NCTS Annual Theory Meeting 2023

Relativistic Shocks in Expanding Media: Simulations and Applications to Compact Objects

- Ishika Palit

NTHU | Institute of Astronomy

Date – 17th December 2023

Research Journey

Postdoctoral Research

National Tsing Hua University, Hsinchu, Taiwan

(Currently working with Prof. Hsiang-Yi Karen Yang)

Postdoctoral Research

Tel Aviv University, Tel Aviv, Israel

(Oct 2021 - June 2023)

Ph.D. in Physics

Centre for Theoretical Physics PAS, Warsaw, Poland

(Oct 2017 - Sep 2021)

Thesis – "Selected aspects of low angular momentum accretion onto a black hole"

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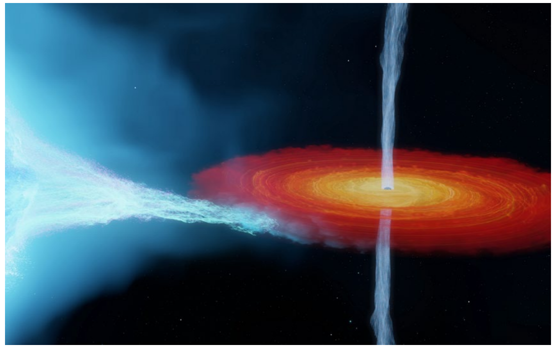
Relativistic shocks around compact object

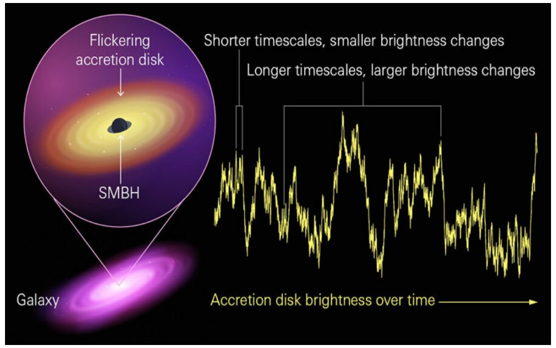
HARM

High Accuracy Relativistic Magnetohydrodynamics

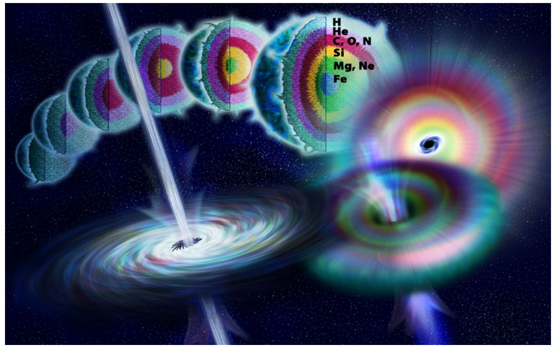
Variability of Magnetically Dominated Jets from Accreting Black Holes



Clumpy Wind Accretion in Cygnus X-1



Effects of adiabatic index on sonic surface and time variability of low angular momentum accretion flows.



Accretion in a Dynamical Spacetime and the Spinning up of the Black Hole in the Gamma-Ray Burst Central Engine.

'Relativistic Spherical Shocks in Expanding Media'

- Taya Govreen-Segal, Noam Youngerman, Ishika Palit, Ehud Nakar, Amir Levinson, Omer Bromberg
School of Physics and Astronomy, Tel Aviv University, Tel Aviv 6997801, Israel

Importance of Relativistic Shocks:

- Crucial in astrophysics (gamma-ray bursts, fast radio bursts, neutron star mergers).

Key Difference:

- Dynamics differ from shocks in stationary media.
- Depends on Lorentz factor distribution shaping density profile.

Research Objective:

- Investigate spherically symmetric shocks in relativistic homologous expanding media.

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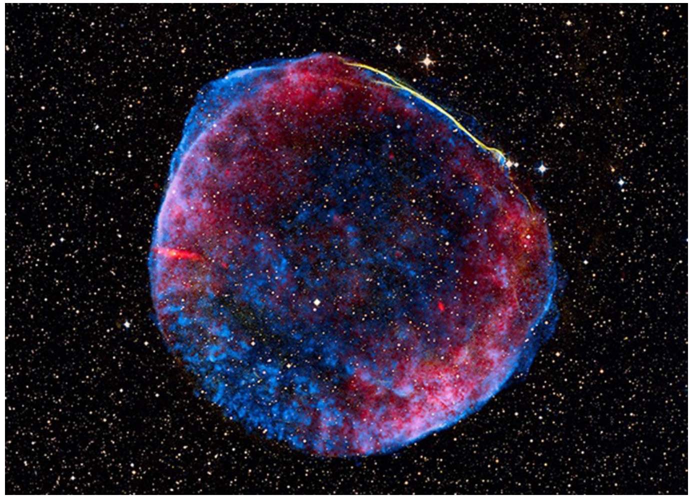
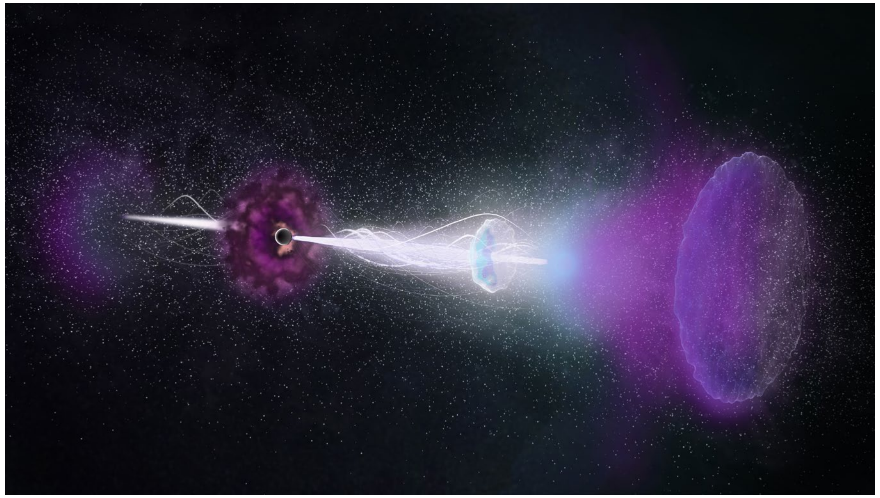
Astrophysical Shocks

An astrophysical shock is a phenomenon where a supersonic flow of plasma interacts with a stationary or slower-moving medium.



Mach 1000 shock wave lights Tycho supernova remnant, credit: NASA

SMBH, Pictor A showing shocked jets in X-ray and Radio. Credit: X-ray: NASA/CXC, Radio: CSIRO/ATNF/ATCA



ALMA Observes Enduring Radio 'Echo' Powered by Jets from Gamma-Ray Burst

Astrophysical Shocks

Relativistic shocks

The shock velocity is a non-negligible fraction of the speed of light.

- Gamma-ray bursts (GRB) afterglows
- Active galactic nuclei (AGN) jets
- Pulsar wind nebulae (PWN)
- Some types of supernovae

Non-relativistic shocks

The shock velocity is much smaller than the speed of light.

- Supernova remnants (SNR)
- Stellar winds
- Accretion disks
- Gamma-ray burst (GRB) precursors

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Self Similar Solution

Self-Similar Solution in PDEs:

- Form of solution in fluid dynamics.
- Similar to itself when variables are appropriately scaled.

Self-Similar Region:

- Describes flow after the shock has moved a significantly large distance.
- Assumes shock wave remains strong.

Astrophysical Context:

- Applicable to understanding shocks in astrophysical phenomena.
- Provides insights into shock dynamics in expansive media.

First type Self Similar Solution

1) Sedov - von Neumann - Taylor solution :

(Non-relativistic regime, K = 0)

It describes an explosion in which a strong shock wave propagates into cold surroundings whose density profile decreases as $\rho \propto r^{-k}$.

Non-dimensional self-similar variable,

$$\xi = \frac{r}{R(t)}$$

First type Self Similar Solution

1) Sedov - von Neumann - Taylor solution :

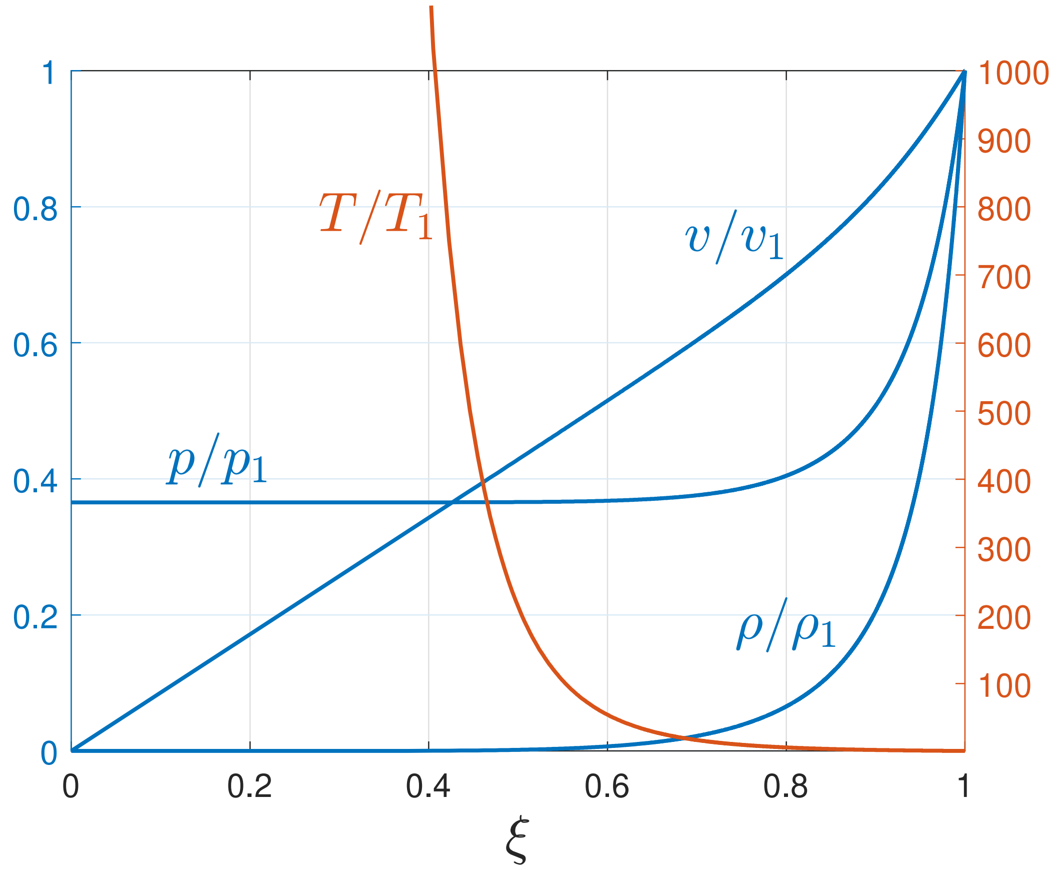
(Non-relativistic regime, K = 0)

density profile - $\rho \propto r^{-k}$

Non-dimensional self-similar variable,

$$\xi = \frac{r}{R(t)}$$

- Density falls to zero very rapidly behind the shock wave.
- Pressure ratio drops rapidly to attain the constant value.
- Since density ratio decays to zero and the pressure ratio is constant, the temperature ratio must become infinite.



First type Self Similar Solution

2) Blandford - McKee solution :

(Ultra-relativistic regime, K < 4)

Valid when the density of the external medium into which the shock propagates varies with the distance r from the origin as $\rho \propto r^{-k}$ for $K < 4$.

Self-similarity variable -

$$\xi = \frac{R-r}{R/\Gamma^2} = (1 - r/R)\Gamma^2.$$

$$\chi = 1 + 2(m+1)\zeta = [1 + 2(m+1)\Gamma^2](1 - r/t).$$

shock Lorentz factor Γ varies as $\Gamma^2 \propto t^{-m}$, $m = 3 - k$

First type Self Similar Solution

2) Blandford - McKee solution :

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Valid when the density of the external medium into which the shock propagates varies with the distance r from the origin as $\rho \propto r^{-k}$ for K < 4.

Pressure, velocity, and density in the shocked fluid as -

$$p = \tfrac{2}{3} w_1 \Gamma^2 f(\chi),$$

$$\gamma^2 = \tfrac{1}{2} \Gamma^2 g(\chi),$$

$$n' = 2 n_1 \Gamma^2 h(\chi).$$

BM Solutions -

$$f = \chi^{-(17-4k)/(12-3k)},$$

$$g = \chi^{-1},$$

$$h = \chi^{-(7-2k)/(4-k)}.$$

Second type Self Similar Solution

1) Best & Sari solution :
(Ultra-relativistic regime, $K > 4.13$)

- ❖ The BM solutions are the ultra-relativistic analogs of the ST solutions.
- ❖ The BM solutions are valid for all $k < 4$, and these new solutions are valid for $k > 5 - \sqrt{3}/4$.
- ❖ No solutions of this type exist for $4 < k < 5 - \sqrt{3}/4$.

  - ➢ Shock Lorentz factor Γ varies as $\Gamma^2 \propto t^{-m}$, $m = (3 - 2\sqrt{3})k - 4(5-3\sqrt{3})$
  - ➢ The flow of the outer region is independent of inner region, so no energy flows from the inner to the outer region.

Second type Self Similar Solution

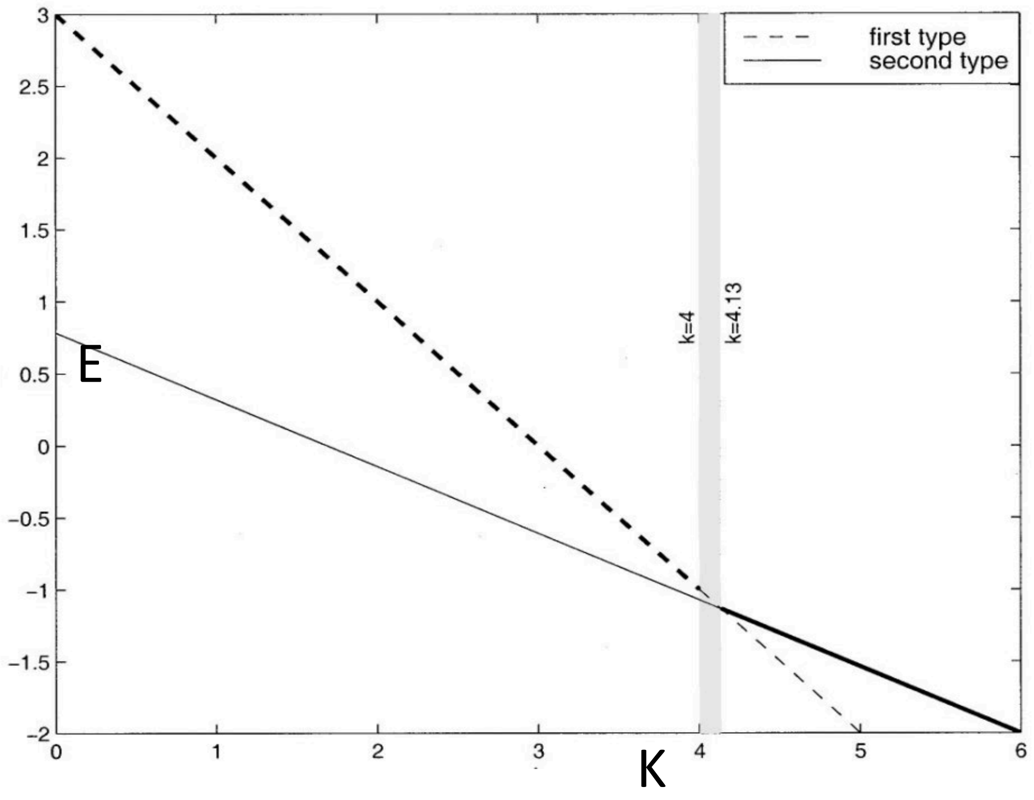
1) **Best & Sari solution :**
(Ultra-relativistic regime, K > 4.13)

❖ The BM solutions are valid for all k < 4, and these new solutions are valid for k > 5 - √3/4.

❖ No solutions of this type exist for 4 < k < 5 - √3/4.

➢ Second-type solution as a solid line and first-type solution as a dashed line,

➢ Heavy lines to denote validity regions (k < 4) and (4 < k < 5 - √3/4), with the 'gap' marked in grey.



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Our model

We investigate the propagation of spherically symmetric shocks in relativistic homologous expanding media with density distributions following a power-law profile in their Lorentz factor.

$$\rho^{ej} \propto t^{-3}\, \Gamma_{ej}(R,t)^{-\alpha}$$

ρ_{ej} - medium proper density,
Γ_{ej} - Lorentz factor,

- Focus on cases with $\alpha > 0$.
- A relativistic ideal EoS with $\gamma = 4/3$.
- Evolution of a spherical blast wave.
- Conditions for shock decays and growth.

Numerical Study - GAMMA

- GAMMA code involves an arbitrary Lagrangian-Eulerian approach.

Solving the following system of equations $$\partial_t \mathbf{U} + \nabla \mathbf{F}(\mathbf{U}) = \mathbf{S}$$

fluid described by state vector of primitive variables $\boldsymbol{V} = (\rho, v, p)^T$, where ρ, p are rest-mass density & pressure in comoving frame, v is fluid velocity in lab frame.

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$$\boldsymbol{U} = \begin{pmatrix} D \\ \vec{m} \\ \tau \end{pmatrix} \equiv \begin{pmatrix} \rho\Gamma \\ \rho h \Gamma^2 \vec{v} \\ \rho h \Gamma^2 - p - D \end{pmatrix} \quad \begin{matrix} \text{(Rest-mass density)} \\ \text{(Momentum)} \\ \text{(Energy)} \end{matrix} ,$$

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GAMMA code

- **Purpose and Application:**
  - Designed for modelling relativistic hydrodynamics and non-thermal emission.
  - Specifically targets gamma-ray burst (GRB) physics applications.

- **Methodology:**
  - Approach involves a Generalized Arbitrary Moving Mesh in relativistic dynamics with local treatment of non-thermal emission.

GAMMA code

- **Purpose and Application:**
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  - Specifically targets gamma-ray burst (GRB) physics applications.

- **Methodology:**
  - Approach involves a Generalized Arbitrary Moving Mesh in relativistic dynamics with local treatment of non-thermal emission.

- **Features:**
  - shock detection, particle injection, consideration of radiative cooling at runtime and local calculation of their evolution.

- **Performance:**
  - Claims to provide accurate broadband GRB afterglow radiation computations from early to late times.

Simulation Setup

- **Initial condition -** BM solutions for density, pressure and velocity.

- **Inner- outer Radii -** $R_0(1-10/\Gamma^2)$ till where the Lorentz factor of the upstream reaches 50.

- **Grid -** Resolution is initially uniform, with 5000 cells,
  Maximum number of cells to 20,000.

- **Boundary -** The inner boundary is reflective, and is stationary throughout the simulation
  The outer boundary is set to outflow and moves at 1.05c.

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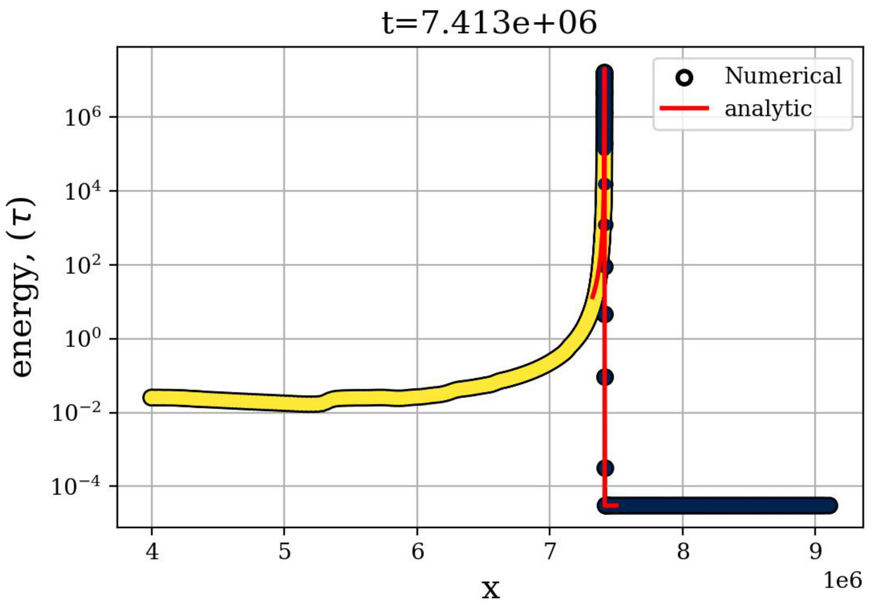
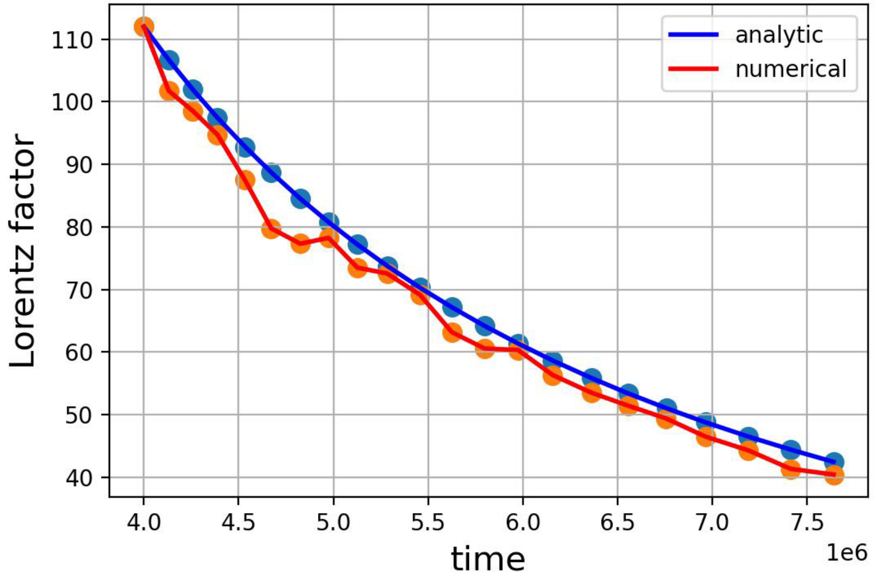
Modeling GRB shocks using Code Gamma

- Dynamics of shocks depend on Lorentz factor distribution of expanding medium affecting its structure, stability, and emission properties.

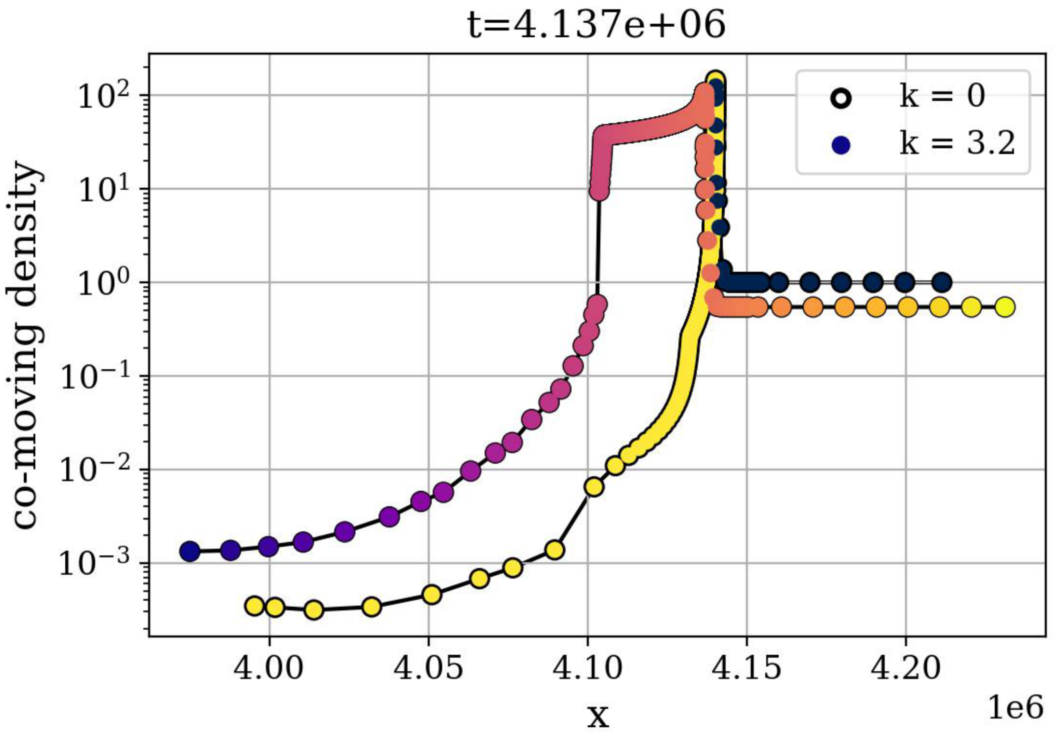
- These solutions can also reveal the critical parameters that determine the shock's evolution.

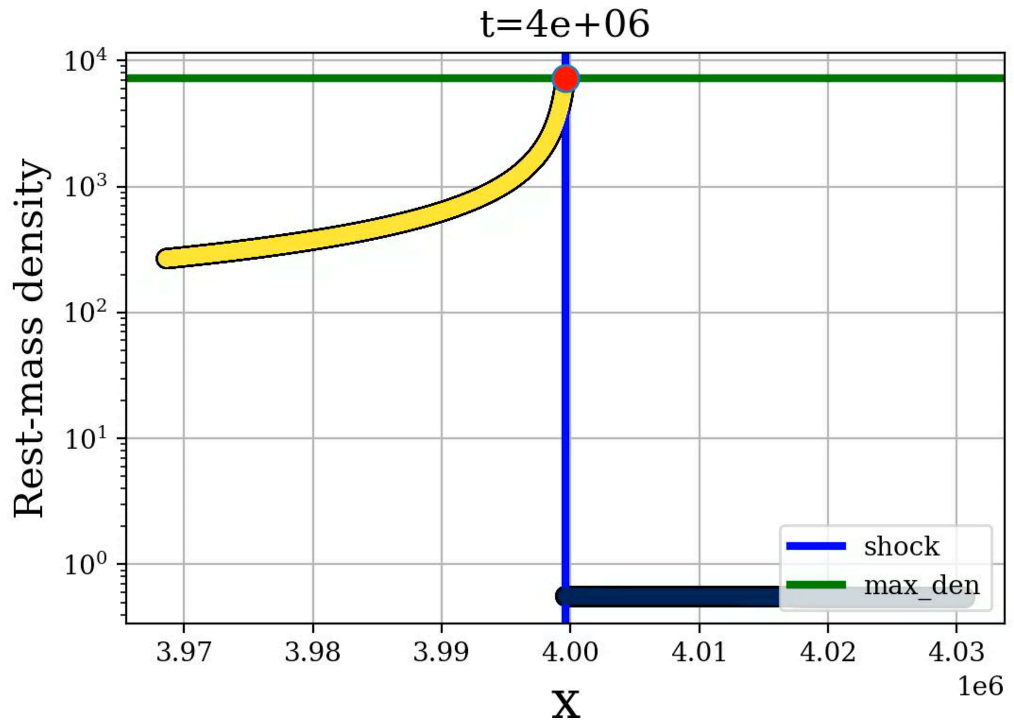
- General shocks that are not self-similar exhibit complex behavior, can have observable consequences for radiation spectra and polarization.





Results





- Density profiles decrease with increasing radius, indicating compression of the ambient medium.
- Applicable to a medium with a - Uniform density profile (Sedov), Power-law density profile (BM).
- The exponent of power-law depends on power-law index of medium and Lorentz factor of shock.

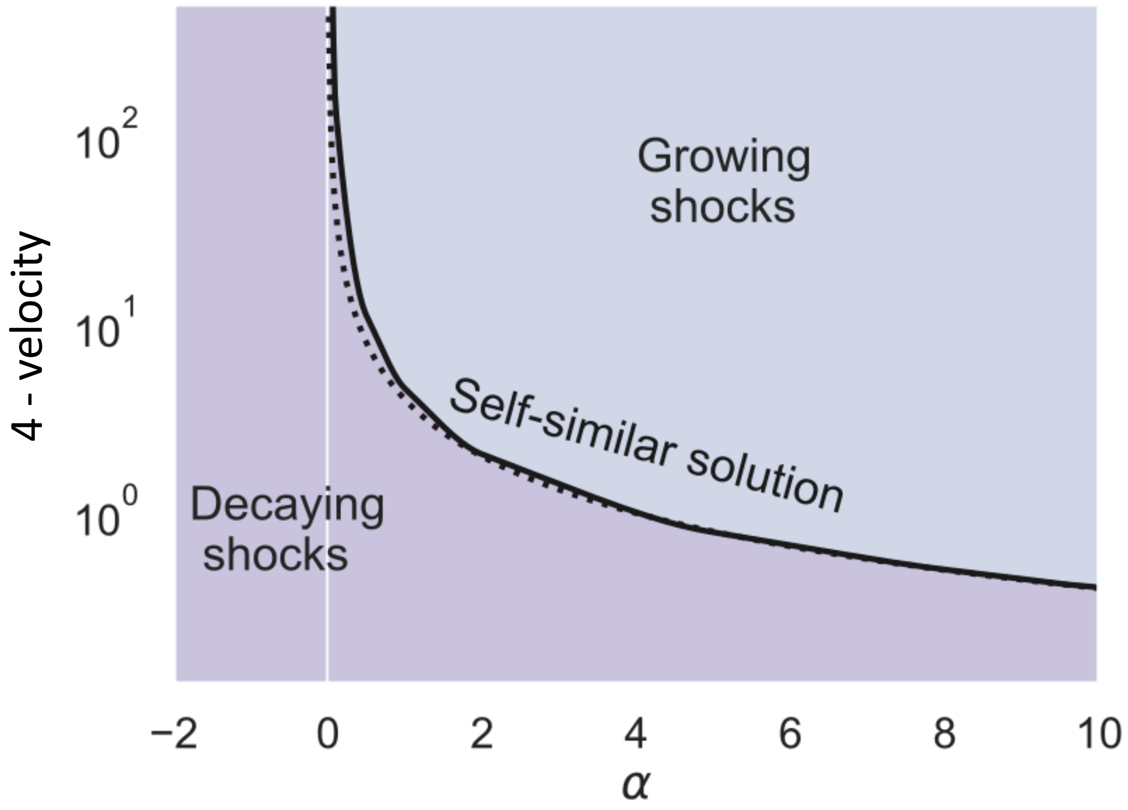
Results

Table showing the simulation data for different test sets.

Alpha	Tests	Γ_{shock} (initial)	$\Gamma_{upstream}$ (initial)	Ratio**	Nature
- 0.5	Test-1	24.7	2.29	17	↓ (slight)
	Test-2	29.35	2.29	20	↔
	Test-3	14.9	2.28	10	↓
	Test-4	17.77	2.29	11	↓
-1	Test-1	5.09	2.25	3.8	↓
	Test-2	6.07	2.26	4.5	↓
	Test-3	8.05	2.27	5.8	↓
	Test-4	10.01	2.28	7	↓ (slight)
-2	Test-1	11.99	2.28	9	↑
	Test-2	8.04	2.27	6	↑
	Test-3	6.07	2.26	4.8	↑ (slight)
	Test-4	4.31	2.23	3.2	↓

Results

The phase space of the analytic solution.

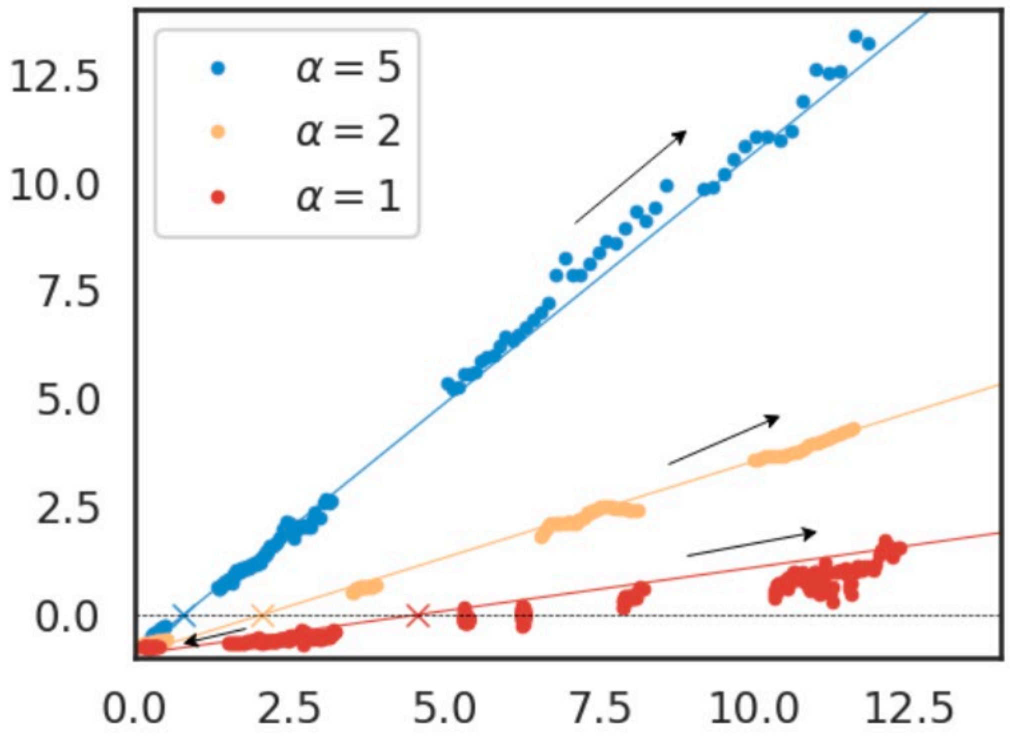


❖ Black line shows dependence of self-similar value of 4-velocity on α, found by requiring a smooth transition through sonic point.

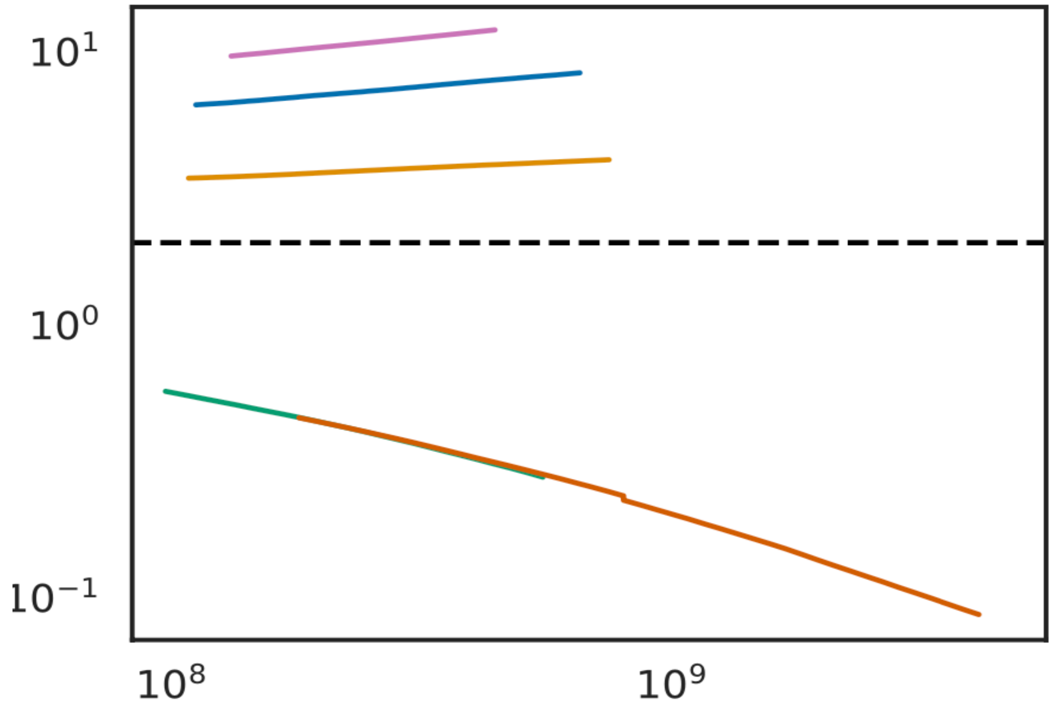
α	r
5	1.47
2	3.06 – 3.08
1	6.5 – 6.6
0.5	15.9 – 16
0.1	196.6 – 196.7
0.01	23,196 – 23,197

Results

dU / d ($\log\Gamma_{ej}$). vs U



4- velocity, U vs time, t



- ❖ Figure 1 - The evolution of the shock strength as it crosses the upstream is plotted for different values of the density profile. The domain below the line corresponds to shocks that decay with time.

- ❖ Figure 2 - The simulations are plotted from the time when the shock reaches a self-consistent structure, independent of the initial conditions.

Conclusions

- We study the propagation of a spherically symmetric shock in a relativistic homologous expanding medium with a power-law density gradient ($\rho^{ej} \propto t^{-3} \Gamma_{ej} (R,t)^{-\alpha}$, $\alpha > 0$).

- 4-velocity measured in the immediate upstream frame may increase monotonously, corresponding to growing shock, or decrease, as shock decays.

- Separating the two regimes there exists an unstable self-similar solution, for which the 4- velocity is constant.

- For every $\alpha > 0$, above a critical value, shock grows and below which it decays.

- The self-similar value diverges as $\alpha \to 0$, and decreases with α, so that for $\alpha \gtrsim 5$, in the self-similar case, U is mildly- relativistic or even Newtonian.

Future works

- Study of Equations of State
- Role of Magnetic Fields and Radiation
- Application to Realistic GRB Afterglow Models
- Comparison with Observational Data

Thank you for your attention!

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