemble mean of the IMFs.
- We successfully applied this algorithm on X-ray light curve of SMC X-1 to trace the evolution of the superorbital modulation
  - We observed a clear time coincidence between the superorbital excursion and spin-up acceleration.
- We successfully applied this algorithm on GWTC-1 data and found that most of their frequency evolution can be well tracked.
- The HHT is an ideal tool for further investigating the time-frequency properties of a signal
  - Powerful in describing the detailed structures in CCSN events