Chiral transport phenomena in core-collapse supernovae

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Chiral magnetic effect

- Chiral fermions (massless fermions) : $J^\mu = J^\mu_{\rm R} + J^\mu_{\rm L}, \quad J^\mu_5 = J^\mu_{\rm R} - J^\mu_{\rm L}.$

  chirality=helicity ($\vec{S} \cdot \hat{p}$)

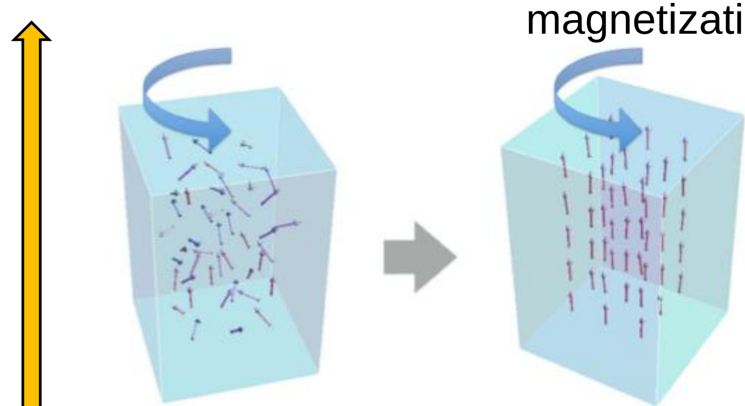
- Chiral magnetic effect (CME) : $J^\mu = \xi_B B^\mu, \quad \xi_B = \frac{\mu_5}{2\pi^2}. \quad (\mu_5 = (\mu_{\rm R} - \mu_{\rm L})/2)$

  parity odd

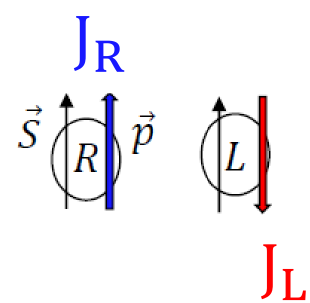
  A. Vilenkin, PRD 22, 3080 (1980)

  K. Fukushima, D. Kharzeev, H. Warringa, PRD78, 074033 (2008)

magnetic field: B



M. Matsuo et al, fphy.2015.00054



$J_V = J_R + J_L \neq 0$

when $n_R - n_L = n_5 \neq 0$

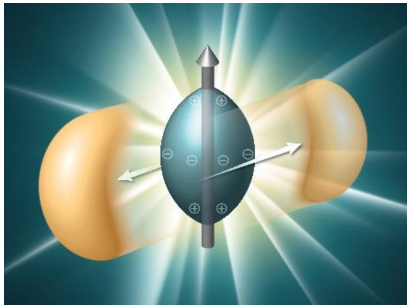
- Chiral anomaly : $\partial_\mu J^\mu_{R/L} = \pm \frac{\mathbf{E} \cdot \mathbf{B}}{4\pi^2} \Rightarrow \partial_\mu J^\mu_5 = \frac{\mathbf{E} \cdot \mathbf{B}}{2\pi^2}$

  S. Adler, J. Bell, R. Jackiw, 69

Where to find chiral transport in the real world?

- Nuclear physics : relativistic heavy ion collisions

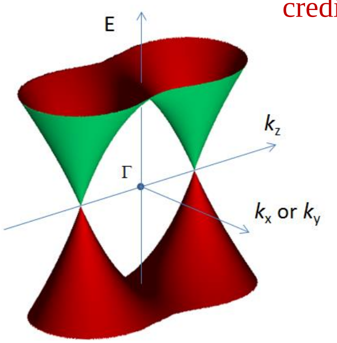
Review : D. Kharzeev, et.al, Prog. Part. Nucl. Phys. 88, 1 (2016)



credit : BNL

- Condensed matter : Weyl semimetals

Textbook : E. V. Gorbar, et.al,
“Electric Properties of Dirac and Weyl Semimetals”
(World Scientific, 2021)



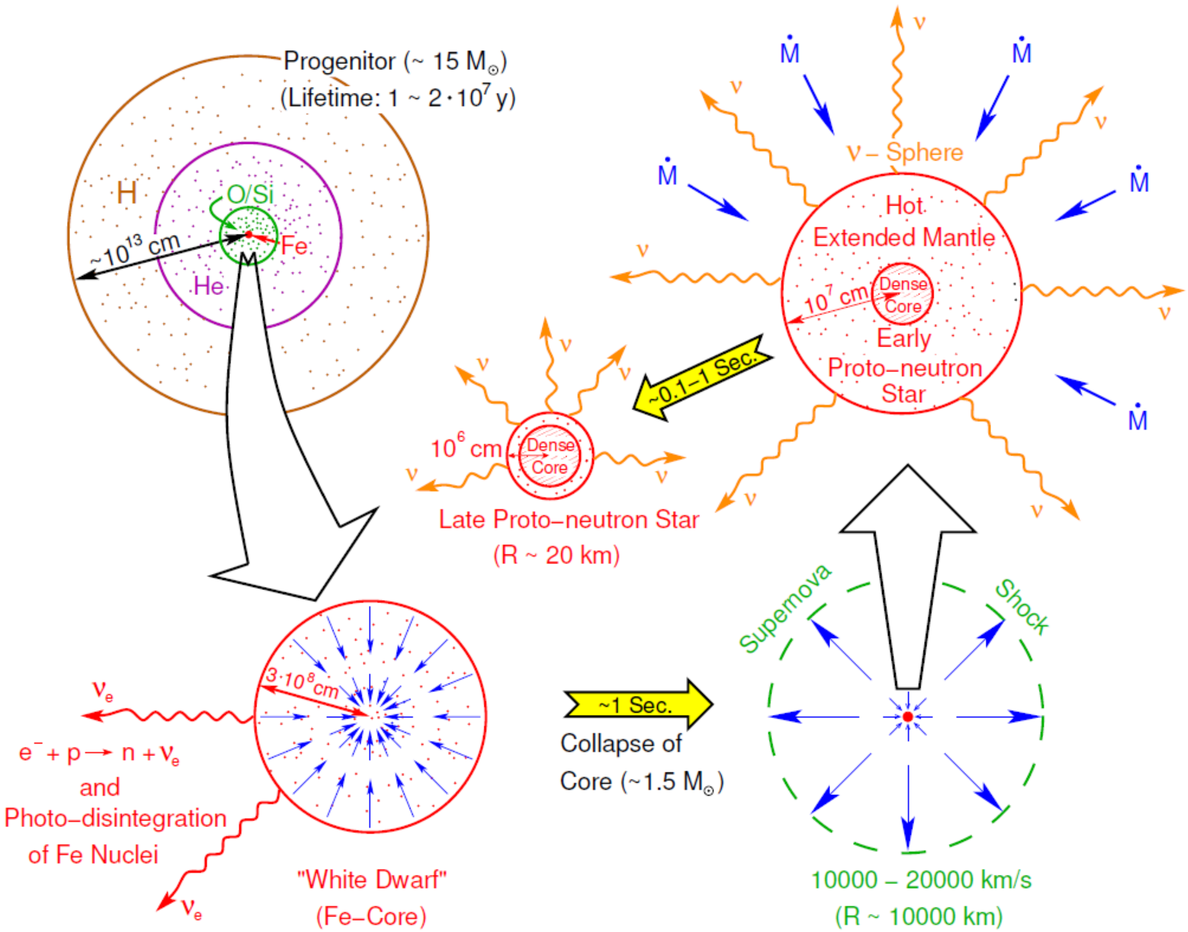
Borisenko et al., Phys. Rev. Lett. 113, 027603 (2014)

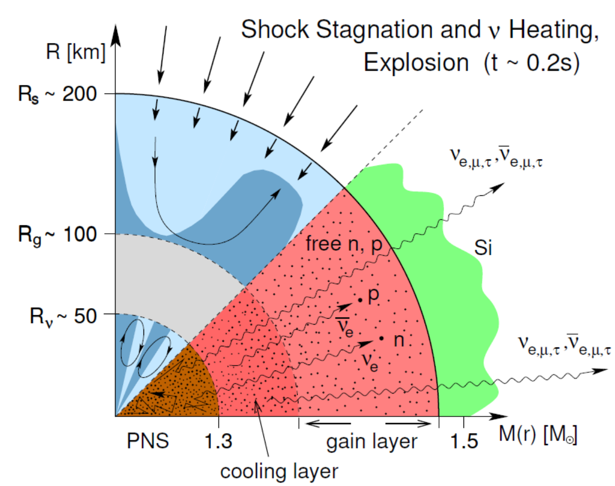
- Astrophysics : core-collapse supernovae

Review : K. Kamada, N. Yamamoto, DY,
Prog. Part. Nucl. Phys. 129 (2023) 104016

credit : RIKEN

Evolution of core-collapse supernovae (CCSN)

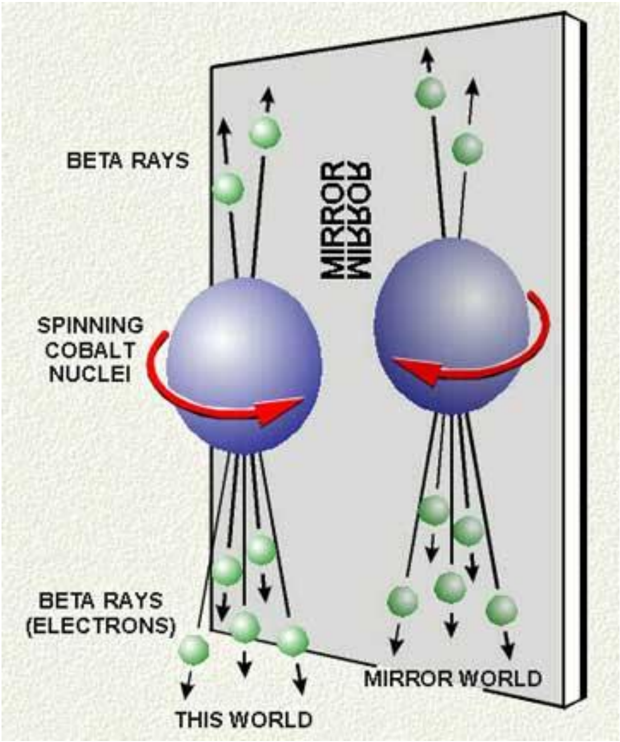




H.-Th. Janka et al., astro-ph/0612072

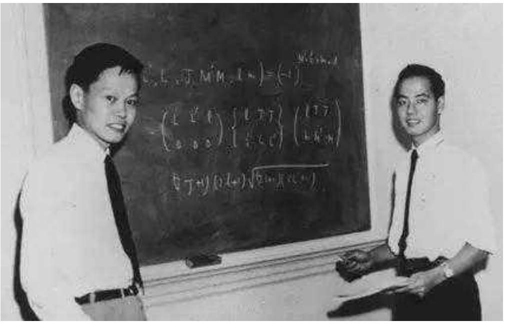
H.-Th. Janka, arXiv:1702.08713

Parity violation & weak interaction



http://physics.nist.gov/GenInt/Parity/cover.html]

$$n \to p + e_{\mathrm{L}} + \bar{\nu}_{\mathrm{R}}$$



Lee & Yang (th) 1956

Wu et al., (exp) 1957

- Global parity violation in weak interaction
- Weak-interaction processes between leptons and nucleons are ubiquitous in CCSN.
- What will be the transport properties for (massless) fermions under parity (chirality) violation?

massless neutrinos & electrons considered ;
other quantum effects neglected : neutrino flavor oscillations

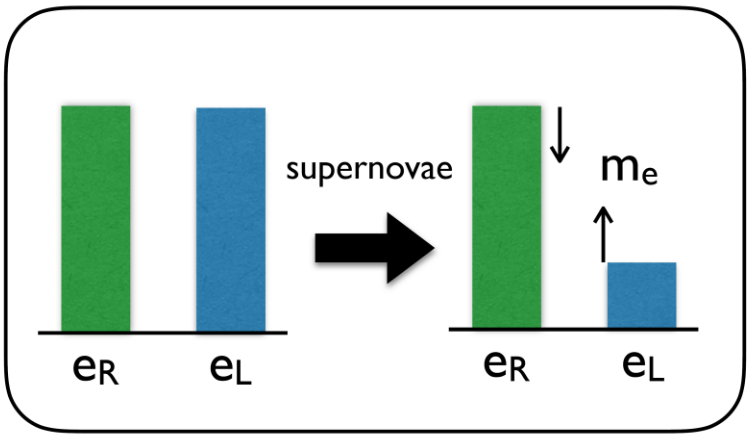
Generation of chirality imbalance

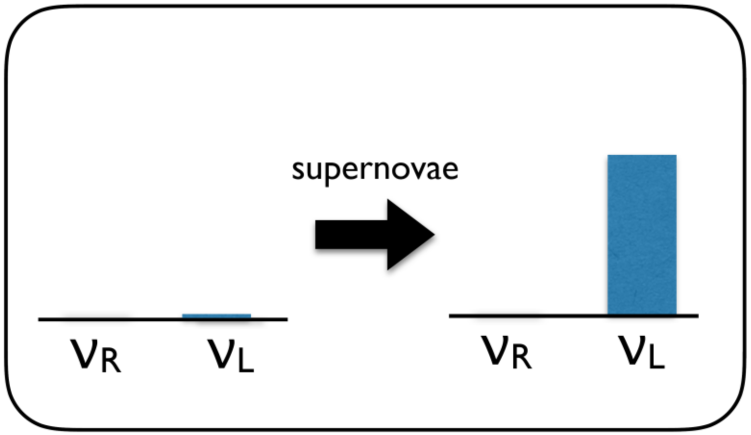
- Electron capture process in supernovae :

A. Ohnishi, N. Yamamoto, 2014, arXiv:1402.4760

an innate lefthander

$$p + e^{\mathrm{L}} \to n + \nu_e^{\mathrm{L}}$$





D. Grabowska, D.B. Kaplan, S. Reddy, PRD 91 (8) (2015) 085035

- Back-reaction from non-equilibrium neutrinos.

$$\nu_{\mathrm{L}}^{\mathrm{e}}(q) + \mathrm{n}(k) \rightleftharpoons \mathrm{e}_{\mathrm{L}}(q') + \mathrm{p}(k')$$

(relativistic electrons/neutrinos will be treated as massless pts.)

Chiral effects on dense stars?

- Three long-standing puzzles :
  - Pulsar kicks : the origin of momentum asymmetry for neutron stars?
  - Magnetars : the origin of strong & stable magnetic fields?
  - Explosions of CCSN with the observed energy?

❖ Chiral effects may provide possible explanations (qualitatively) in a consistent framework. (disclaimer : not the only solution)

- Radiation hydrodynamics : e.g. S. W. Bruenn, Astrophys. Jl. Suppl. 58 (1985) 771.

  magneto-hydrodynamics + relativistic kinetic theory (Boltzmann eq.)

  matter (e, n, p) in equilibrium   radiation (ν) out of equilibrium

- Constructing chiral radiation hydrodynamics :

  chiral magneto-hydrodynamics + chiral kinetic theory

  review : K. Kamada, N. Yamamoto, DY, PPNP 129 (2023) 104016

Chiral radiation transport equation

- CKT for neutrinos : $\Box_{\rm i} f^{(\nu)}_q = \frac{1}{E_{\rm i}}\left[(1-f^{(\nu)}_q)\Gamma^{<}_q - f^{(\nu)}_q \Gamma^{>}_q\right]$

Boltzmann eq. in the inertial frame

collision term with quantum corrections

$\Box_{\rm i} \equiv q\cdot D/E_{\rm i}$

N. Yamamoto & DY, APJ 895 (2020), 1

- Neutrino absorption : $\bar{\Gamma}^{(\rm ab)\lessgtr}_q \approx \bar{\Gamma}^{(0)\lessgtr}_q + \hbar\bar{\Gamma}^{(\omega)\lessgtr}_q(q\cdot\omega) + \hbar\bar{\Gamma}^{(B)\lessgtr}_q(q\cdot B)$

$\nu^{\rm e}_{\rm L}(q) + {\rm n}(k) \rightleftharpoons {\rm e}_{\rm L}(q') + {\rm p}(k')$

Fermi's EFT for weak int.
isoenergetic approx.:
NR approx., $M_{\rm n}\approx M_{\rm p}\approx M$
small-energy transf.

vorticity & magnetic field corrections :
breaking spherical symmetry & axisymmetry

analytic expressions :

$$\bar{\Gamma}^{(0)>}_q \approx \frac{1}{\pi\hbar^4c^4}(g_{\rm V}^2+3g_{\rm A}^2)G_{\rm F}^2(q\cdot u)^3(1-f^{(\rm e)}_{0,q})\left(1-\frac{3q\cdot u}{Mc^2}\right)\frac{n_{\rm n}-n_{\rm p}}{1-{\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}},$$

$$\bar{\Gamma}^{(B)>}_q \approx \frac{1}{2\pi\hbar^4c^4M}(g_{\rm V}^2+3g_{\rm A}^2)G_{\rm F}^2(q\cdot u)(1-f^{(\rm e)}_{0,q})\left(1-\frac{8q\cdot u}{3Mc^2}\right)\frac{n_{\rm n}-n_{\rm p}}{1-{\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}},$$

$$\bar{\Gamma}^{(\omega)>}_q \approx \frac{1}{2\pi\hbar^4c^4}(g_{\rm V}^2+3g_{\rm A}^2)G_{\rm F}^2(q\cdot u)^2(1-f^{(\rm e)}_{0,q})\left(\frac{2}{E_{\rm i}}+\beta f^{(\rm e)}_{0,q}\right)\frac{n_{\rm n}-n_{\rm p}}{1-{\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}}$$

$\bar{\Gamma}^{(0)>}_q$: S. Reddy, M. Prakash, J. M. Lattimer, PRD58:013009,1998

Neutrino flux driven by magnetic fields

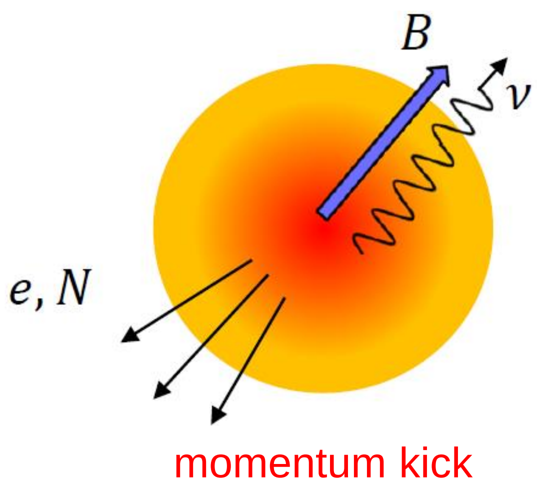
- Energy-momentum tensor & neutrino current : N. Yamamoto & DY, PRD 104, 123019 (2021)

$$T^{\mu\nu}_{(\nu)} = \int_q 4\pi\delta(q^2)\left(q^\mu q^\nu f^{(\nu)}_q - \hbar c q^{\{\mu} S^{\nu\}\rho}_q \mathcal{D}_\rho f^{(\nu)}_q\right),$$

$$J^\mu_{(\nu)} = \int_q 4\pi\delta(q^2)\left(q^\mu f^{(\nu)}_q - \hbar c S^{\mu\rho}_q \mathcal{D}_\rho f^{(\nu)}_q\right), \qquad \mathcal{D}_\mu f^{(\nu)}_q \equiv D_\mu f^{(\nu)}_q - \mathcal{C}_\mu[f^{(\nu)}_q]$$

- The momentum kick from neutrinos near equilibrium : $\Delta T^{i0}_\nu = -\kappa(\boldsymbol{\nabla}\cdot\boldsymbol{v})\mu_\nu B^i$

$$\kappa = \frac{1}{72\pi M G_{\rm F}^2(g_{\rm V}^2 + 3g_{\rm A}^2)}\frac{{\rm e}^{2\beta(\mu_{\rm n}-\mu_{\rm p})}}{n_{\rm n}-n_{\rm p}}$$



momentum con. :

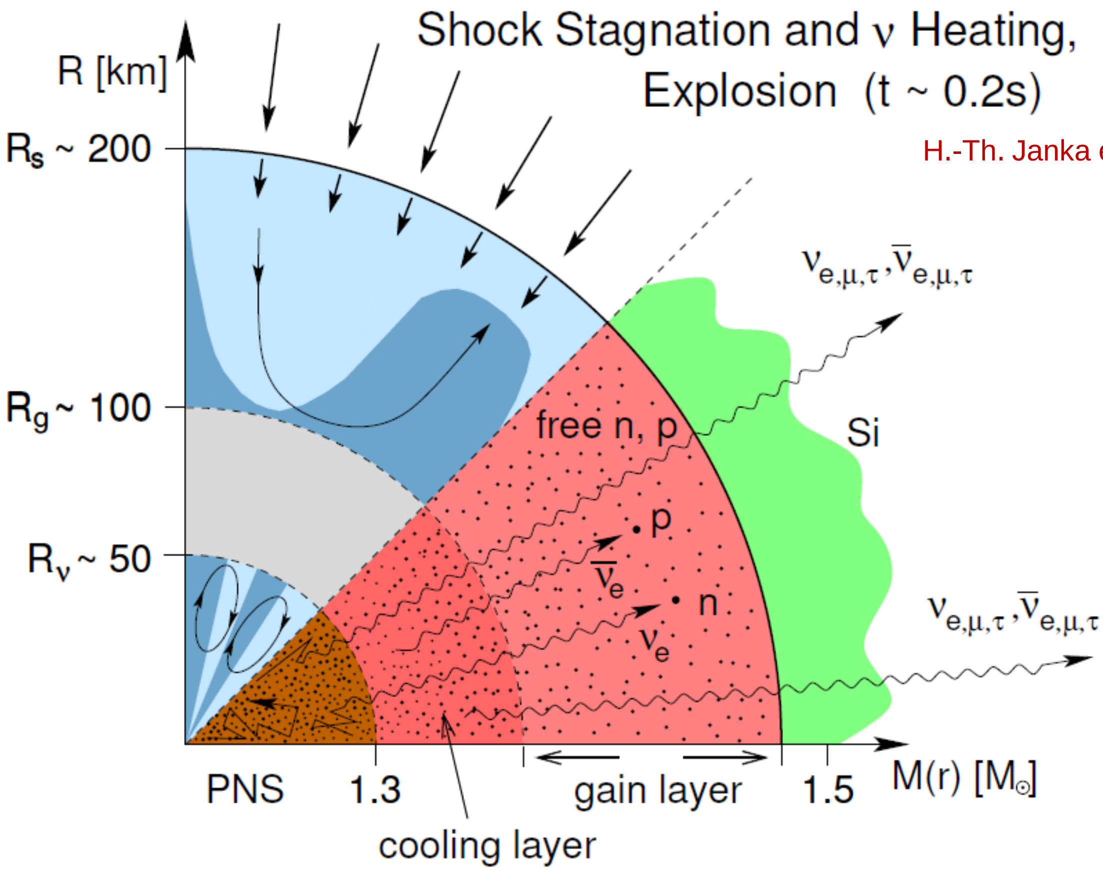
$$\Delta T^{i0}_{\rm e} = -\Delta T^{i0}_\nu, \qquad \Delta T^{i0}_{\rm e} = \mu_{\rm e}\Delta j^i_{\rm e}$$

$$\Rightarrow \quad \Delta J^i_{\rm e} = \xi_B B^i, \quad \text{(effective CME)}$$

$$\xi_B = -\kappa(\boldsymbol{\nabla}\cdot\boldsymbol{v})\frac{\mu_\nu}{\mu_{\rm e}}.$$

Shock revival by neutrino heating

- Heating by non-equilibrium neutrinos :



H.-Th. Janka et al., astro-ph/0612072

❖ How to have chiral effects from non-equilibrium neutrinos in the gain region?

Effective CME from neutrinos out of equilibrium

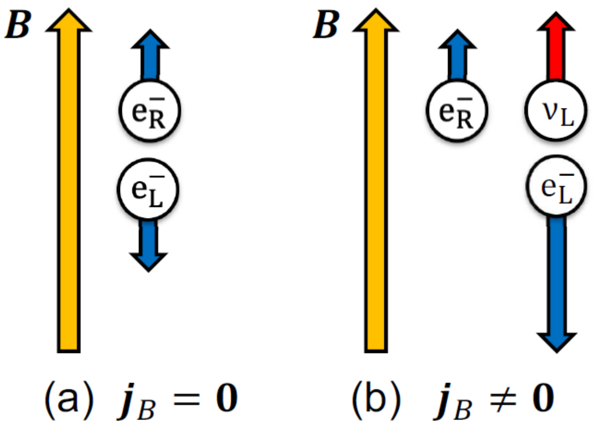
- Effective CME from neutrino radiation :

N. Yamamoto, DY, PRL 131, 012701 (2023)

$$j_B^\mu \approx \hbar e^2 \int \frac{d^4q}{(2\pi)^3}\frac{\delta(q^2)}{q_0}\left(B^\mu q\cdot\partial_q - q\cdot B\partial_{\bar{q}}^\mu\right)\delta f_{\rm W}^{(e)} \equiv \xi_B B^\mu$$

$$\delta f_{\rm W}^{(e)}(q,x) = -\frac{1}{q_0}\int_0^{x_0} dx'_0 F_{\rm W}(q,x')|_c\,,$$

$$F_{\rm W} \approx \frac{(q\cdot u)^3}{\pi}(g_V^2+3g_A^2)G_F^2(n_p-n_n)\left[\frac{\bar{f}_q^{(e)}(1-f_q^{(\nu)})}{1-e^{\beta(\mu_n-\mu_p)}} + \frac{(1-\bar{f}_q^{(e)})f_q^{(\nu)}}{1-e^{\beta(\mu_p-\mu_n)}}\right]$$



- Approximate upper bounds (in the gain region) :

$\xi_B^{\rm tot} \approx -0.5\,{\rm MeV}$ for $x_0 = 0.1\,{\rm s}$ (neutrino emission time)

- Kick velocity : $v_{\rm kick} \sim \frac{|T_{B,\rm tot}^{i0}|}{\rho_{\rm core}} \approx \left(\frac{eB}{10^{13\text{-}14}\,{\rm G}}\right){\rm km/s}\,.$

$B \sim 10^{15-16}$ G for $v_{\rm kick} \sim 10^2$ km/s

see also A. Vilenkin, Astrophys. J. 451, 700 (1995).

Chiral plasma instability

- Chiral plasma instability (CPI) :

M. Joyce, M. E. Shaposhnikov, PRL **79**, 1193 (1997)

Y. Akamatsu, N. Yamamoto, PRL 111, 052002 (2013)

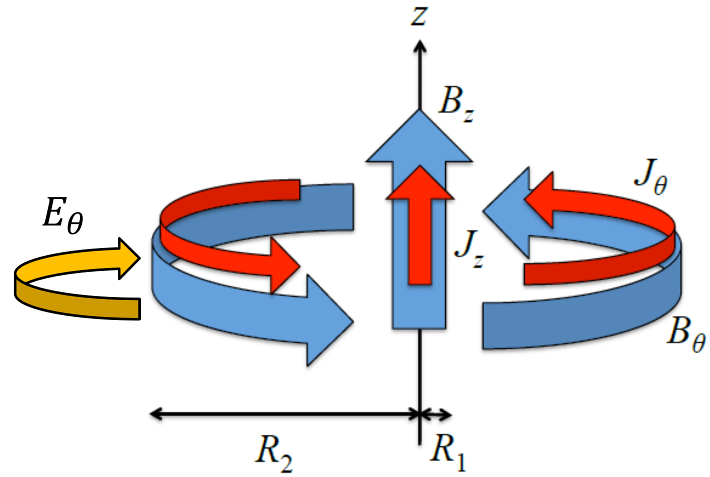
- Anomalous Maxwell's eq. :

$$\partial_t \boldsymbol{B} = -\boldsymbol{\nabla} \times \boldsymbol{E}, \quad \boldsymbol{\nabla} \times B = \eta^{-1} \boldsymbol{E} + \xi_B \boldsymbol{B}.$$

CME

$$\Rightarrow \frac{\partial \boldsymbol{B}}{\partial t} = \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta\, \boldsymbol{\nabla} \times (\xi_B \boldsymbol{B})$$

diffusion CME (instability)



Y. Akamatsu, N. Yamamoto, PRD 90, 125031 (2014)

- Unstable modes at long wavelength : $\delta \boldsymbol{B} \propto \mathrm{e}^{\sigma t + \mathrm{i} \boldsymbol{k} \cdot \boldsymbol{x}}$

$$\sigma = \eta k (\xi_B - k)$$ (for small k, long wavelength)

- Helicity conservation :

$$\frac{dH_{\rm tot}}{dt} = 0,$$

$$H_{\rm tot} \equiv N_5 + \frac{H_{\rm mag}}{4\pi^2},$$

$$N_5 \equiv \int d^3x\, n_5, \quad H_{\rm mag} \equiv \int d^3x\, \boldsymbol{A} \cdot \boldsymbol{B}.$$

exchange btw the (effective) axial charge & magnetic helicity

Y. Hirono, D. Kharzeev, Y. Yin, PRD 92, 125031 (2015)

Chiral magnetohydrodynamics

- Chiral magnetohydrodynamics (MHD) equations:

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0 \,, \quad (17)$$

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot \left[\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \left(P + \frac{B^2}{2}\right)\mathbf{I}\right] = \boldsymbol{S} \,, \quad (18)$$

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \frac{1}{\Gamma - 1}P + \frac{B^2}{2}\right) + \boldsymbol{\nabla} \cdot \left[\left(\frac{1}{2}\rho v^2 + \frac{\Gamma}{\Gamma - 1}P\right)\boldsymbol{v} + \boldsymbol{E} \times \boldsymbol{B}\right] = -\boldsymbol{S} \cdot \boldsymbol{v} - \Delta \boldsymbol{J} \cdot \boldsymbol{E} \,, \quad (19)$$

$$\partial_\nu F^{\nu\mu} = J^\mu$$

$$\partial_\mu J_5^\mu = \frac{\mathbf{E} \cdot \mathbf{B}}{2\pi^2}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta \, \boldsymbol{\nabla} \times (\xi_B \boldsymbol{B}) \,, \quad (20)$$

$$\frac{\partial n_{5,\mathrm{eff}}}{\partial t} = \frac{1}{2\pi^2} \boldsymbol{E} \cdot \boldsymbol{B} \,, \quad (21)$$

$$\boldsymbol{S} = \rho \nu \boldsymbol{\nabla}^2 \boldsymbol{v} + \frac{1}{3} \rho \nu \boldsymbol{\nabla} (\boldsymbol{\nabla} \cdot \boldsymbol{v})$$

J. Matsumoto, N. Yamamoto, DY, PRD 105 (2022) 12, 123029

See also Y. Masada, K. Kotake, T. Takiwaki, and N. Yamamoto, PRD 98, 083018 (2018)

Time evolution of the magnetic field

- Numerical simulations :

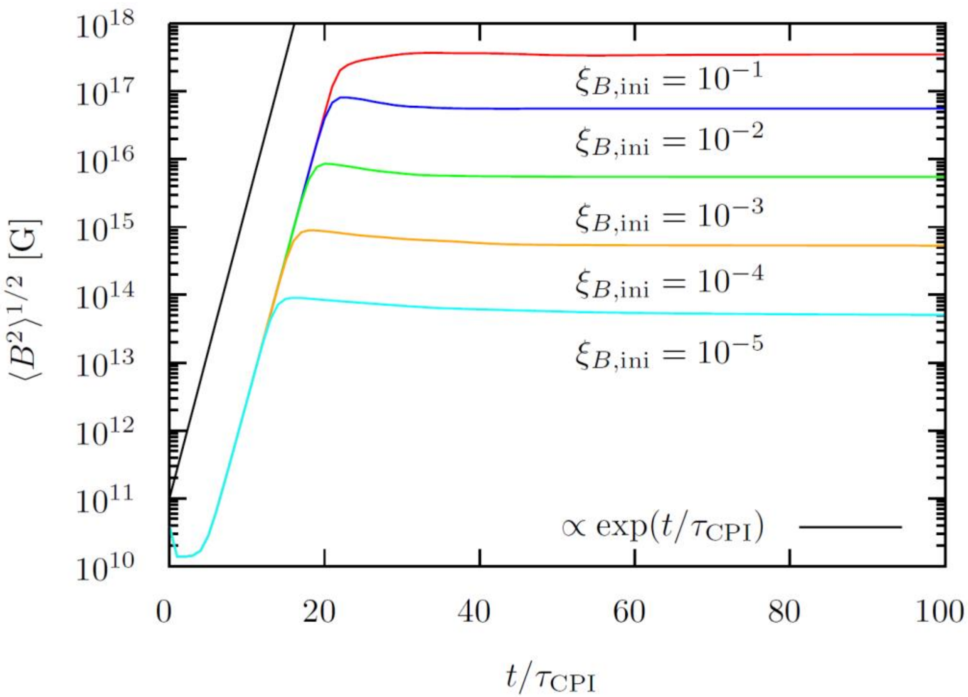
resistivity : $\eta = 1$ viscosity : $\nu = 0.01$

in the units of $100\,\mathrm{MeV} = 1$

J. Matsumoto, N. Yamamoto, DY, PRD 105 (2022) 12, 123029

TABLE I. Summary of the simulation runs.

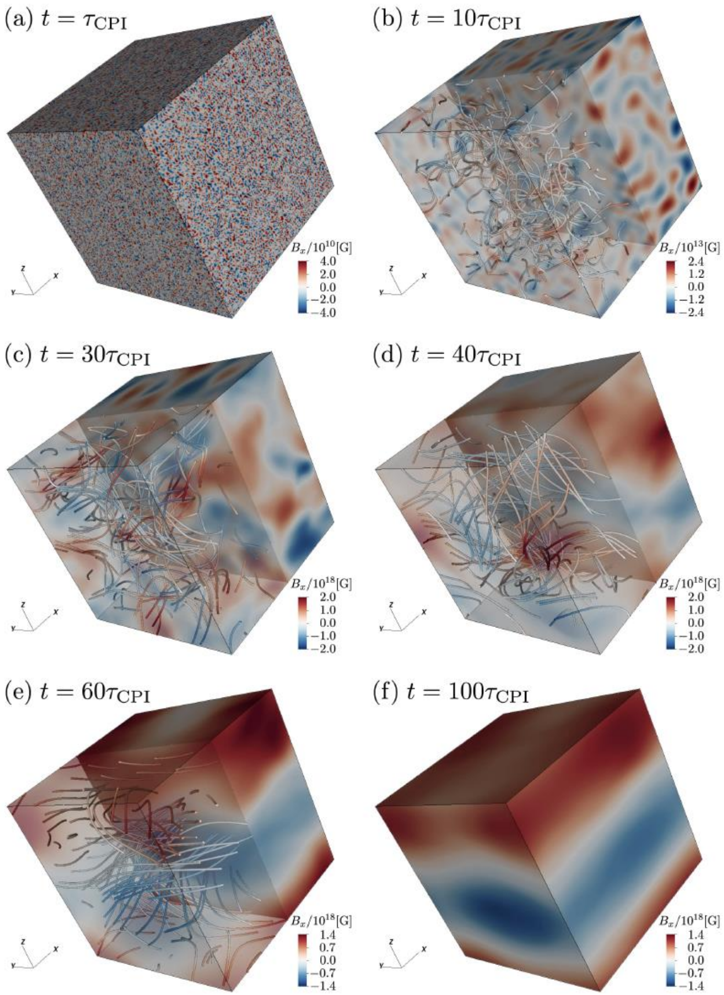
Name	L	$\xi_{B,\mathrm{ini}}$	τ_{CPI}
Model 1	8×10^2	10^{-1}	4×10^2
Model 2	8×10^3	10^{-2}	4×10^4
Model 3	8×10^4	10^{-3}	4×10^6
Model 4	8×10^5	10^{-4}	4×10^8
Model 5	8×10^6	10^{-5}	4×10^{10}



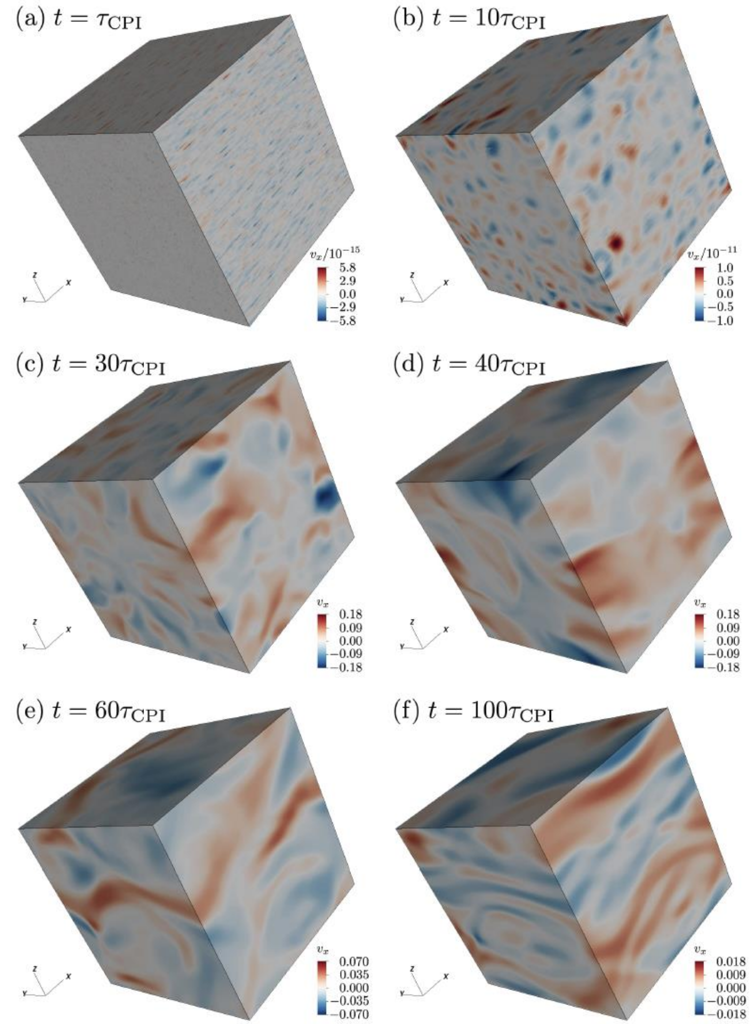
$B_{\mathrm{CPI}} \sim \mu_{5,\mathrm{eff}}^2$

Inverse cascade in local ChMHD simulations

- Inverse cascade : J. Matsumoto, N. Yamamoto, DY, PRD 105 (2022) 12, 123029



magnetic field



fluid velocity

Summary & outlook

❖ Summary

- ✓ The chiral effect for leptons due to “parity violation” from weak int. could qualitatively affect the supernova evolution.
- ✓ Neutrino radiation could induce an “effective CME”, which may result in pulsar kicks and dynamically generate strong magnetic fields in magnetars from “CPI“.
- ✓ Local ChMHD simulations show the “inverse cascade” through chiral effects, which may favor explosions of CCSN.

❖ Outlook

- ❑ Ultimate goal : comprehensive simulations for chiral radiation hydro.
- ❑ Chiral effects from vorticity, temperature/chemical-potential gradients, etc.
- ❑ Chiral effects + flavor oscillation for neutrinos

Thank you!

Chiral kinetic theory from QFT

- Wigner functions : $\grave{S}^{<}_{\mathrm{L}}(q,x) \equiv \int_y \mathrm{e}^{-\frac{iq\cdot y}{\hbar}} \langle \psi^{\dagger}_{\mathrm{L}}(x,y/2)\psi_{\mathrm{L}}(x,-y/2)\rangle \equiv \sigma^{\mu}\mathcal{L}^{<}_{\mu}(q,x)$

see e.g. Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017)

review : Y. Hidaka S. Pu, Q, Wang, DY, PPNP 127 (2022) 103989

- Perturbative solution up to $\mathcal{O}(\hbar)$: ($\sim\partial/q$: gradient expansion)

$$\mathcal{L}^{<\mu} = 2\pi\Big[\delta(q^2)\big(q^{\mu} - \hbar S^{\mu\nu}_{(n)}\mathcal{D}_{\nu}\big) - \hbar\tilde{F}^{\mu\nu}q_{\nu}\delta'(q^2)\Big] f_{\mathrm{L}}\,,$$

$$\mathcal{D}_{\mu}\mathcal{L}^{<}_{\nu} \equiv \big(\nabla_{\mu} - \Gamma^{\lambda}_{\mu\rho}q^{\rho}\partial_{q\lambda} + F_{\rho\mu}\partial^{\rho}_{q}\big)\mathcal{L}^{<}_{\nu} - \Sigma^{<}_{\mu}\mathcal{L}^{>}_{\nu} + \Sigma^{>}_{\mu}\mathcal{L}^{<}_{\nu}, \quad S^{\mu\nu}_{(n)} = \frac{\epsilon^{\mu\nu\alpha\beta}q_{\alpha}n_{\beta}}{2q\cdot n}.$$

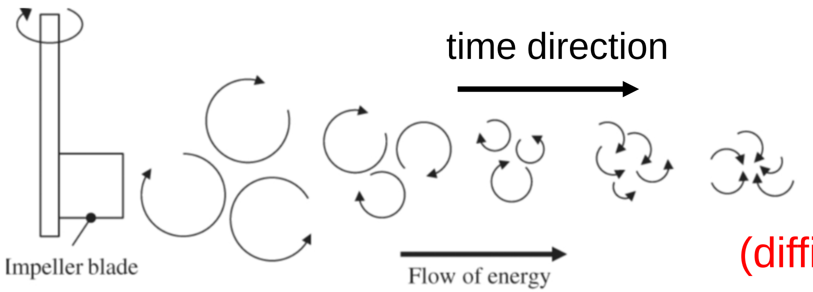
- Chiral kinetic equation :

$$0 = \delta\big(q^2 - \hbar F_{\alpha\beta}S^{\alpha\beta}_{(n)}\big)\Bigg\{\Bigg[q\cdot\mathcal{D} - \hbar\left(\frac{S^{\mu\nu}_{(n)}F_{\mu\rho}n^{\rho}}{q\cdot n} + \big(D_{\mu}S^{\mu\nu}_{(n)}\big)\right)\mathcal{D}_{\nu} - \hbar S^{\mu\nu}_{(n)}\big(\nabla_{\mu}F^{\lambda}{}_{\nu} - q^{\rho}R^{\lambda}_{\rho\mu\nu}\big)\partial_{q\lambda}\Bigg] f_{\mathrm{L}}$$
$$-\hbar\frac{\epsilon^{\mu\nu\alpha\beta}n_{\nu}}{2q\cdot n}\big((1-f_{\mathrm{L}})\Delta^{>}_{\alpha}\Sigma^{<}_{\beta} - f_{\mathrm{L}}\Delta^{<}_{\alpha}\Sigma^{>}_{\beta}\big)\Bigg\}.$$

- Current and EM tensor : $J^{\mu} = 2\int_q \mathcal{L}^{<\mu},\quad T^{\mu\nu} = \int_q \big(\mathcal{L}^{<\mu}q^{\nu} + \mathcal{L}^{<\nu}q^{\mu}\big).$

Direct & inverse energy cascades

- 3D supernova simulations are more difficult to achieve explosion than 2D
- Turbulence in 3D : direct energy cascade

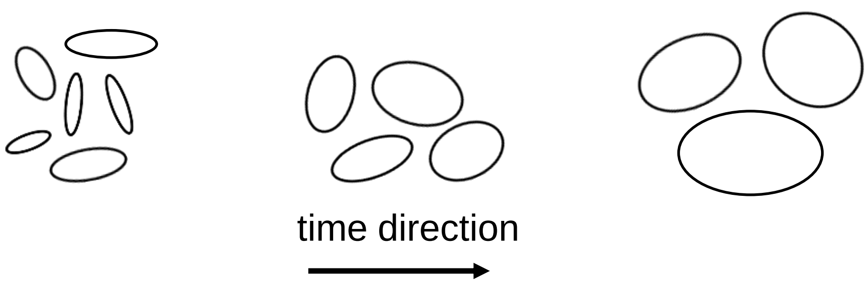


https://doi.org/10.1515/htmp-2016-0043

Kolmogorov 1941

(difficult to explode)

- Turbulence in 2D : inverse energy cascade



Kraichnan, Leith, Batchelor
1967-1969

(easier to explode)

- Chiral effects in 3D : inverse cascade