Helicity amplitudes without gauge cancellation

Kentarou Mawatari
健太郎　馬渡

with Junmou Chen (Jinan U.)
Kaoru Hagiwara (KEK)
Junichi Kanzaki (Kavli IPMU)
Ya-Juan Zheng (Iwate U.)

arXiv: 2003.03003(EPJC), 2203.10440, 2211.14562

@ Iwate U. campus

A warm-up survey: what is your field ?

arXiv

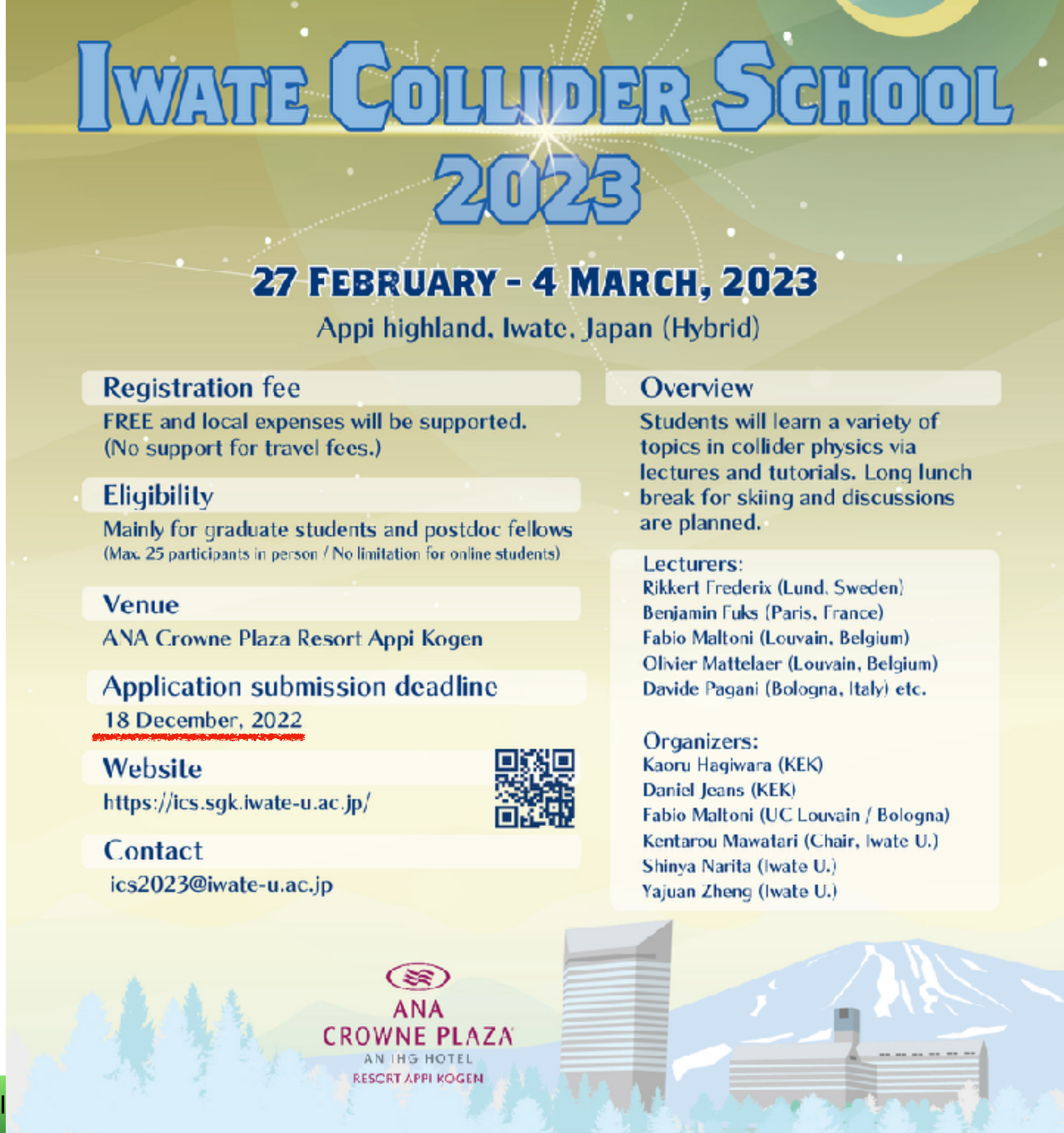
Physics

- Astrophysics (**astro-ph** new, recent, search)
  includes: Astrophysics of Galaxies; Cosmology and Nongalactic Astrophysics
  Solar and Stellar Astrophysics
- Condensed Matter (**cond-mat** new, recent, search)
  includes: Disordered Systems and Neural Networks; Materials Science; Mesos
  Correlated Electrons; Superconductivity
- General Relativity and Quantum Cosmology (**gr-qc** new, recent, search)
- High Energy Physics - Experiment (**hep-ex** new, recent, search)
- High Energy Physics - Lattice (**hep-lat** new, recent, search)
- High Energy Physics - Phenomenology (**hep-ph** new, recent, search)
- High Energy Physics - Theory (**hep-th** new, recent, search)
- Mathematical Physics (**math-ph** new, recent, search)
- Nonlinear Sciences (**nlin** new, recent, search)
  includes: Adaptation and Self-Organizing Systems; Cellular Automata and La
- Nuclear Experiment (**nucl-ex** new, recent, search)
- Nuclear Theory (**nucl-th** new, recent, search)
- Physics (**physics** new, recent, search)
  includes: Accelerator Physics; Applied Physics; Atmospheric and Oceanic Phy
  Physics; Data Analysis, Statistics and Probability; Fluid Dynamics; General Phy
  Society; Physics Education; Plasma Physics; Popular Physics; Space Physics
- Quantum Physics (**quant-ph** new, recent, search)

A warm-up question: where is Iwate ?

International Linear Collider

Iwate University
岩手大学


IWATE COLLIDER SCHOOL
2023
27 FEBRUARY - 4 MARCH, 2023
Appi highland, Iwate, Japan (Hybrid)
Registration fee
FREE and local expenses will be supported.
(No support for travel fees.)
Eligibility
Mainly for graduate students and postdoc fellows
(Max. 25 participants in person / No limitation for online students)
Venue
ANA Crowne Plaza Resort Appi Kogen
Application submission deadline
18 December, 2022
Website
https://ics.sgk.iwate-u.ac.jp/
Contact
ics2023@iwate-u.ac.jp
Overview
Students will learn a variety of topics in collider physics via lectures and tutorials. Long lunch break for skiing and discussions are planned.
Lecturers:
Rikkert Frederix (Lund, Sweden)
Benjamin Fuks (Paris, France)
Fabio Maltoni (Louvain, Belgium)
Olivier Mattelaer (Louvain, Belgium)
Davide Pagani (Bologna, Italy) etc.
Organizers:
Kaoru Hagiwara (KEK)
Daniel Jeans (KEK)
Fabio Maltoni (UC Louvain / Bologna)
Kentarou Mawatari (Chair, Iwate U.)
Shinya Narita (Iwate U.)
Yajuan Zheng (Iwate U.)
ANA
CROWNE PLAZA
AN IHG HOTEL
RESORT APPI KOGEN

Iwate Collider School 2023

Moriond-like

Program

All the lectures will be given at the venue, but you can also participate online. The tutorials are only for the participants in person.

Feb. 27 (Mon)	Feb. 28 (Tue)	Mar. 1 (Wed)	Mar. 2 (Thu)	Mar. 3 (Fri)	Mar. 4 (Sat)
	L2	L4	L6	L8	L10
	L3	L5	L7	L9	Departure
	Lunch / Ski break	Lunch / Ski break	Lunch / Ski break	Lunch / Ski break	
Arrival	T1	T3	T5	T7	
L1	T2	T4	T6	T8	

編集

8:30-11:30 : lectures (including break)

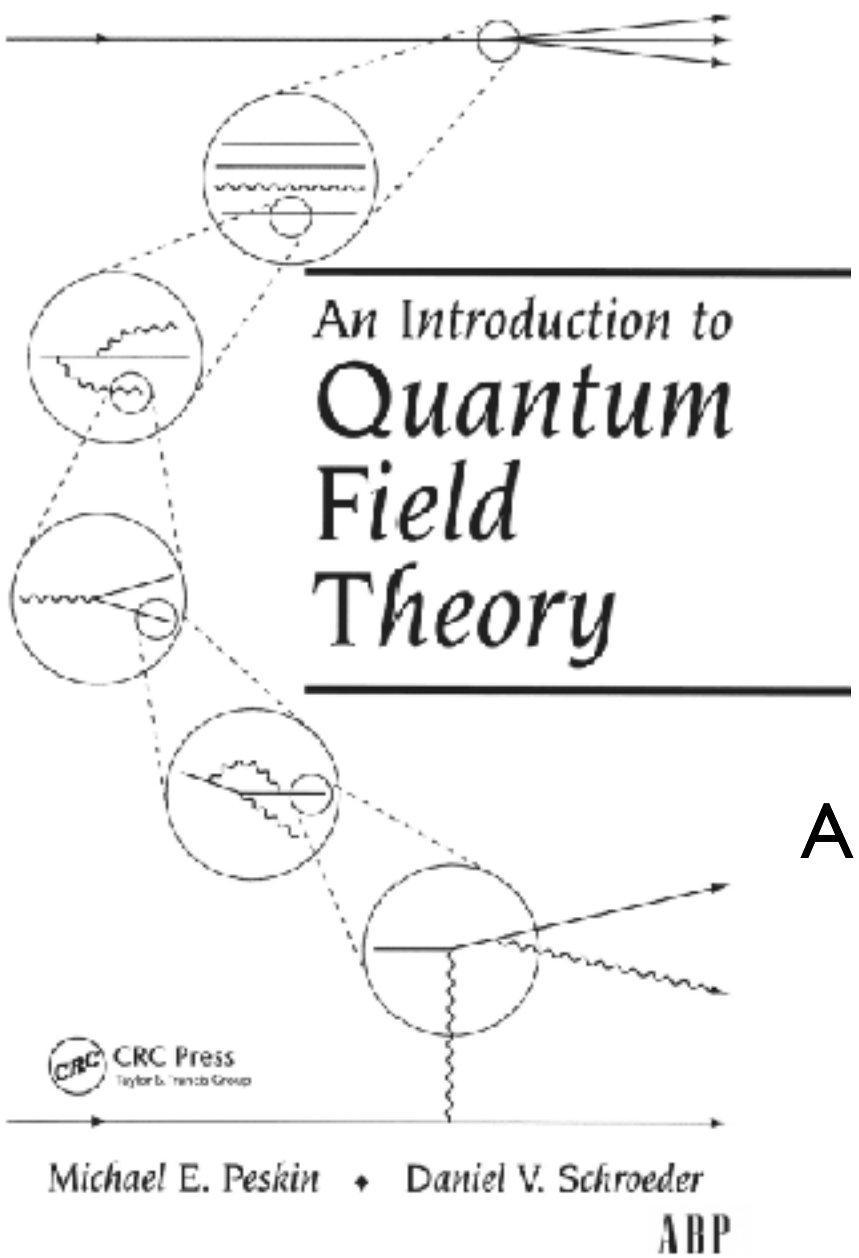
11:30-16:30 : lunch/ski break

16:30-19:30 : lectures / tutorials (including break)

19:30 : dinner


Iwate Collider School 2022 (ICS2022)
Mar/21-26 @Appi ski resort in Iwate
APPI
APPI
APPI
APPI
APPI
We are waiting for your application!

When I was a PhD student, …



744 Chapter 21 Quantization of Spontaneously Broken Gauge Theories

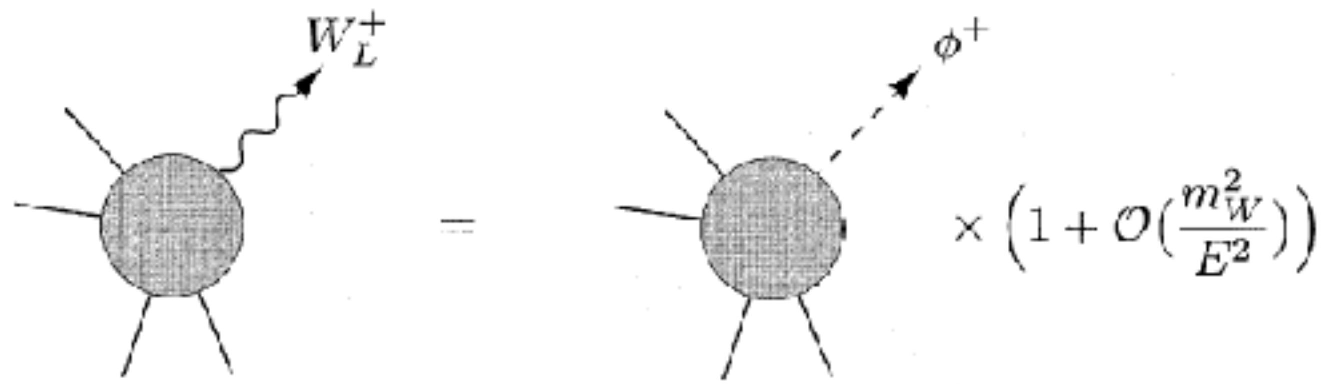


Figure 21.3. The Goldstone boson equivalence theorem. At high energy, the amplitude for emission or absorption of a longitudinally polarized massive gauge boson becomes equal to the amplitude for emission or absorption of the Goldstone boson that was eaten by the gauge boson.

A gauge boson (massless; v=c)

➡eats a Nambu-Goldstone boson (NGB)

➡becomes fat (massive; v<c)

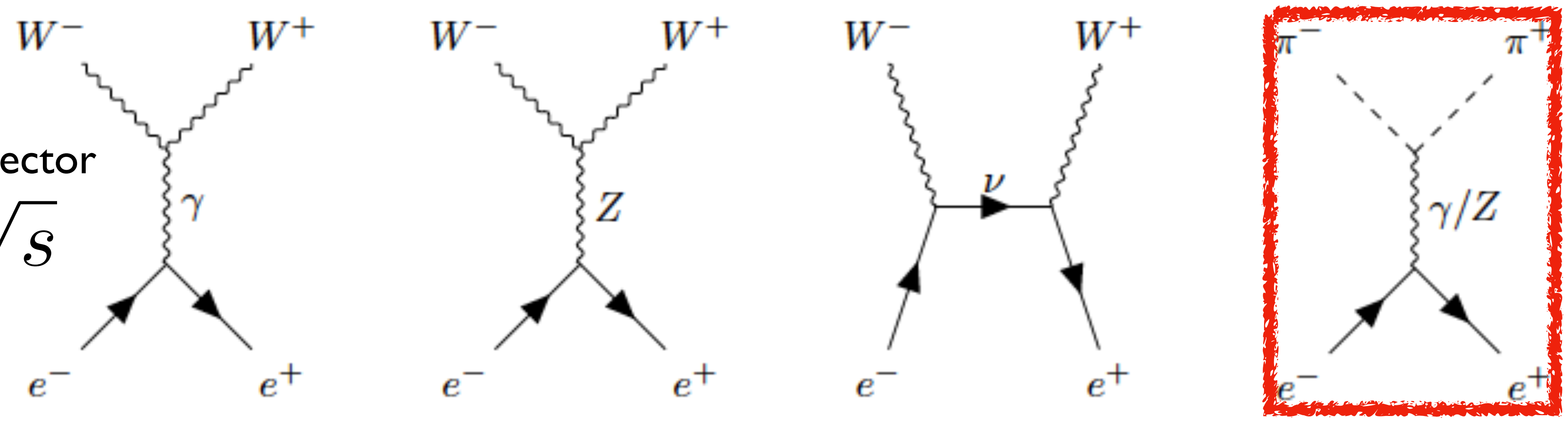
➡cannot digest the NGB well

➡throw up the NGB … **NGB equivalence theorem**

Peskin&Schroeder (Chapter 21) tells …

longitudinal polarization vector

$$\epsilon_L^\mu \propto p^\mu \sim \sqrt{s}$$



$$\mathcal{M}(e^+e^- \to W_L^+ W_L^-)_{\gamma,Z,\nu} \propto (\sqrt{s})^2$$

$$\cancel{\Longrightarrow} \sigma(e^+e^- \to W_L^+ W_L^-) \propto s$$ Unitarity violation ???

$$\Longrightarrow \sigma(e^+e^- \to W_L^+ W_L^-) \propto 1/s$$ Miracle gauge cancellation among the 3 amplitudes !!!

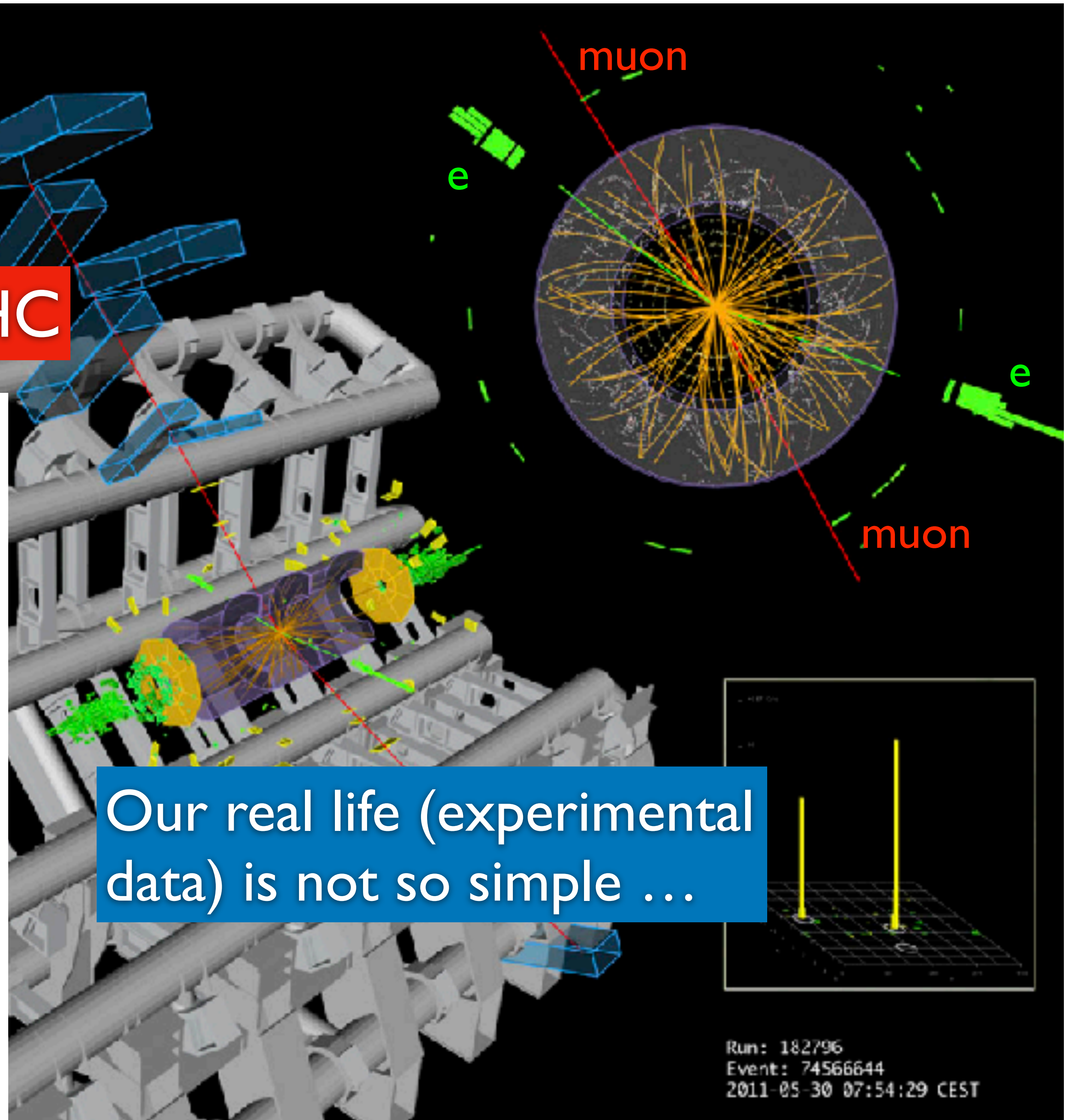
$$\approx \sigma(e^+e^- \to \pi^+\pi^-)$$ Goldstone-boson equivalence theorem

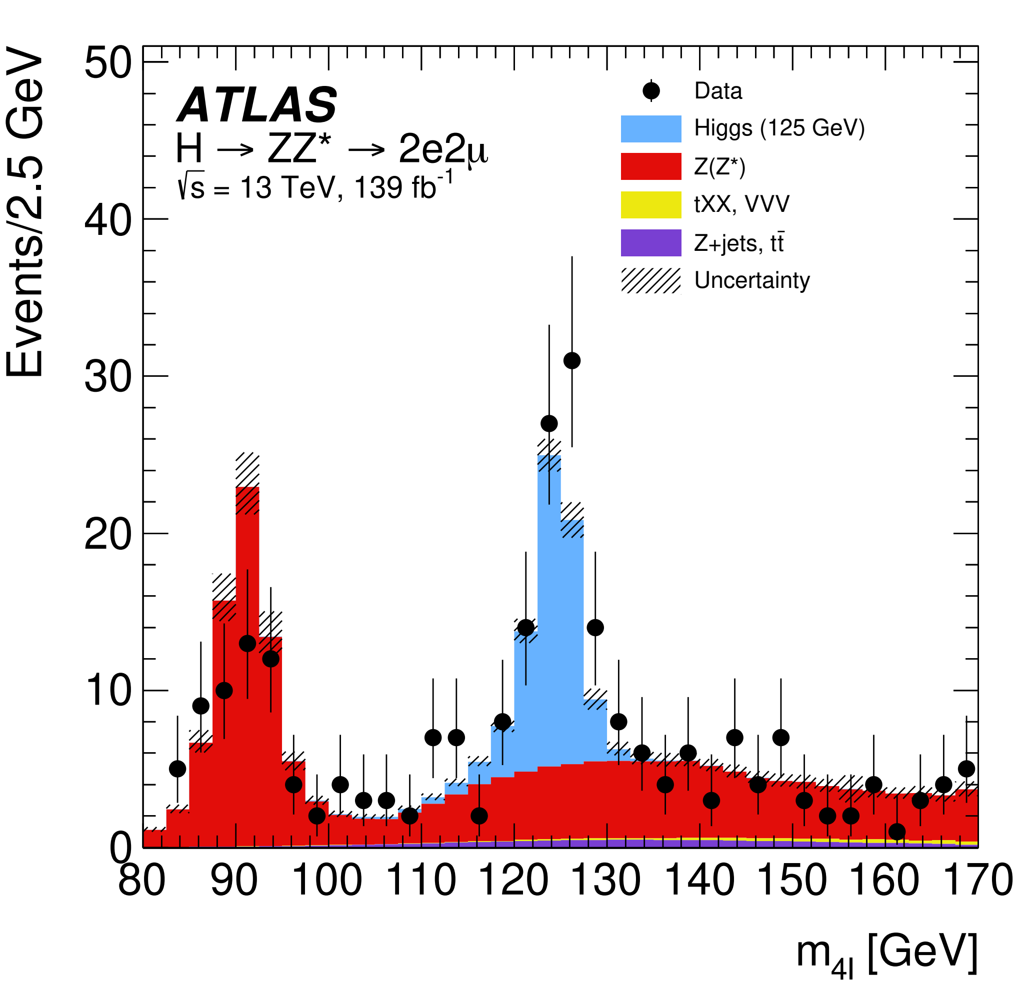
in the high-energy limit

Beauty of the spontaneously broken EW gauge theory

On the other hand, we need experimental tests to proof theories.

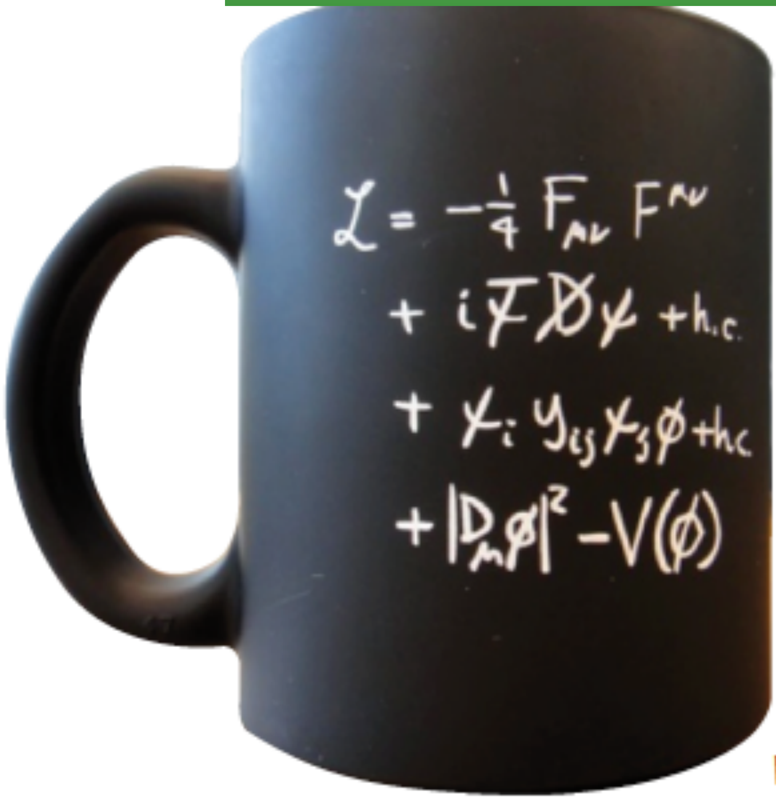
e.g. Higgs production @ LHC





Our real life (experimental data) is not so simple …

Lagrangian (TH) ⇔ Data (EXP)

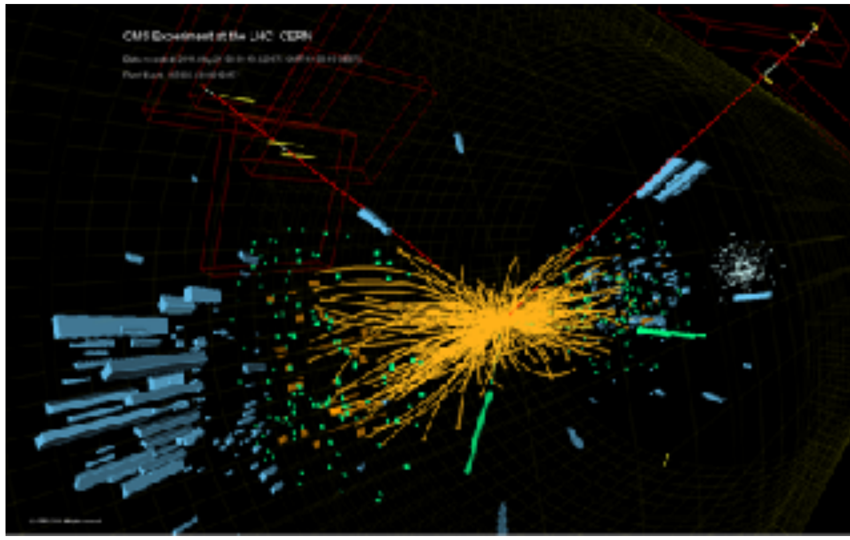


simulation tools

prediction

interpretation

MadGraph is one of such tools to generate events, based on numerical evaluation of helicity amplitudes.



HELAS (HELicity Amplitude Subroutines) plays a main role in MadGraph to evaluate helicity amplitudes numerically, invented by K.Hagiwara, H.Murayama and I.Watanabe in 1991.

Eur. Phys. J. C
DOI 10.1140/epjc/s10052-008-0663-x

THE EUROPEAN
PHYSICAL JOURNAL C

Special Article - Tools for Experiment and Theory

[arXiv:0805.2554]

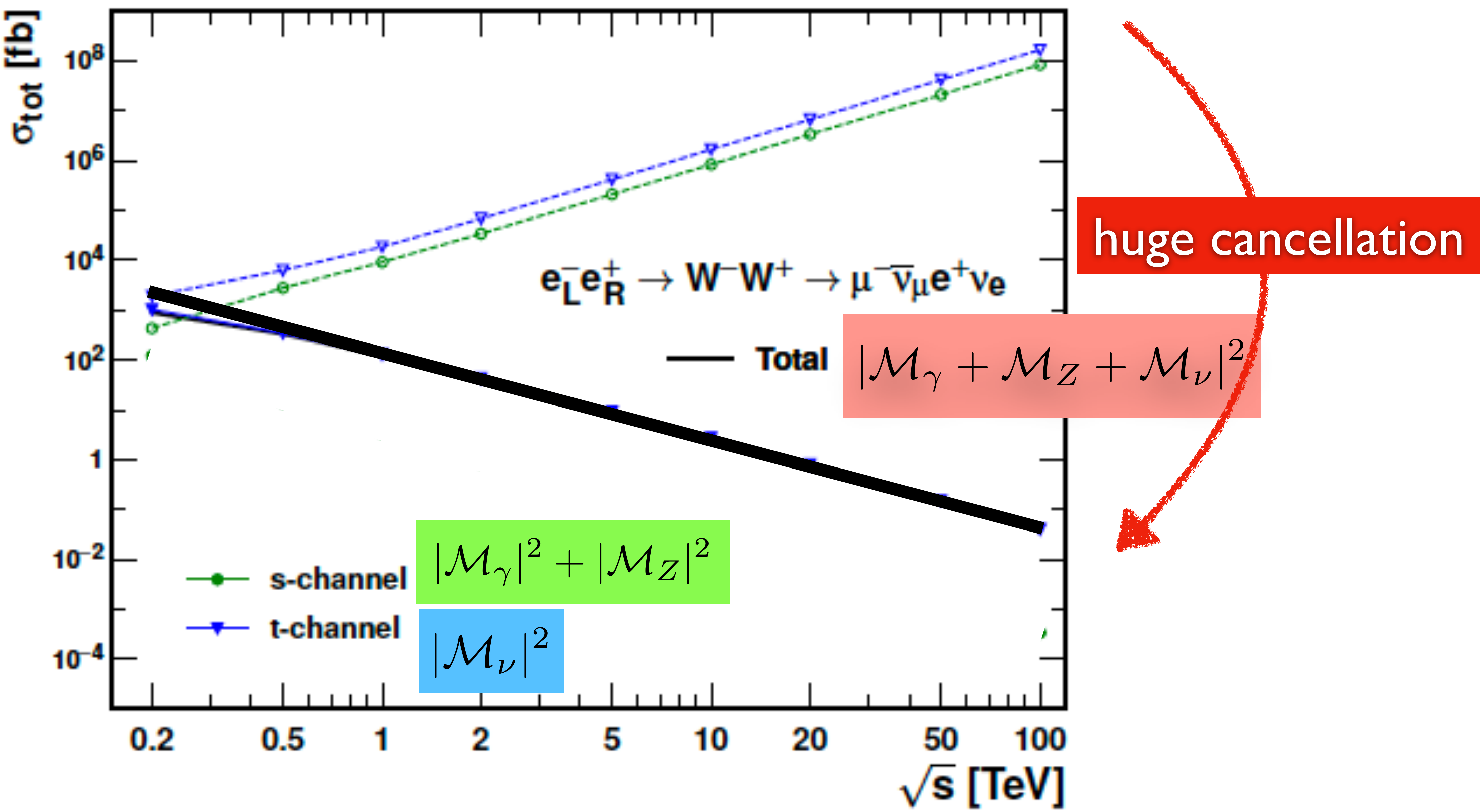
HELAS and MadGraph/MadEvent with spin-2 particles

K. Hagiwara[1], J. Kanzaki[2,a], Q. Li[3,b], K. Mawatari[4,c]
[1]KEK, Theory Division and Sokendai, Tsukuba 305-0801, Japan
[2]KEK, Tsukuba 305-0801, Japan
[3]Institut für Theoretische Physik, Universität Karlsruhe, Postfach 69
[4]School of Physics, Korea Institute for Advanced Study, Seoul 130-7

Vertex	Inputs	Output	Subroutine
SST	SST	Amplitude	SSTXXX
	ST	S	HSTXXX
	SS	T	USSXXX
FFT	FFT	Amplitude	IOTXXX
	FT	F	FTIXXX, FTOXXX
	FF	T	UIOXXX
VVT	VVT	Amplitude	VVTXXX
	VT	V	JVTXXX
	VV	T	UVVXXX
FFVT	FFVT	Amplitude	IOVTXX
	FVT	F	FVTIXX, FVTOXX
	FFT	V	JIOTXX
	FFV	T	UIOVXX
VVVT	VVVT	Amplitude	VVVTXX
	VVT	V	JVVTXX
	VVV	T	UVVVXX
VVVVT	GGGGT	Amplitude	GGGGTX
	GGGT	G	JGGGTX
	GGGG	T	UGGGGX

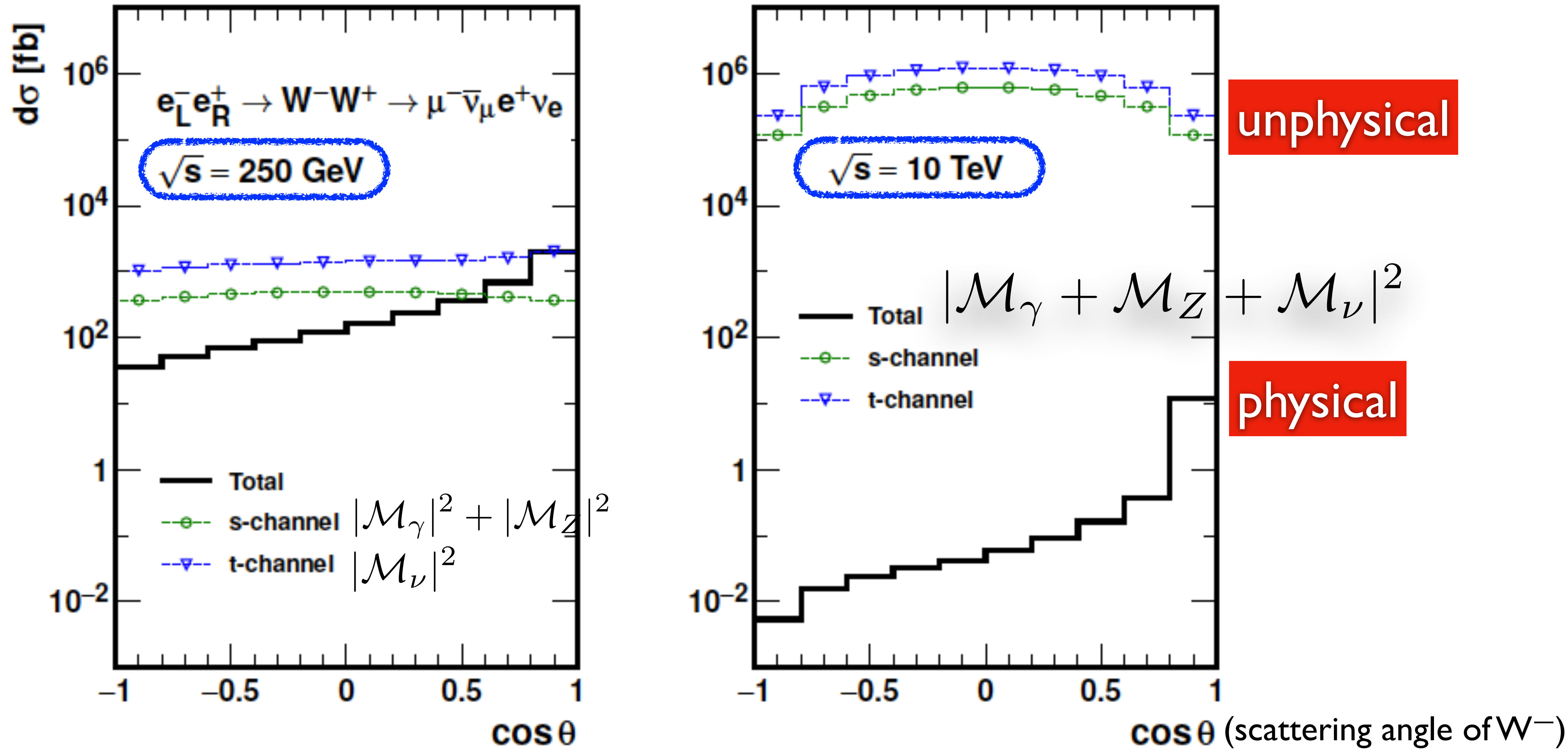
```
      TKK = TKK - T12*(pv1(1)*pv2(2) + pv1(2)*pv2(1))
     &          - T13*(pv1(1)*pv2(3) + pv1(3)*pv2(1))
     &          - T14*(pv1(1)*pv2(4) + pv1(4)*pv2(1))
     &          + T23*(pv1(2)*pv2(3) + pv1(3)*pv2(2))
     &          + T24*(pv1(2)*pv2(4) + pv1(4)*pv2(2))
     &          + T34*(pv1(3)*pv2(4) + pv1(4)*pv2(3))

      TK1V2 = TK1V2 - T12*(pv1(1)*v2(2) + pv1(2)*v2(1))
     &              - T13*(pv1(1)*v2(3) + pv1(3)*v2(1))
     &              - T14*(pv1(1)*v2(4) + pv1(4)*v2(1))
     &              + T23*(pv1(2)*v2(3) + pv1(3)*v2(2))
     &              + T24*(pv1(2)*v2(4) + pv1(4)*v2(2))
     &              + T34*(pv1(3)*v2(4) + pv1(4)*v2(3))

      TVV = TVV - T12*(v1(1)*v2(2) + v1(2)*v2(1))
     &          - T13*(v1(1)*v2(3) + v1(3)*v2(1))
     &          - T14*(v1(1)*v2(4) + v1(4)*v2(1))
     &          + T23*(v1(2)*v2(3) + v1(3)*v2(2))
     &          + T24*(v1(2)*v2(4) + v1(4)*v2(2))
     &          + T34*(v1(3)*v2(4) + v1(4)*v2(3))

      TK2V1 = TK2V1 - T12*(v1(1)*pv2(2) + v1(2)*pv2(1))
     &              - T13*(v1(1)*pv2(3) + v1(3)*pv2(1))
     &              - T14*(v1(1)*pv2(4) + v1(4)*pv2(1))
     &              + T23*(v1(2)*pv2(3) + v1(3)*pv2(2))
     &              + T24*(v1(2)*pv2(4) + v1(4)*pv2(2))
     &              + T34*(v1(3)*pv2(4) + v1(4)*pv2(3))

      vertex = (ft(1,1)-ft(2,2)-ft(3,3)-ft(4,4))*( K1V2*K2V1 - V1V2*F )
     &       + F*TVV + V1V2*TKK - K2V1*TK1V2 - K1V2*TK2V1
```

Numerical check for gauge cancellation



At very high energies, event generators encounter numerical problems due to cancellation of significant digits …

Moreover ...



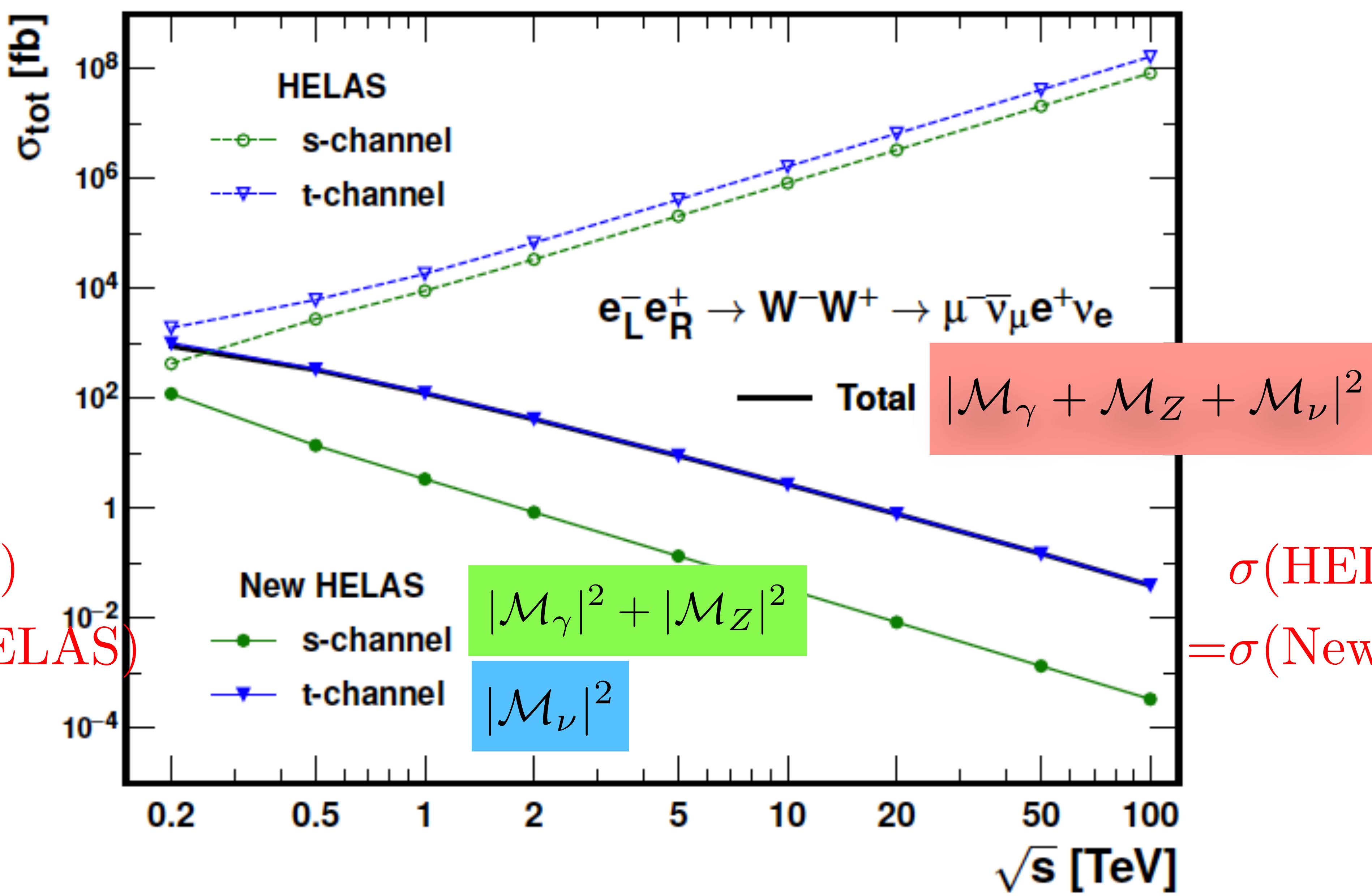
Each amplitude is (too) unphysical ...

Do we have any choice?

YES!!!

Highlight of New HELAS (total cross section)

$\mathcal{M}_i(\text{HELAS})$
$\neq \mathcal{M}_i(\text{New HELAS})$



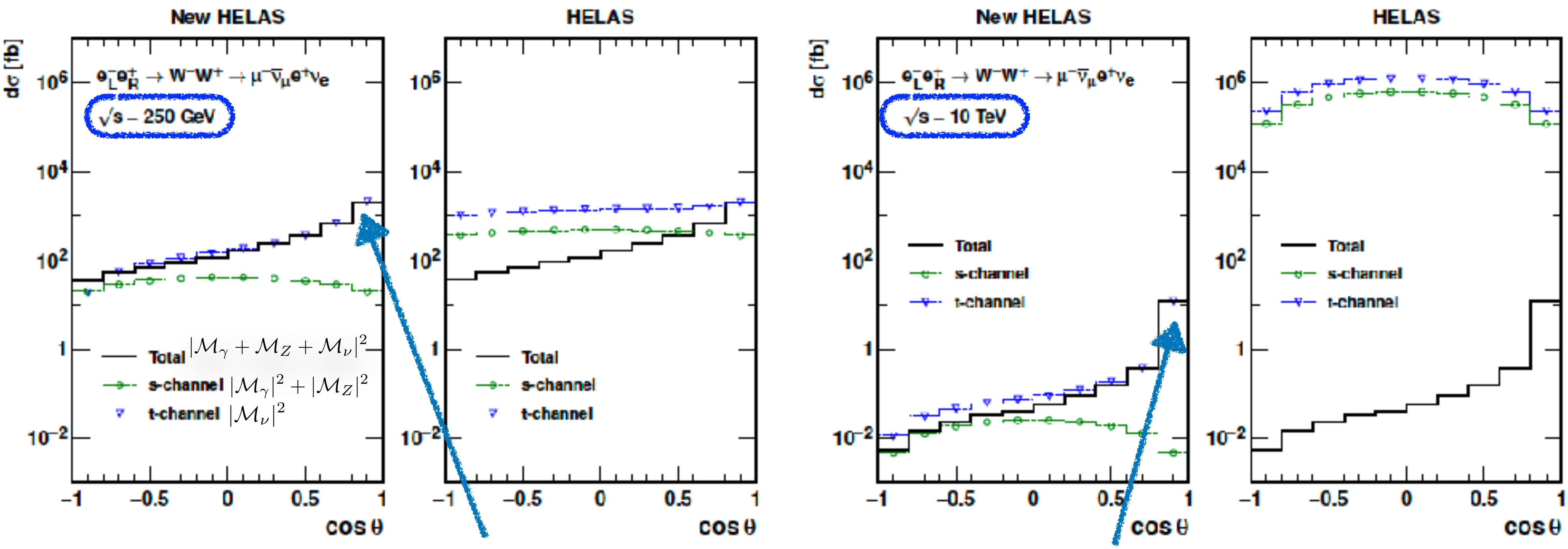
$\sigma(\text{HELAS})$
$= \sigma(\text{New HELAS})$

In the New HELAS calculation, No unphysical energy growth for each amplitude!

Chen, Hagiwara, Kanzaki, Mawatari, 2203.10440

Highlight of New HELAS (distribution)

Feynman-Diagram gauge | Unitary gauge | Feynman-Diagram gauge | Unitary gauge



Enhancement from the t-channel amplitude

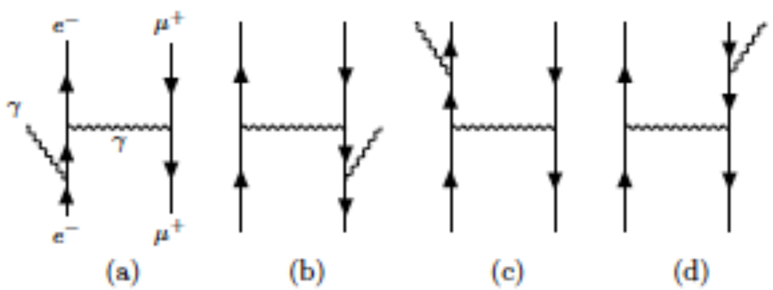
In the New HELAS calculation:

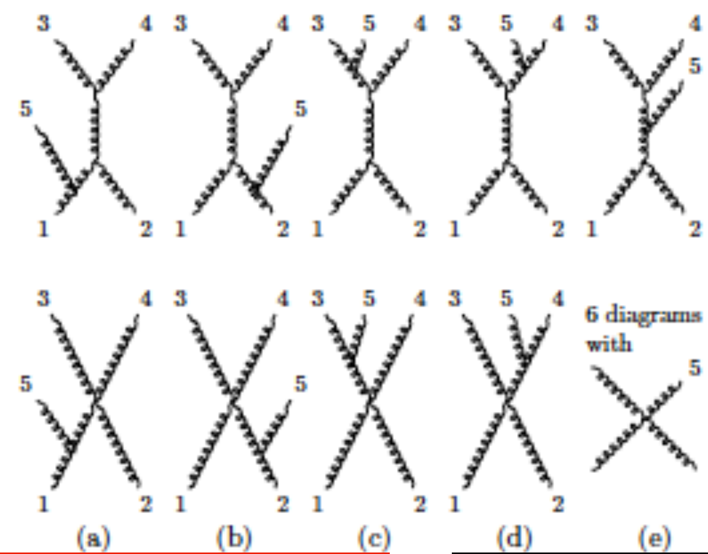
- No subtle cancellation among the amplitudes!
- Each squared amplitude describes the physical distributions!

What is new? Gauge choice

Hagiwara, Kanzaki, Mawatari, 2003.03003
Chen, Hagiwara, Kanzaki, Mawatari, Zheng, 2211.14562

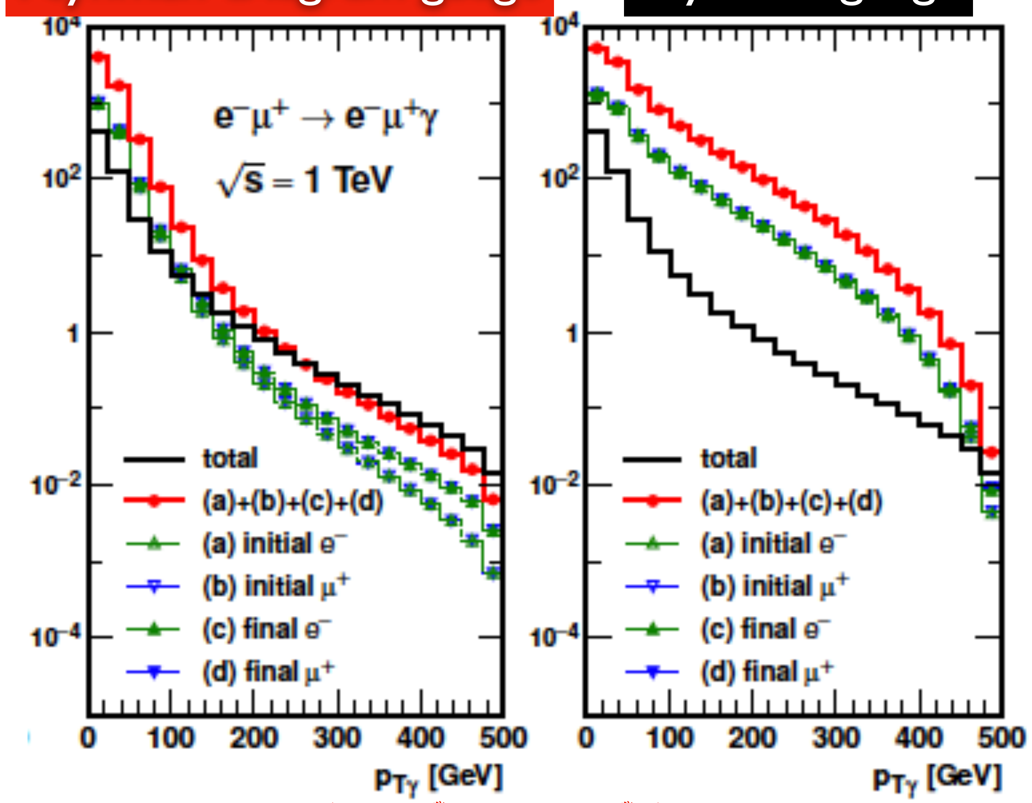
Not only EW, but also QED/QCD





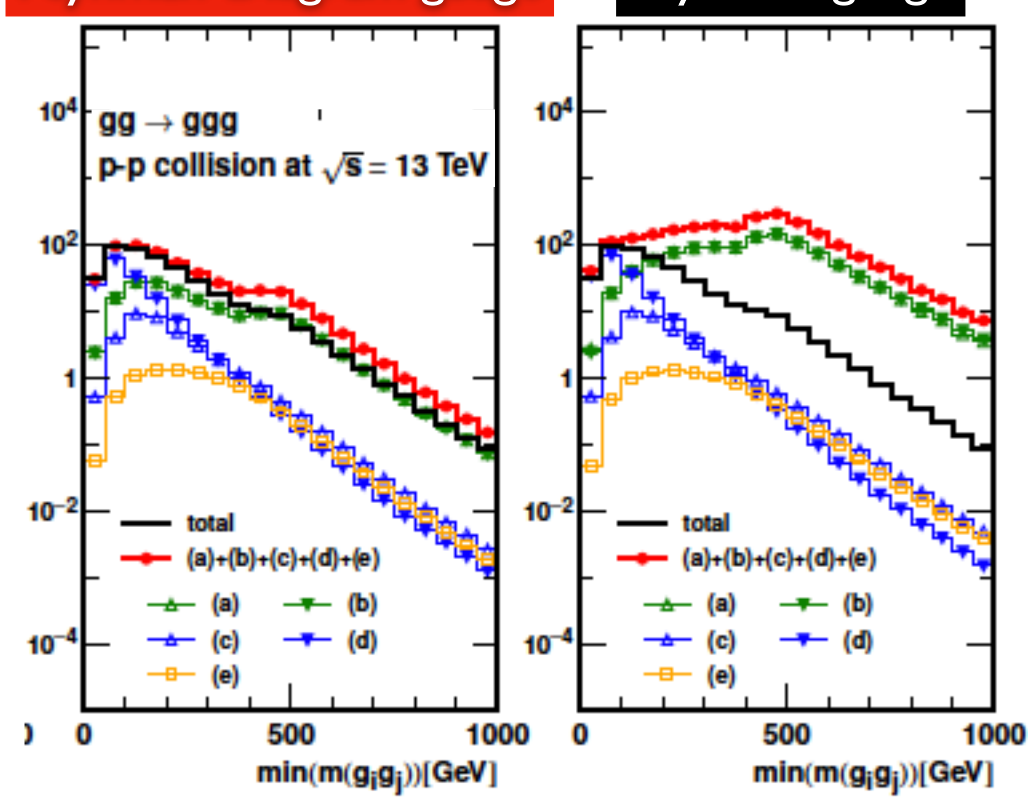
Feynman-Diagram gauge

Feynman gauge



Feynman-Diagram gauge

Feynman gauge



What is the Feynman-Diagram gauge ?

Key idea of Feynman-Diagram (FD) gauge

Kunszt, Soper (1988)
Wulzer (2014), Cuomo, Vecchi, Wulzer (2020)
Chen, Han, Tweedie (2017), etc

Hagiwara, Kanzaki, Mawatari, 2003.03003
Chen, Hagiwara, Kanzaki, Mawatari, 2203.10440
Chen, Hagiwara, Kanzaki, Mawatari, Zheng, 2211.14562

- 5-component polarization vector

$$\mathcal{M}_0 = \epsilon_{V\mu}(q,0)\, T_V^\mu \Longrightarrow \tilde{\epsilon}_{V\,M}(q,0)\, T_V^M \;\; \text{with } M = (\mu, 4)$$

$\tilde{\epsilon}_V^M(q,0) = (\tilde{\epsilon}_V^\mu(q,0),\, i)$ [reduced pol. vector] $\tilde{\epsilon}_V^\mu(q,0) = \epsilon_V^\mu(q,0) - \dfrac{q^\mu}{m_V}$

$T_V^4 = T^{\pi_V}$ [BRST identities] $iq_\mu T_V^\mu = m_V T^{\pi_V}$

- 5x5 component propagator

$$\mathcal{M} = T_1^\mu \frac{P^U_{V\,\mu\nu}}{q^2 - m_V^2 + i\varepsilon} T_2^\nu \Longrightarrow T_1^M \frac{\tilde{P}_{V\,MN}}{q^2 - m_V^2 + i\varepsilon} T_2^N$$

$$\tilde{P}_{V\,MN} = \begin{pmatrix} -\tilde{g}_{\mu\nu} & -i\tilde{\epsilon}_\mu(q,0)^* \dfrac{m_V}{Q} \\ i\tilde{\epsilon}_\nu(q,0)\dfrac{m_V}{Q} & 1 \end{pmatrix}$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{n_\mu q_\nu + q_\mu n_\nu}{n \cdot q}$$

$$n^\mu = (\mathrm{sgn}(q^0), -\vec{q}/|\vec{q}|)$$

Massive gauge boson = QM superposition [reduced polarization vector + its associate NGB]

We can systematically remove unphysical terms which grow with energy from all the diagrams.

Derivation of the FD gauge propagator from the Lagrangian

Chen, Hagiwara, Kanzaki, Mawatari, Zheng, 2211.14562

- The free Lagrangian in the Light-Cone (LC) gauge:

$$\mathcal{L} = -\frac{1}{4}(\partial^\mu Z^\nu - \partial^\nu Z^\mu)^2 + \frac{1}{2}m^2 Z^\mu Z_\mu + \frac{1}{2}(\partial^\mu \pi)^2 + mZ^\mu \partial_\mu \pi - \frac{1}{2\xi}(n^\mu Z_\mu)^2$$
$$= \frac{1}{2}\begin{pmatrix} Z^\mu & \pi \end{pmatrix} \begin{pmatrix} (\partial^2 + m^2)g_{\mu\nu} - \partial_\mu \partial_\nu - \frac{n_\mu n_\nu}{\xi} & m\partial_\mu \\ -m\partial_\nu & -\partial^2 \end{pmatrix} \begin{pmatrix} Z^\nu \\ \pi \end{pmatrix}$$

- The equation of motion in the momentum space:

$$\begin{pmatrix} (-q^2 + m^2)g^\mu{}_\nu + q^\mu q_\nu - \frac{n^\mu n_\nu}{\xi} & -imq^\mu \\ imq_\nu & q^2 \end{pmatrix} \begin{pmatrix} \tilde{Z}^\nu(q) \\ \tilde{\pi}(q) \end{pmatrix} = O^M_N \tilde{Z}^N(q) = 0$$

- All the 5-component of $\tilde{Z}^N(q)$ have a common mass.
- The LC gauge propagator:

$$G_{\mathrm{LC}}{}^M_N = \frac{P_{\mathrm{LC}}{}^M_N}{q^2 - m^2 + i\varepsilon} \qquad P_{\mathrm{LC}}{}^M_N = \begin{pmatrix} -g^\mu{}_\nu + \frac{n^\mu q_\nu + q^\mu n_\nu}{n\cdot q} & im\frac{n^\mu}{n\cdot q} \\ -im\frac{n_\nu}{n\cdot q} & 1 \end{pmatrix}$$

- The FD gauge propagator is obtained by setting

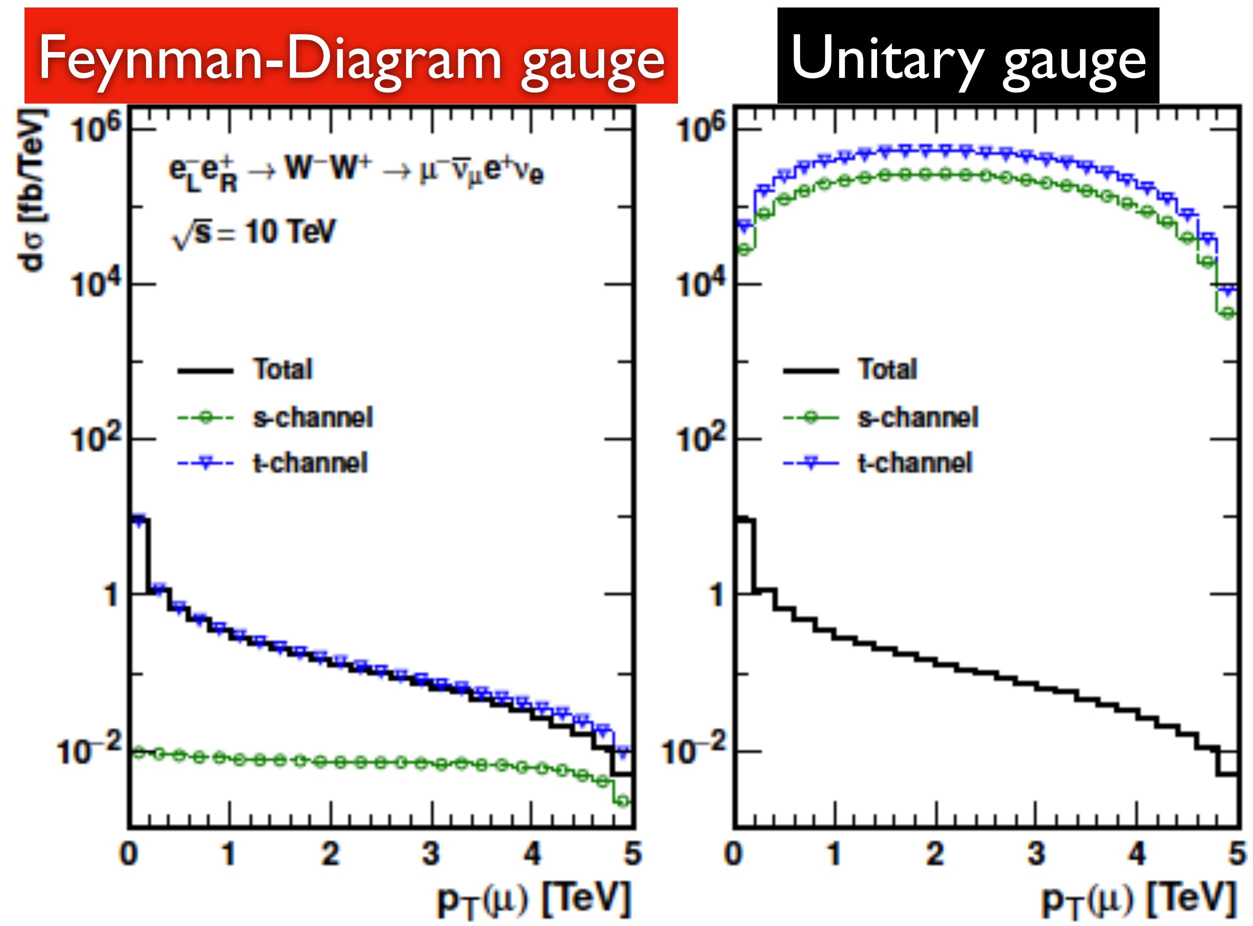
$$n^\mu \equiv n^\mu_{\mathrm{FD}} = \left(\mathrm{sgn}(q^0), -\vec{q}/|\vec{q}|\right)$$

*The new HELAS subroutines are publicly available on http://madgraph.ipmu.jp/IPMU/Software/HELAS/

Key features of Feynman-Diagram (FD) gauge

- All the numerical problems associated with so-called gauge cancellation is absent.
- Helicity amplitudes for individual Feynman diagram have definite physical interpretation as

  [all the invariant Feynman propagator factors for the connecting lines]
  x
  [the splitting amplitudes at each vertex (=Wigner's d-function)]



Because of these properties, we believe that amplitudes evaluated in the FD gauge give us deeper insights on scattering processes, and give improved efficiency in event generations.