Shock waves in holographic EPR pair

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Chen-Pin Yeh
National Dong Hwa University , Hualien, Taiwan

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(1) Entanglement and Wormholes.

(2) Wormhole on String Worldsheet.

(3) Perturb the Worldsheet Wormhole by Shocks.

(4) Discussion

ER=EPR

The close relation between quantum entanglement (EPR pair) and wormhole (ER bridge)

"Cool Horizons for entangled black holes", J. Maldacena and L. Susskind (2013)

- Two parts have non-local connections, but no signal exchange over speed of light.
- Manipulations in one side may change the state in the other side.
- Entanglement decreases when the bridge length increases.

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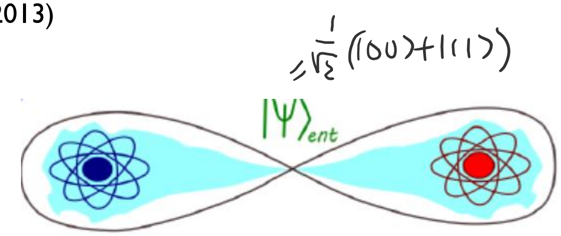
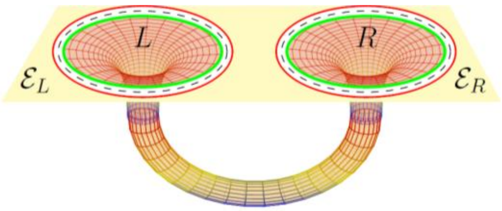


EPR pair



Gravity picture of an EPR pair

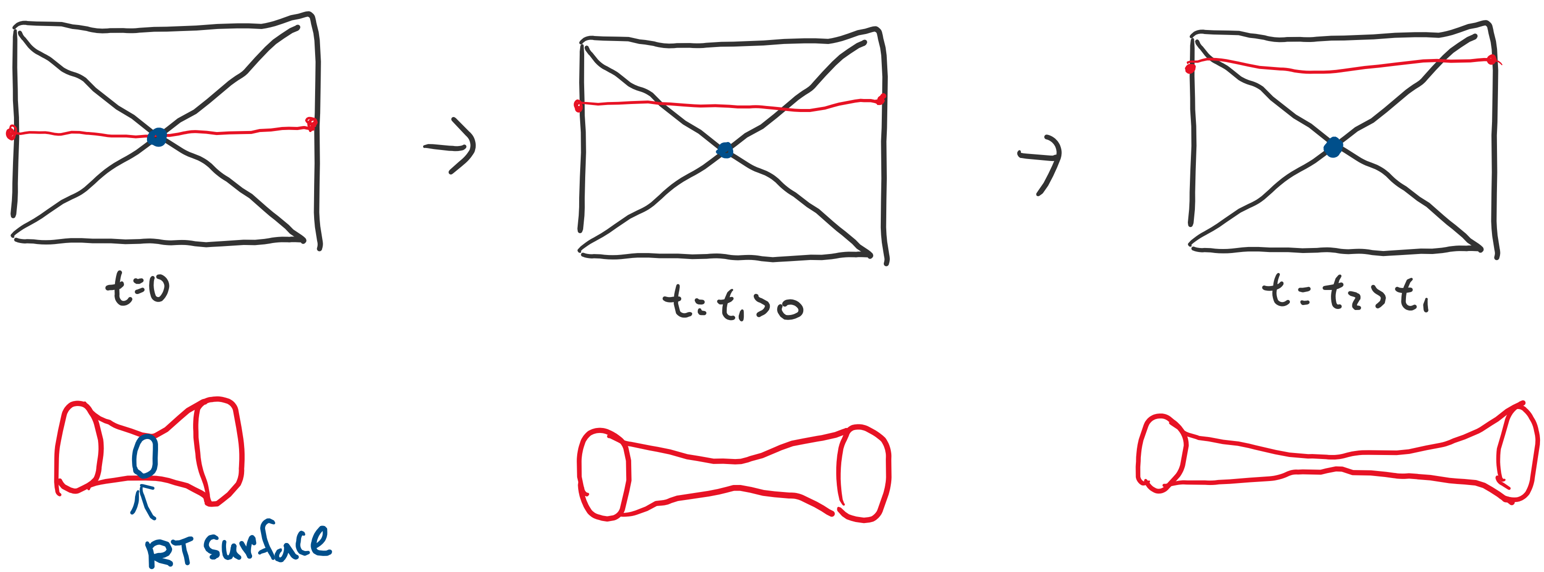
Two-sided AdS Black Hole

Let the boundary state be the thermofield double at t=0:

$$|TFD\rangle = (Z_T)^{-1/2} \sum_n e^{-\frac{E_n}{2T}} |n\rangle_L \otimes |n\rangle_R$$

"Eternal AdS Black holes",
J. Maldacena (2003)

Then we evolve the state by the Hamiltonian: $\hat{H}_R + \hat{H}_L$



As the bridge stretches, the total entanglement entropy remains the same (the Ryu-Taganayagi surface also remains unchanged)

Another Setting: Accelerating Quarks

A minimal surface from Nambu-Goto action of string in AdS:

$$x_b(t, z) = \pm\sqrt{t^2 + b^2 - z^2}$$

"On the exact solution of the accelerating string In AdS5 space", B.-W. Xiao(2013)

- From the quarks' point of view, CFT is in a thermofield double state.
- The worldsheet encodes the quarks' entanglement.
- Quarks lost entanglement due to interaction with CFT fields.

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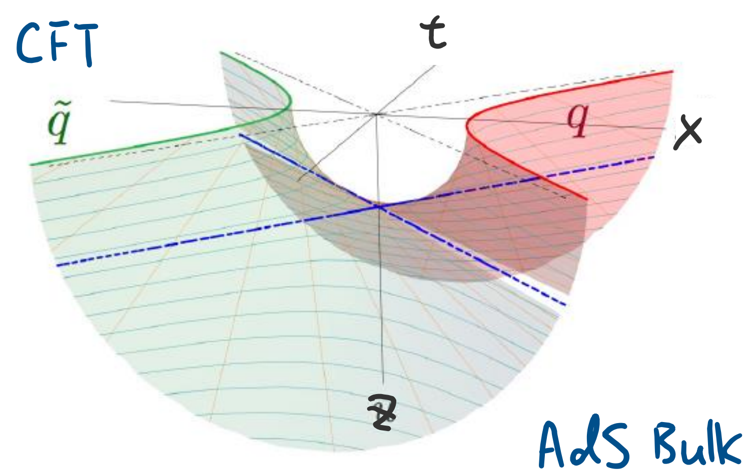
The worldsheet geometry changes.



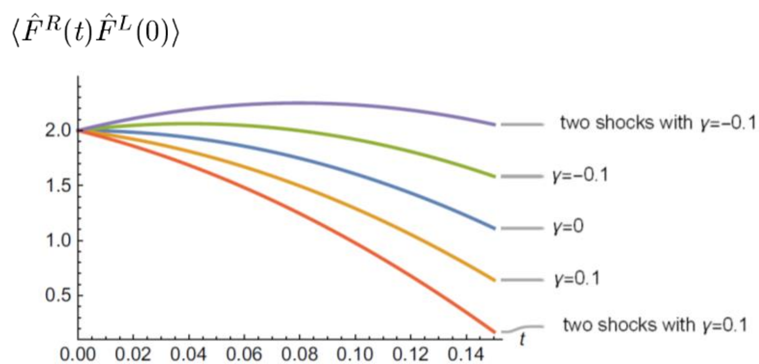
String worldsheet

Findings

- The worldsheet induced metric also describes a two-sided AdS BH.
  (Can we make the worldsheet dynamics like JT gravity?)

- We still don't know how to calculate (define) quark-quark entanglement entropy, and thus see its relation to worldsheet geometry.

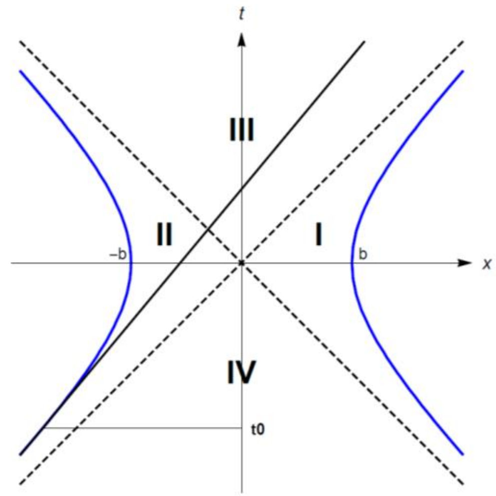
- Instead, we can see how the force-force correlators changes due to the shock waves on worldsheet geometry.



Worldsheet Geometry

The induced metric on "accelerating string":

$$\frac{1}{(t^2+b^2-x^2)^2}\left((x^2-b^2)dt^2-2txdtdx+(t^2+b^2)dx^2\right)$$



To see that is a two-sided (worldsheet) AdS black hole, we do the following (in region I):

1, $u=\frac{b^2t+xb\sqrt{b^2+t^2-x^2}}{x^2-t^2}$, $v=\frac{b^2t-xb\sqrt{b^2+t^2-x^2}}{x^2-t^2}$

with $-\infty<v<u<\infty$

2, $\tilde{u}=b\sinh\frac{u}{b}$ $\tilde{v}=b\sinh\frac{v}{b}$

Then we have: $ds^2=-\frac{d\tilde{u}d\tilde{v}}{b^2\sinh^2\frac{\tilde{u}-\tilde{v}}{2b}}$, a standard form of AdS2 black hole metric with temperature $T=\frac{1}{2\pi b}$

The global coordinates:

To obtain the Kruskal-like coordinate, we take (in region I)

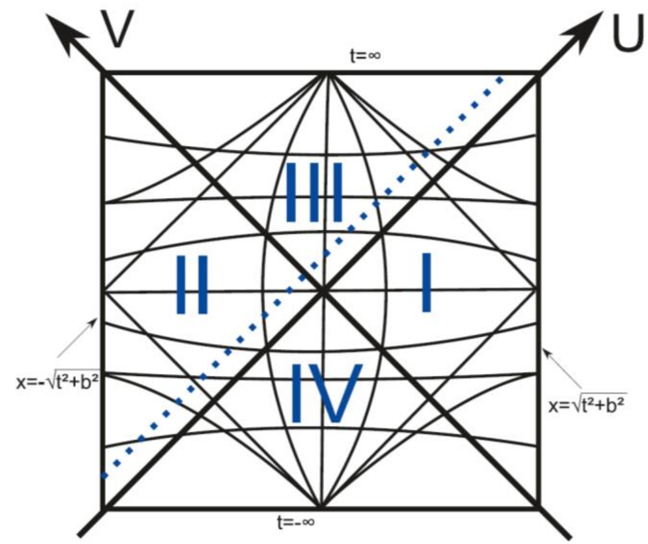
1, $u' = 2b \tanh \frac{\tilde{u}}{2b}$ and $v' = 2b \tanh \frac{\tilde{v}}{2b}$

with $-\infty < v' < u' < \infty$

2, $u'' = 2b \tan^{-1} \frac{u'}{2b}$ and $v'' = 2b \tan^{-1} \frac{v'}{2b}$

$U = v'' + \frac{b\pi}{2}$, $V = u'' - \frac{b\pi}{2}$

with $b\pi > U > 0, -b\pi < V < 0$



Then we have a global metric: $\dfrac{-dU\,dV}{b^2 \cos^2 \frac{U-V}{2b}}$

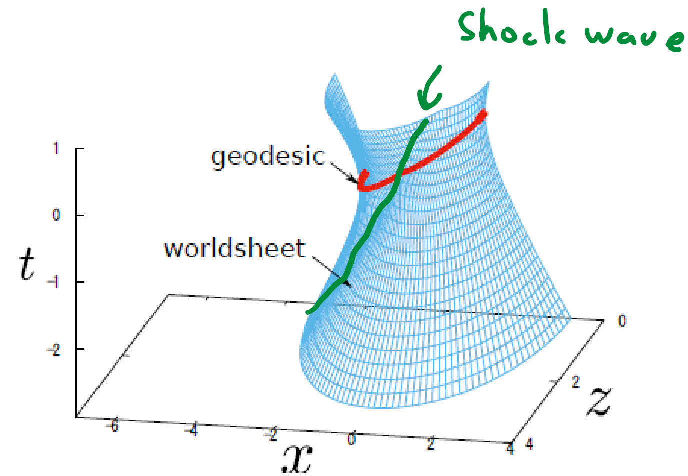
Shock Waves (matching two worldsheets of quarks accelerations, b and bs)

Matching conditions at $\tilde{u} = \tilde{u}_0 \equiv -b \sinh^{-1} \frac{t_0}{b}$

1, $\tilde{u}_s = \tilde{u} = \tilde{u}_0$

$$(V_s = V)$$

2, $$b_s \tanh \frac{\tilde{u}_0 - \tilde{v}_s}{2b_s} = b \tanh \frac{\tilde{u}_0 - \tilde{v}}{2b}$$



"Fast scrambling in holographic EPR pair", K. Murata (2017)

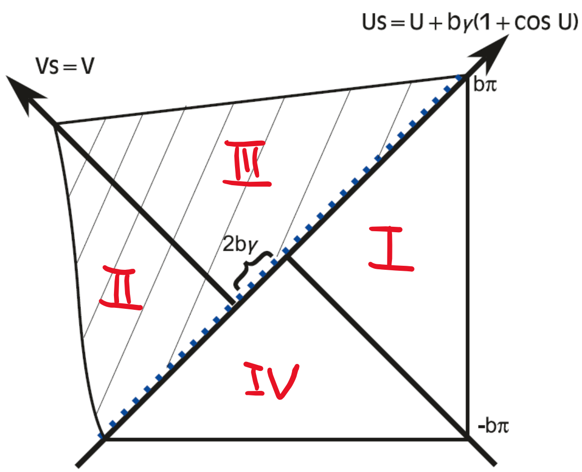
Taking the double scaling limit: $b - b_s \to 0$ and $\tilde{u}_0 \to \infty$

and keep $$\frac{1}{2b}(b - b_s)e^{\frac{\tilde{u}_0}{b}} \equiv \gamma$$ fixed

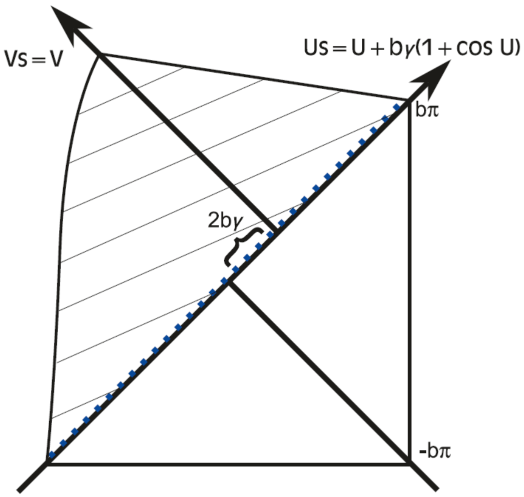
Then the Matching Condition 2 becomes:

$$U_s = U + b\gamma\left(1 + \cos\frac{U}{b}\right)$$

Shock Waves in double scaling limit (shock waves approach V=0)



(a) Shock wave with positive γ. The wormhole remains non-traversable



(b) Shock wave with negative γ. The wormhole becomes traversable

Action and mode functions

Quadradic action: $$S_q = -R^2 T_0 \int dt\, dz \frac{1}{z^2} \left(\frac{q'^2 - \dot{q}^2 - \dot{x}_b^2 q'^2 - x_b'^2 \dot{q}^2 + 2\dot{x}_b x_b' \dot{q} q'}{2\sqrt{1 - \dot{x}_b^2 + x_b'^2}} \right)$$

Solution of the linearized equation of motion:

$$q^i(U,V) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f^i(\omega) Q^i_\omega(U,V) + g^i(\omega) P^i_\omega(U,V)\,,$$

where $i = \mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}$, labels the four regions on the worldsheet. And we have

$$Q^{\mathrm{I}}_\omega(U,V) = \frac{1 - i\omega z(U,V)}{(1 - i\omega z_m) e^{i\omega b \tanh^{-1} \frac{z_m}{b}}} e^{-i\omega b \ln \tan \frac{U}{2b}}\,,$$

and

$$P^{\mathrm{I}}_\omega(U,V) = \frac{1 - i\omega z(U,V)}{(1 - i\omega z_m) e^{-i\omega b \tanh^{-1} \frac{z_m}{b}}} e^{i\omega b \ln \tan \frac{-V}{2b}}\,, \qquad z(U,V) = b\frac{\cos \frac{U-V}{2b}}{\cos \frac{U+V}{2b}}$$

"Field correlators from holographic EPR pairs",
K. Shoichi, D. -S. Lee and C. -P. Yeh (2022)

Match the mode functions

1, Across U=0, $f^{\mathrm{I}}(\omega) = e^{-\pi b\omega} f^{\mathrm{IV}}(\omega)$ and $g^{\mathrm{I}}(\omega) = g^{\mathrm{IV}}(\omega)$

2, Across V=0, $g^{\mathrm{II}}(\omega) = e^{-\pi b\omega} g^{\mathrm{IV}}(\omega)$ and

$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f^{\mathrm{II}}(\omega) H(\omega) e^{-i\omega b \ln\tan\left(\frac{-U}{2b} + \frac{\gamma}{2}\left(1+\cos\frac{-U}{b}\right)\right)} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f^{\mathrm{IV}}(\omega) H(\omega) e^{-i\omega b \ln\tan\frac{-U}{2b}}$$

$\gamma \ll 1$ ⟹ $$f^{\mathrm{IV}}(\omega') = f^{\mathrm{II}}(\omega') + \frac{1}{H(\omega')} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f^{\mathrm{II}}(\omega) H(\omega) \Gamma(ib(\omega-\omega'))(-i\omega\gamma)^{-ib(\omega-\omega')}$$

⟹ $$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f^{IV}(\omega) e^{-i\omega t} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} f^{II}(\omega)\left(2 - e^{-i\gamma b\omega e^{-\frac{t}{b}}}\right) e^{-i\omega t}$$

Early time and late time limits

As $t \gg b$ ⟹ $f^{\mathrm{IV}}(\omega) = f^{\mathrm{II}}(\omega)$ ⟹ Shock wave effect is negligible

As $t \ll b$ ⟹ $f^{\mathrm{IV}}(\omega) = f^{\mathrm{II}}(\omega)(1 + i\gamma b\omega)$

Then together with $f^{\mathrm{I}}(\omega) + g^{\mathrm{I}}(\omega) = \tilde{q}^R(\omega), \quad f^{\mathrm{II}}(\omega) + g^{\mathrm{II}}(\omega) = \tilde{q}^L(\omega)$ (quark position fluctuations)

, we obtain the on-shell action:

$$
\begin{aligned}
\ln Z(\tilde{q}^L, \tilde{q}^R) &= S_q(q^i(U,V)) \\
&= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \tilde{q}^R(\omega) \left(\mathrm{Re}G_R(\omega) - \frac{1 - \frac{i\gamma\omega}{2\pi T} + e^{-\frac{\omega}{T}}}{1 - \frac{i\gamma\omega}{2\pi T} - e^{-\frac{\omega}{T}}} i\mathrm{Im}G_R(\omega) \right) \tilde{q}^R(-\omega) \\
&\quad + \tilde{q}^R(\omega) \left(\frac{2 - \frac{i\gamma\omega}{\pi T}}{e^{\frac{\omega}{2T}}(1 - \frac{i\gamma\omega}{2\pi T}) - e^{-\frac{\omega}{2T}}} i\mathrm{Im}G_R(\omega) \right) \tilde{q}^L(-\omega) \\
&\quad + \tilde{q}^L(\omega) \left(\frac{2}{e^{\frac{\omega}{2T}}(1 - \frac{i\gamma\omega}{2\pi T}) - e^{-\frac{\omega}{2T}}} i\mathrm{Im}G_R(\omega) \right) \tilde{q}^R(-\omega) \\
&\quad + \tilde{q}^L(\omega) \left(-\mathrm{Re}G_R(\omega) - \frac{1 - \frac{i\gamma\omega}{2\pi T} + e^{-\frac{\omega}{T}}}{1 - \frac{i\gamma\omega}{2\pi T} - e^{-\frac{\omega}{T}}} i\mathrm{Im}G_R(\omega) \right) \tilde{q}^L(-\omega) \,,
\end{aligned}
$$

where $G_R(\omega) \equiv \frac{R^2 T_0}{2b^2} \frac{b^2 - z_m^2}{z_m^2} Q^{\mathrm{I}}_\omega(z = z_m) \partial_z Q^{\mathrm{I}}_\omega(z = z_m)$, and is given by

$$
G_R(\omega) = \frac{R^2 T_0}{2b^2} \left(\frac{b^2 - z_m^2}{z_m} \frac{\omega^2}{1 + \omega^2 z_m^2} + i \frac{\omega + b^2\omega^3}{1 + \omega^2 z_m^2} \right) .
$$

Force-Force Correlators

$$G^{ab}(\omega) = \frac{\delta^2}{\delta\tilde{q}^a \delta\tilde{q}^b} S_q(q^i(U,V))$$

$$\langle \hat{F}^R(t)\hat{F}^R(t')\rangle = G^{RR}(t,t') = \frac{b}{\sqrt{b^2+t^2-z_m^2}} \frac{b}{\sqrt{b^2+t'^2-z_m^2}} \times \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} G^{RR}(\omega) e^{-i\omega(\tau(t)-\tau(t'))} .$$

$$\langle \hat{F}^R(t)\hat{F}^L(t')\rangle = G^{RL}(t,t') = \frac{b}{\sqrt{b^2+t^2-z_m^2}} \frac{b}{\sqrt{b^2+(t'+b\gamma)^2-z_m^2}} \times \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} G^{RL}(\omega) e^{-i\omega(\tau(t)+\tau(t'+b\gamma))} ,$$

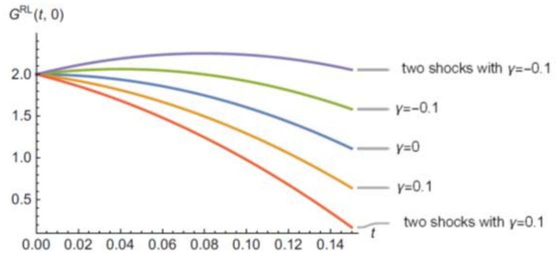
where $\tau(t) = b \tanh^{-1} \frac{t}{\sqrt{b^2+t^2-z_m^2}}$.

Force-Force Correlators in the limit, t<<b and γ <<1:

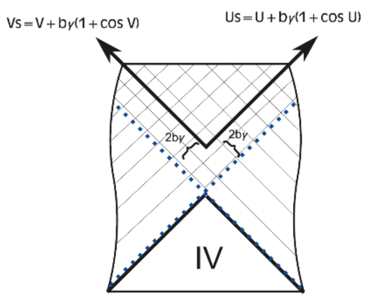
- Effects on GRR (GLL) is quadratic in γ .

- GRL=GLR to leading order, and

$$G^{RL}(t,0) \propto 2 - 10\pi\gamma T t - (2\pi T t)^2$$



-For two shock wave, the cross correlators in this limit have the same form with $\gamma \to 2\gamma$



Discussion

(a) Probing quantum chaos by OTOCs?

(b) Probing high energy scattering amplitudes in AdS?

(c) Entanglement entropy of quarks from holography?

Discussion

(a) Probing quantum chaos by OTOCs?

$$G^{RL}(0,0) = \langle TFD|\hat{W}_L^\dagger(t_0)\hat{F}^R(0)\hat{F}^L(0)\hat{W}_L(t_0)|TFD\rangle$$

$$= \langle \hat{W}^\dagger(t_0)\hat{F}(0)\hat{W}(t_0)\hat{F}\left(\frac{i}{2T}\right)\rangle_T$$

$$\gamma \ll 1 \qquad \simeq \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}G^{RL}(\omega)\mid_{\gamma=0} - \frac{i\gamma}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{\frac{\omega}{T}}{1-e^{-\frac{\omega}{T}}}G^{RL}(\omega)\mid_{\gamma=0}\,;$$

$$\propto \quad 1 - constant \times e^{\frac{\tilde{u}_0}{b}}$$

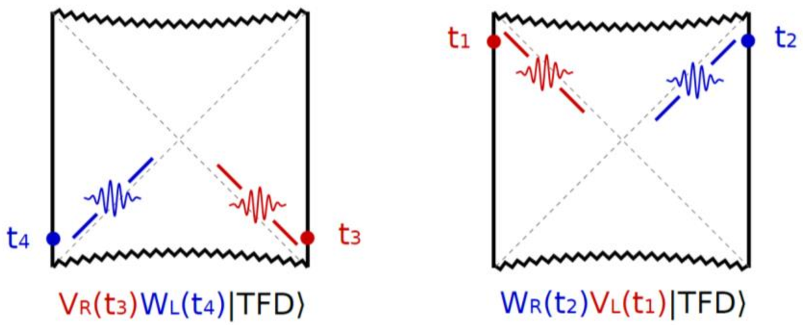
Note1: $\frac{1}{2b}(b-b_s)e^{\frac{\tilde{u}_0}{b}} \equiv \gamma$

Note2: $t_0 = b\sinh\frac{-\tilde{u}_0}{b}$

Discussion

(b) Probing high energy scattering amplitudes in AdS?

$$\langle V_{x_1}(t_1)W_{x_2}(t_2)V_{x_3}(t_3)W_{x_4}(t_4)\rangle = \frac{a_0^4}{(4\pi)^2}\int e^{i\delta(s,|x-x'|)}\left[p_1^u\psi_1^*(p_1^u,x)\psi_3(p_1^u,x)\right]\left[p_2^v\,\psi_2^*(p_2^v,x')\psi_4(p_2^v,x')\right]$$



"Stringy effects in scrambling",
S. Shenker and D. Stanford (2015)

Discussion

(c) Entanglement entropy of quarks from holography?
Do it reflect the change in the wormhole geometry?

An example from free fields

$$\ddot{\hat{\chi}}_1 + 2\gamma_1 \dot{\hat{\chi}}_1 + \Omega^2 \hat{\chi}_1 + \sigma \hat{\chi}_2 = \frac{\eta_{T_1}}{m},$$
$$\ddot{\hat{\chi}}_2 + 2\gamma_2 \dot{\hat{\chi}}_2 + \Omega^2 \hat{\chi}_2 + \sigma \hat{\chi}_1 = \frac{\eta_{T_2}}{m}.$$

$$V_{ij} = \frac{1}{2}\langle \hat{X}_i \hat{X}_j + \hat{X}_j \hat{X}_i \rangle - \langle \hat{X}_i \rangle \langle \hat{X}_j \rangle$$

$$\hat{\mathbf{X}} = (\hat{\chi}_1, \hat{p}_1, \hat{\chi}_2, \hat{p}_2)$$

Reduced Density Matrix

$$G_{R,T_a}(s,s') = -\frac{1}{2\pi} g_a^2 \theta(s-s')\delta'(s-s'),$$
$$G_{H,T_a}(s,s') = \frac{g_a^2}{4\pi}\int \frac{d\omega}{2\pi}\omega \coth\left[\frac{\omega}{2T_a}\right] e^{-i\omega(s-}$$

"Entanglement of quantum oscillators coupled to different heat baths",
W. -C. Syu, D. -S. Lee and C. -P. Yeh (2021)