Sequential Quantum Maximum Confidence Discrimination

Joonwoo Bae

Korea Advanced Institute of Science and Technology (KAIST)

Joint work with

Hanwool Lee (Jyvasklya University, Finland)
Kieran Flatt (KAIST, Korea)

KAIST

Based on the manuscripts: "Sequential Quantum Maximum Confidence Discrimination" arXiv:2411.2550

Introduction: Quantum State Discrimination (single-shot)

Motivation
- Sequential quantum information processing: Bell nonlocality, State discrimination
- Monogamy: Entanglement, Nonlocality, …

Sequential Quantum Maximum Confidence Discrimination (Sequential MCD)
- Main result 1: sequential quantum MCD for linearly independent states
- Main result 2: sequential quantum MCD for linearly dependent states

Future Directions

Quantum State Discrimination

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quantum state cannot be discriminated

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AI Overview

Quantum states cannot be perfectly discriminated because of limitations in quantum mechanics. This is especially true for nonorthogonal states, which are common in quantum computing and quantum communication.

Why can't quantum states be discriminated perfectly?

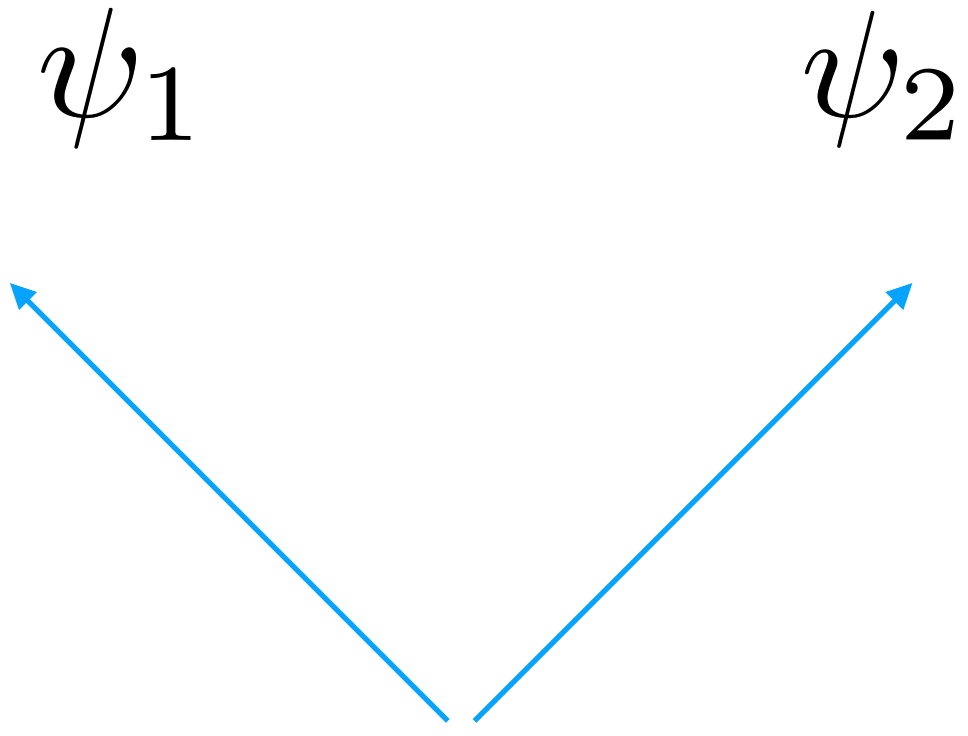
- Quantum mechanics limits the ability to determine the state of a quantum system.
- Nonorthogonal states cannot be perfectly discriminated, even if they are known.

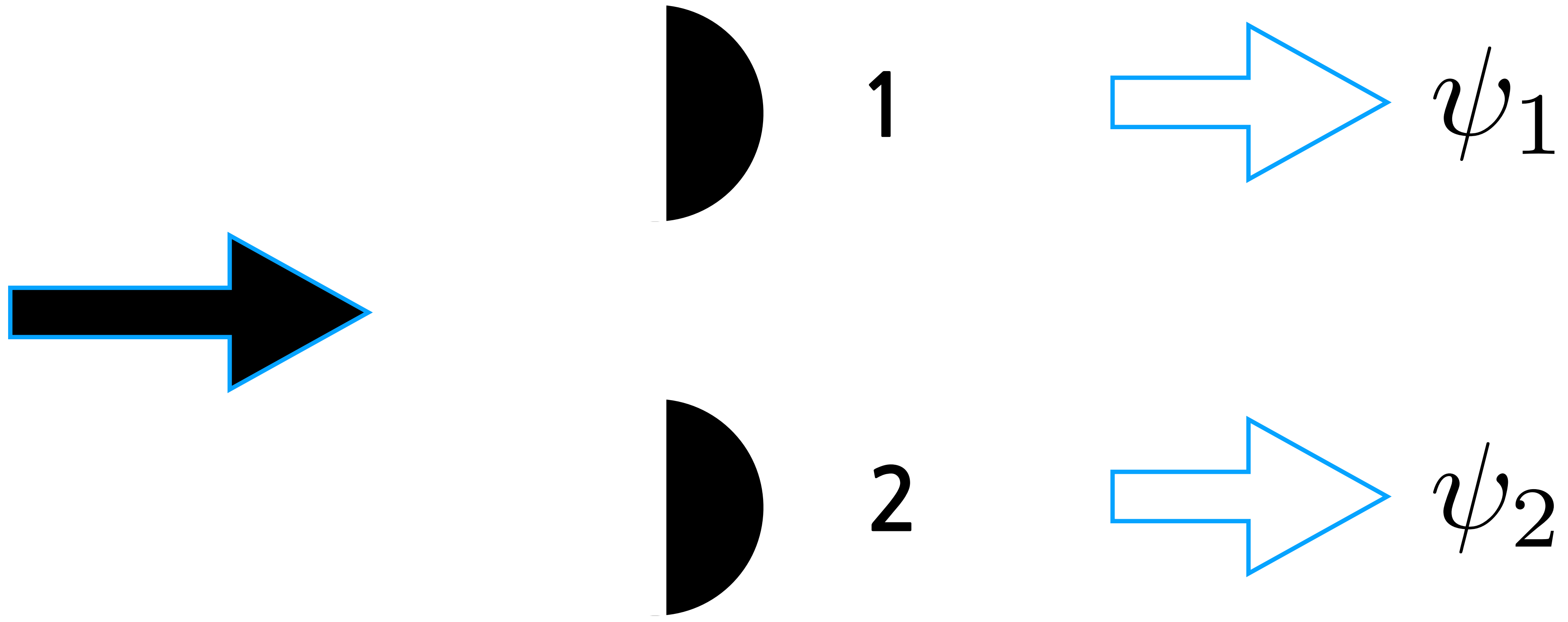
How to discriminate quantum states?

- Use a figure of merit to design a discrimination scheme and optimize the measurement setting.
- Minimize the average error in the discrimination scheme.
- Incorporate inconclusive outcomes to unambiguously determine the prepared state.
- Use projective quantum measurement.
- Use generalized measurement.

Applications of quantum state discrimination quantum communication, quantum computing, cryptography, and probabilistic algorithms.

Q - C scenario



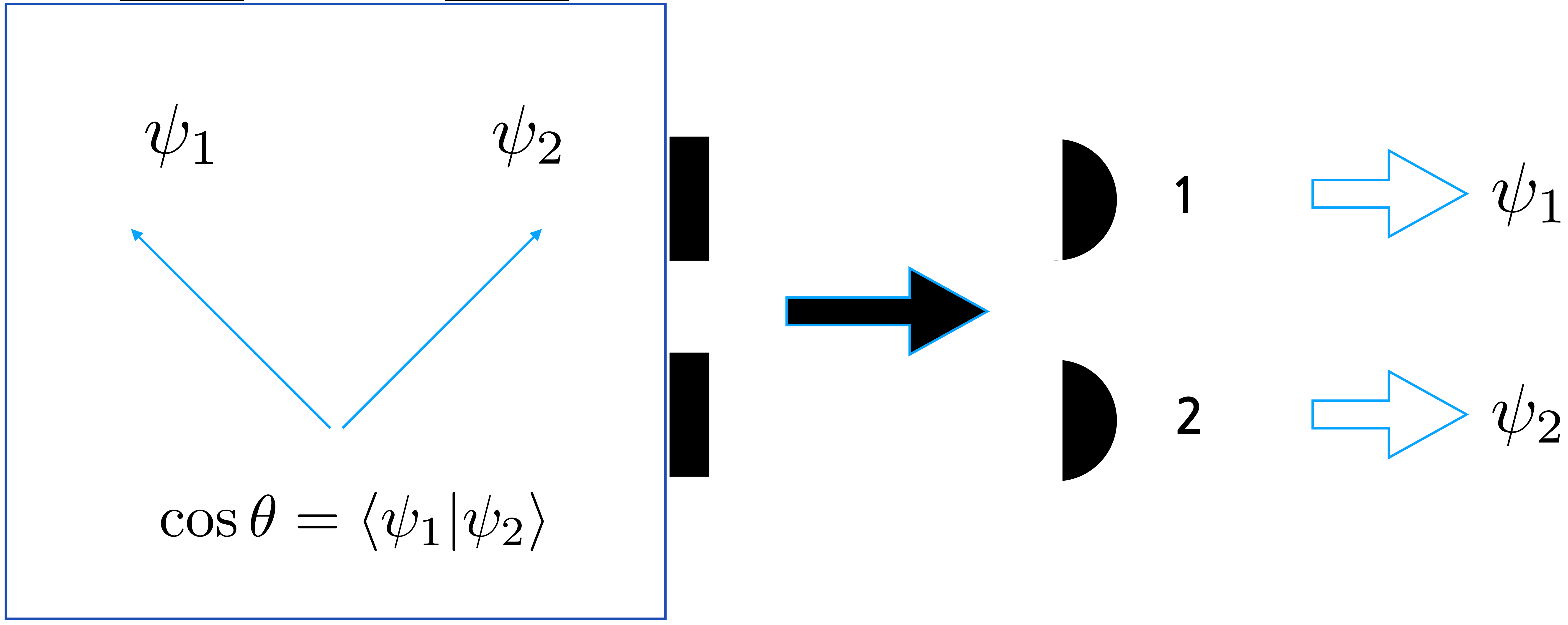


Quantum State Discrimination

$$\cos\theta = \langle \psi_1 | \psi_2 \rangle$$

C - Q - C scenario

Quantum State Discrimination

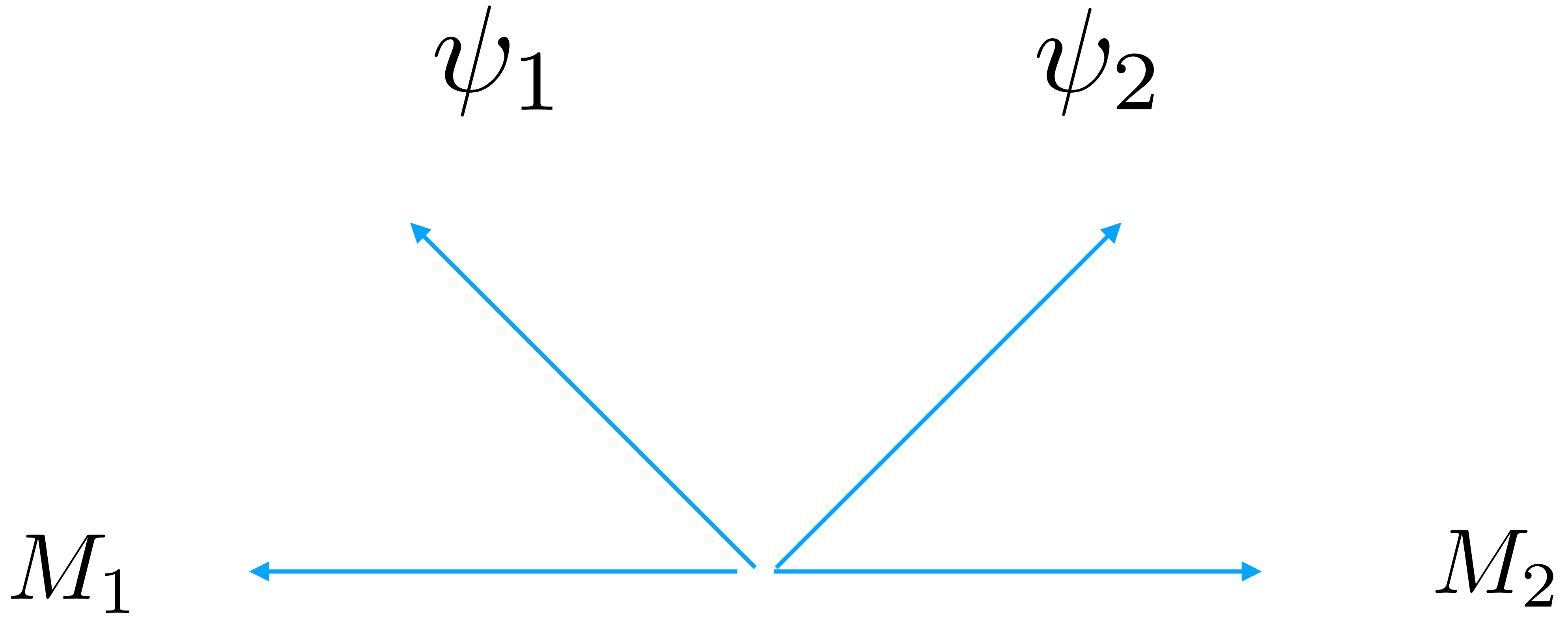


Success probability $$p_{succ} = \frac{1}{2}p(0|\psi_1) + \frac{1}{2}p(0|\psi_2)$$

Optimization problem $$\max \quad \frac{1}{2}\mathrm{tr}[\psi_1 M_1] + \frac{1}{2}\mathrm{tr}[\psi_2 M_2]$$

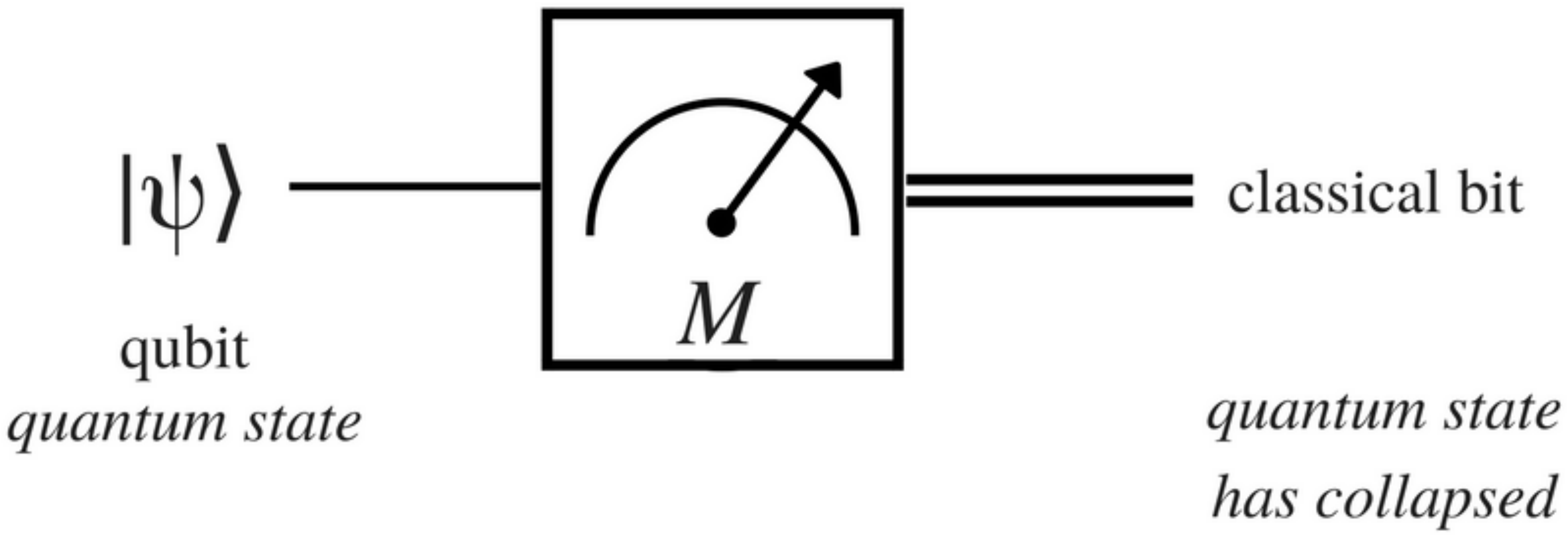
Optimization problem $\max \quad \frac{1}{2}\mathrm{tr}[\psi_1 M_1] + \frac{1}{2}\mathrm{tr}[\psi_2 M_2]$

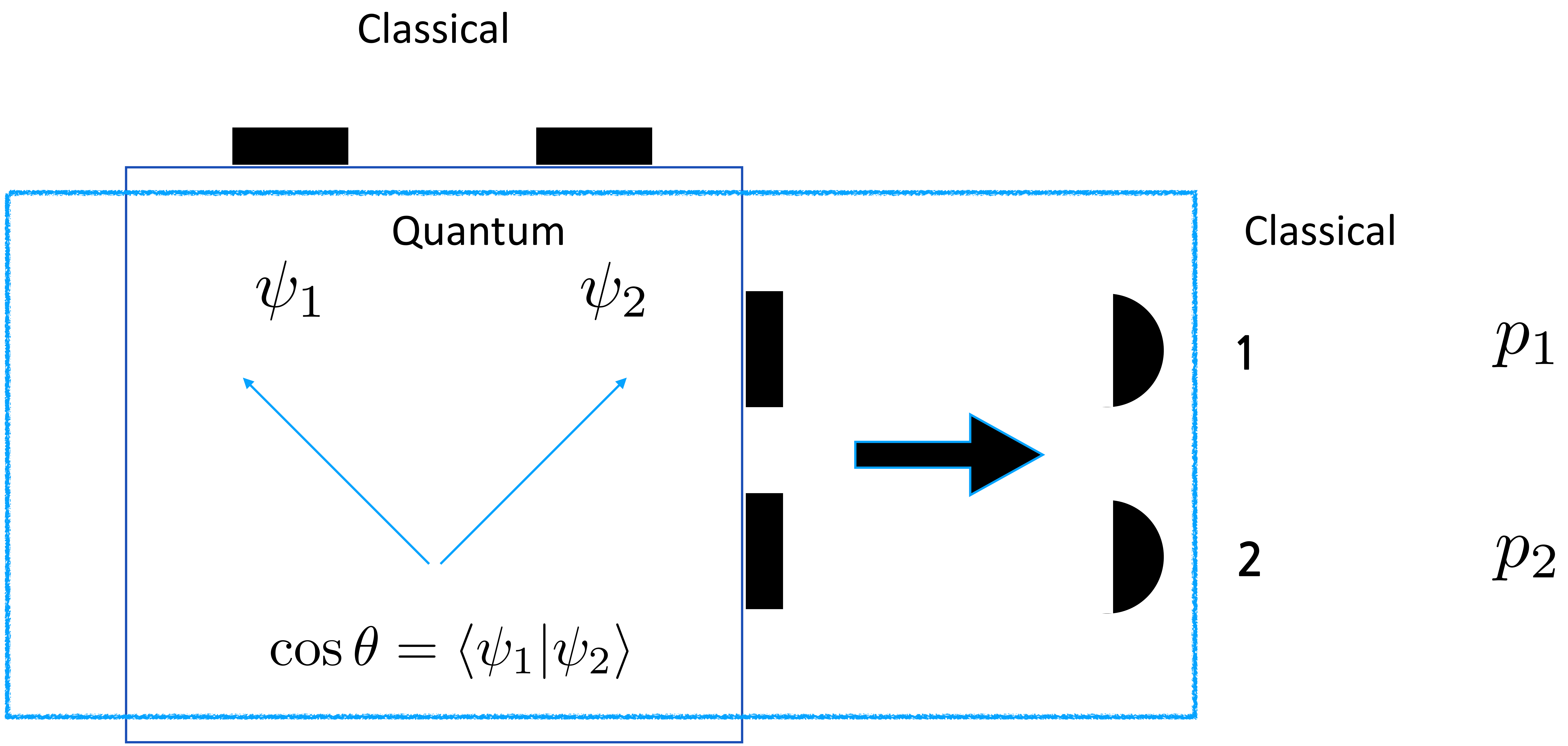
Quantum State Discrimination



Applications: the BB 1984 protocol, Quantum computing

Transmitting station bit	0	1	1	0	1	0	0	1
Transmitting station basis	+	+	X	+	X	X	X	+
Polarization	↑	→	↖	↑	↖	↗	↗	→
Receiving station basis	+	X	X	X	+	X	+	+
Receiving measurement	↑	↗	↖	↗	→	↗	→	→
Open channel discussion								
Shared key	0		1			0		1




Classical
Quantum
ψ_1
ψ_2
$\cos\theta = \langle\psi_1|\psi_2\rangle$
Classical
1
2
p_1
p_2

Classical

Classical

1 p_1

2 p_2

Can there be a classical model for the scenario?

$$s_q \equiv \frac{1}{2}\mathrm{Tr}[E_{g_\phi}|\phi\rangle\langle\phi|] + \frac{1}{2}\mathrm{Tr}[E_{g_\psi}|\psi\rangle\langle\psi|].$$

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Contextual Advantage for State Discrimination

David Schmid* and Robert W. Spekkens

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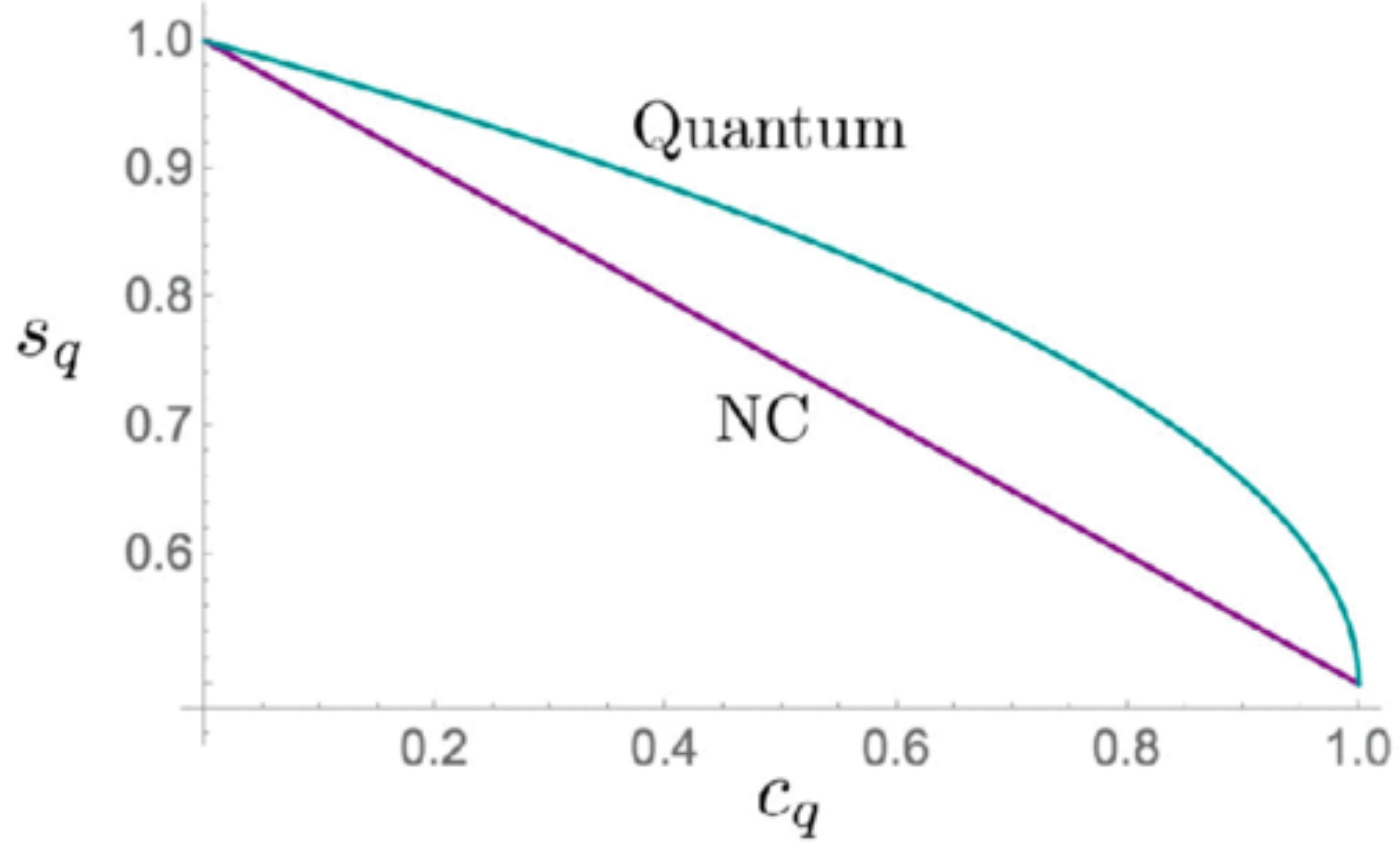
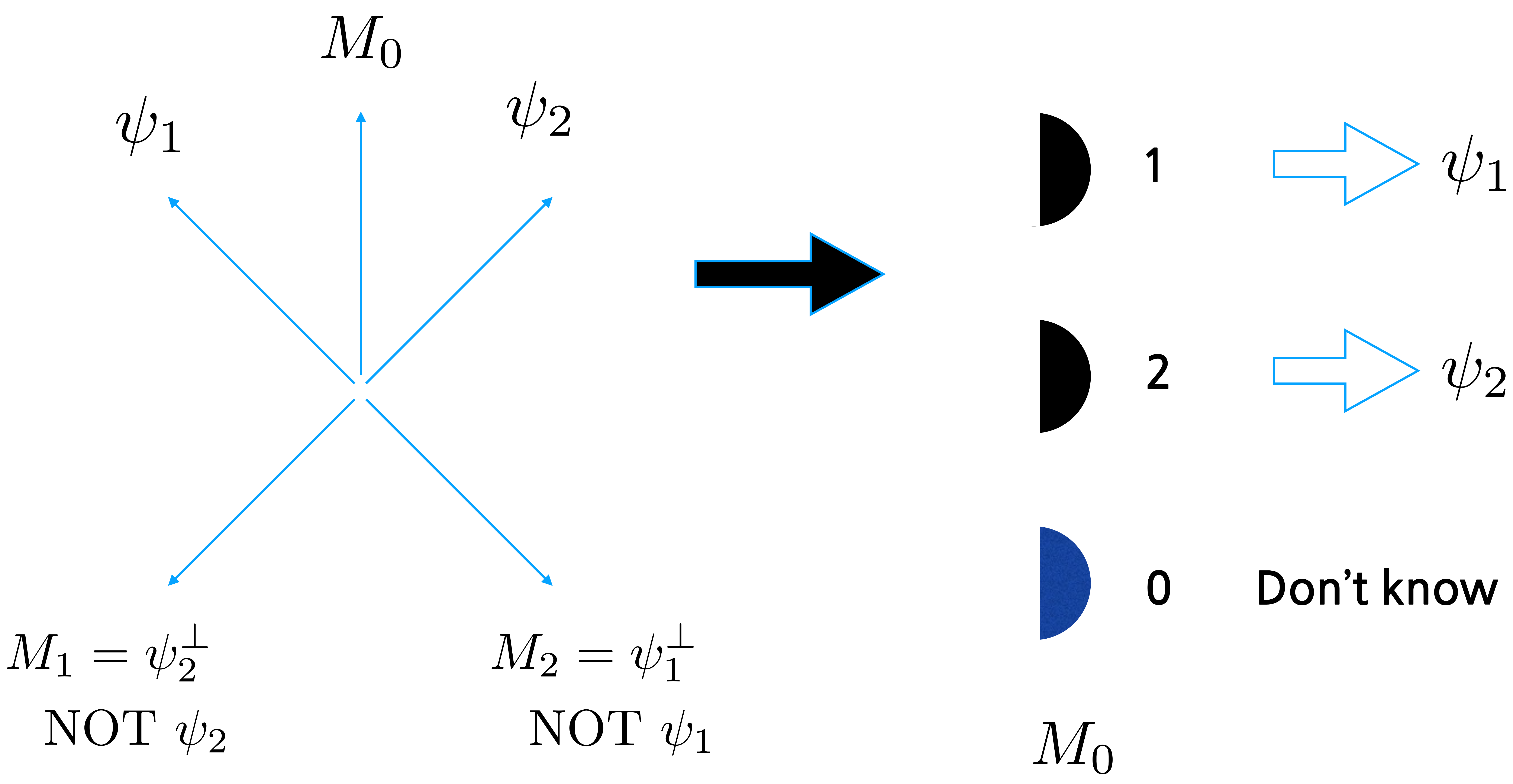
Phys. Rev. X **8**, 011015 – **Published 2 February, 2018**

DOI: https://doi.org/10.1103/PhysRevX.8.011015



$$c_q = \int_{\mathrm{supp}[\mu_\phi(\lambda)]} d\lambda \mu_\psi(\lambda).$$

Quantum State Discrimination



Unambiguity in outcomes $\mathrm{tr}[\psi_2 M_1] = 0$ $\mathrm{tr}[\psi_1 M_2] = 0$

Unambiguous state discrimination (USD)

Quantum cryptography using any two nonorthogonal states

[Charles H. Bennett](#)

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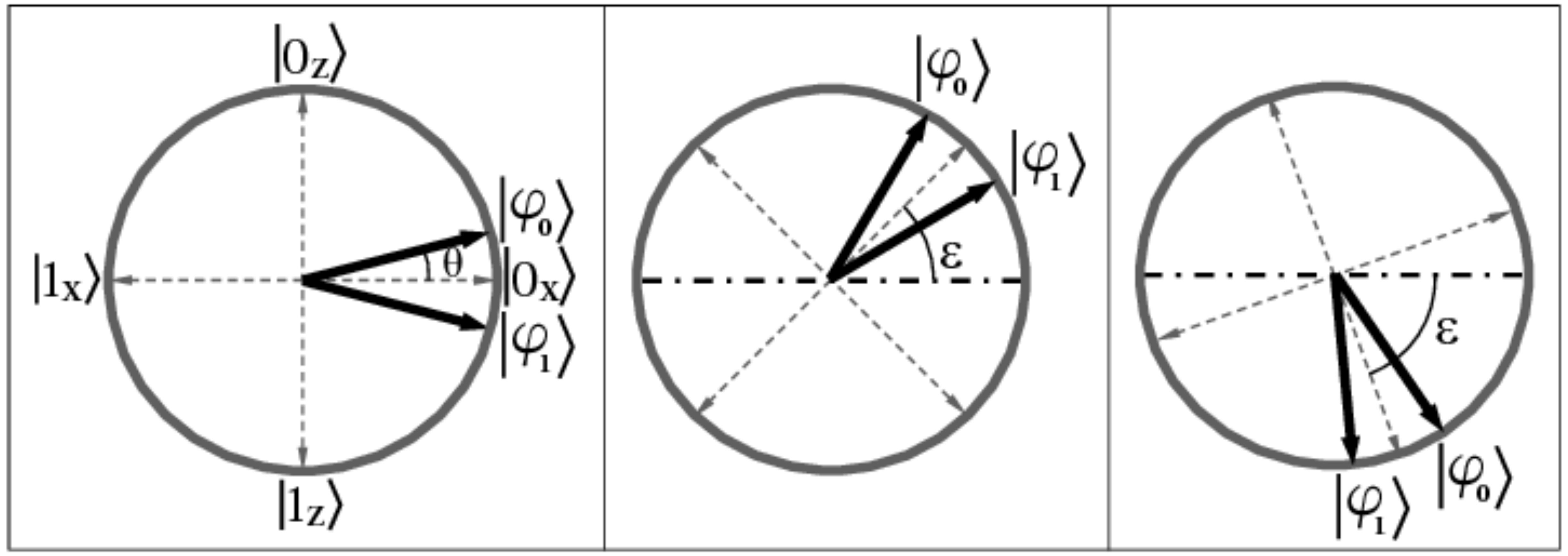
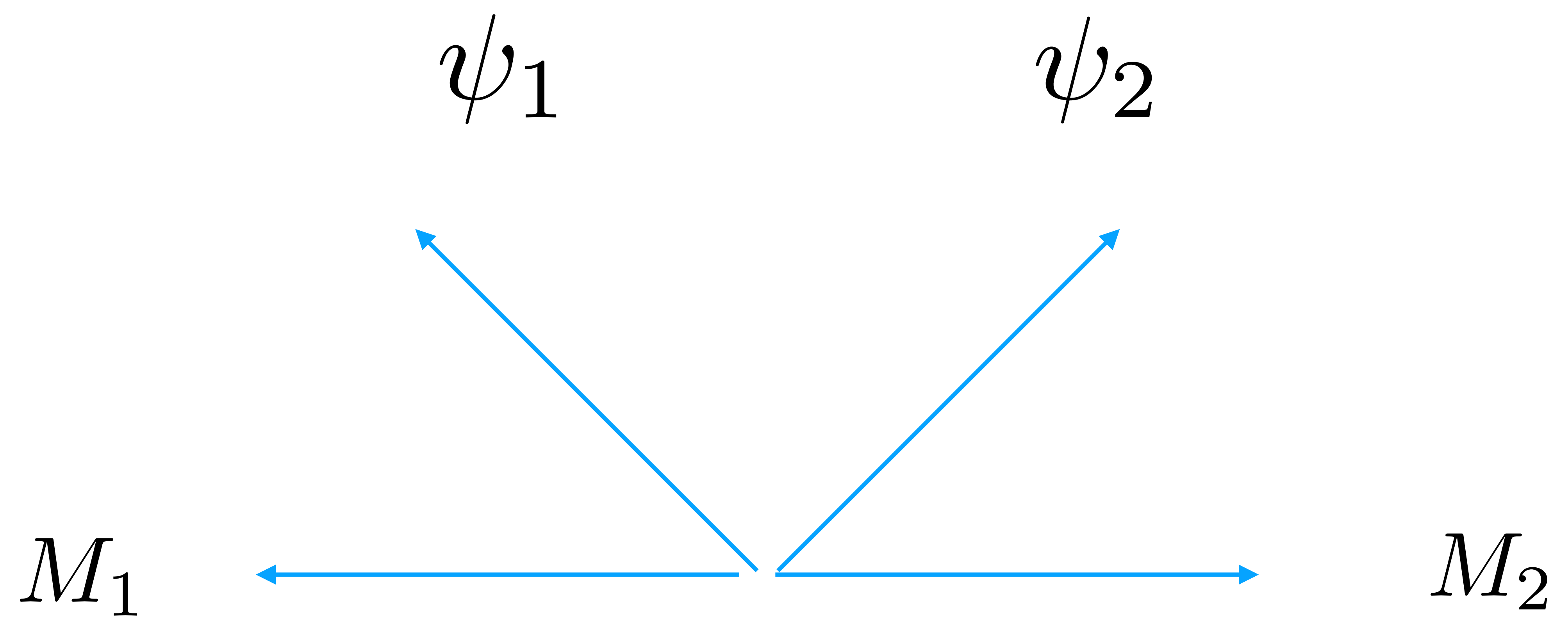
Phys. Rev. Lett. **68**, 3121 – **Published 25 May, 1992**

DOI: https://doi.org/10.1103/PhysRevLett.68.3121



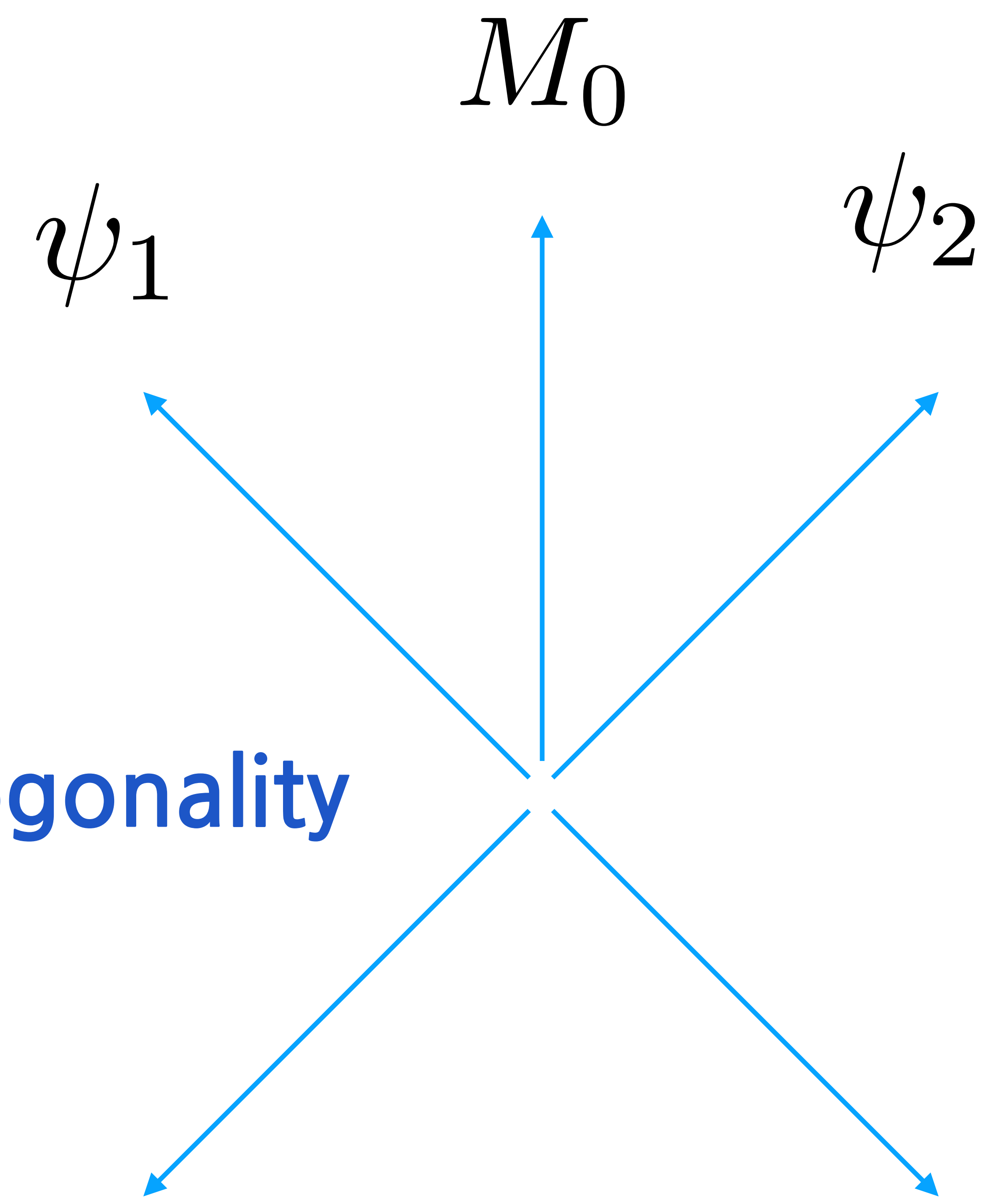
Criticism about Quantum State Discrimination

No outcomes are inconclusive $M_0 = 0$



$$M_1 + M_2 = I$$

Perfect orthogonality



$M_1 = \psi_2^{\perp}$ NOT ψ_2

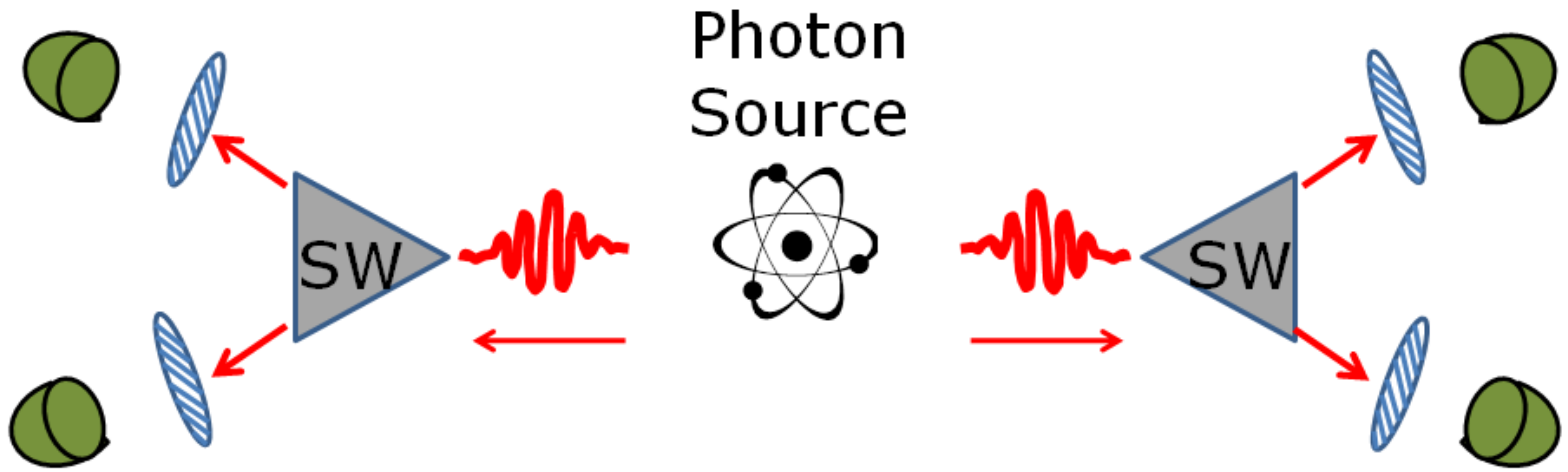
$M_2 = \psi_1^{\perp}$ NOT ψ_1

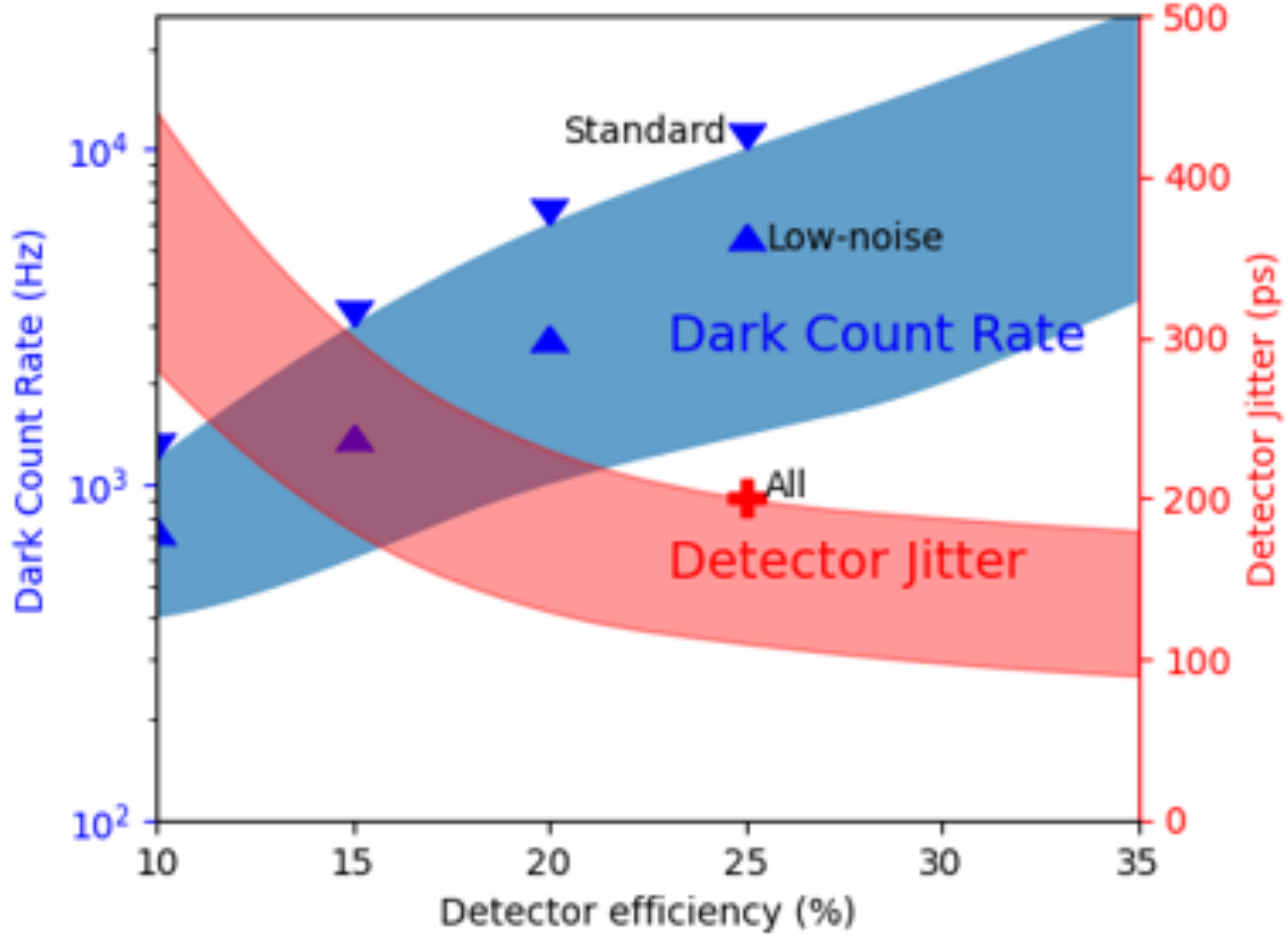
$$\mathrm{tr}[\psi_2 M_1] = 0 \qquad \mathrm{tr}[\psi_1 M_2] = 0$$

$$M_0 \neq 0$$

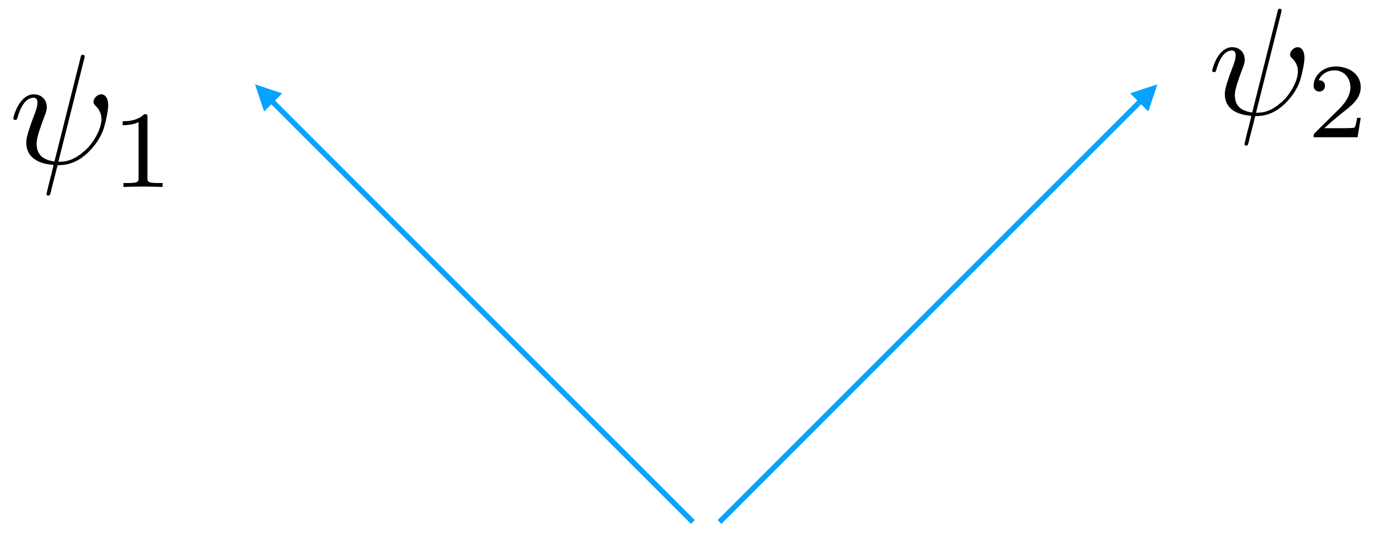
$$\mathrm{tr}[\rho M] \neq 0$$

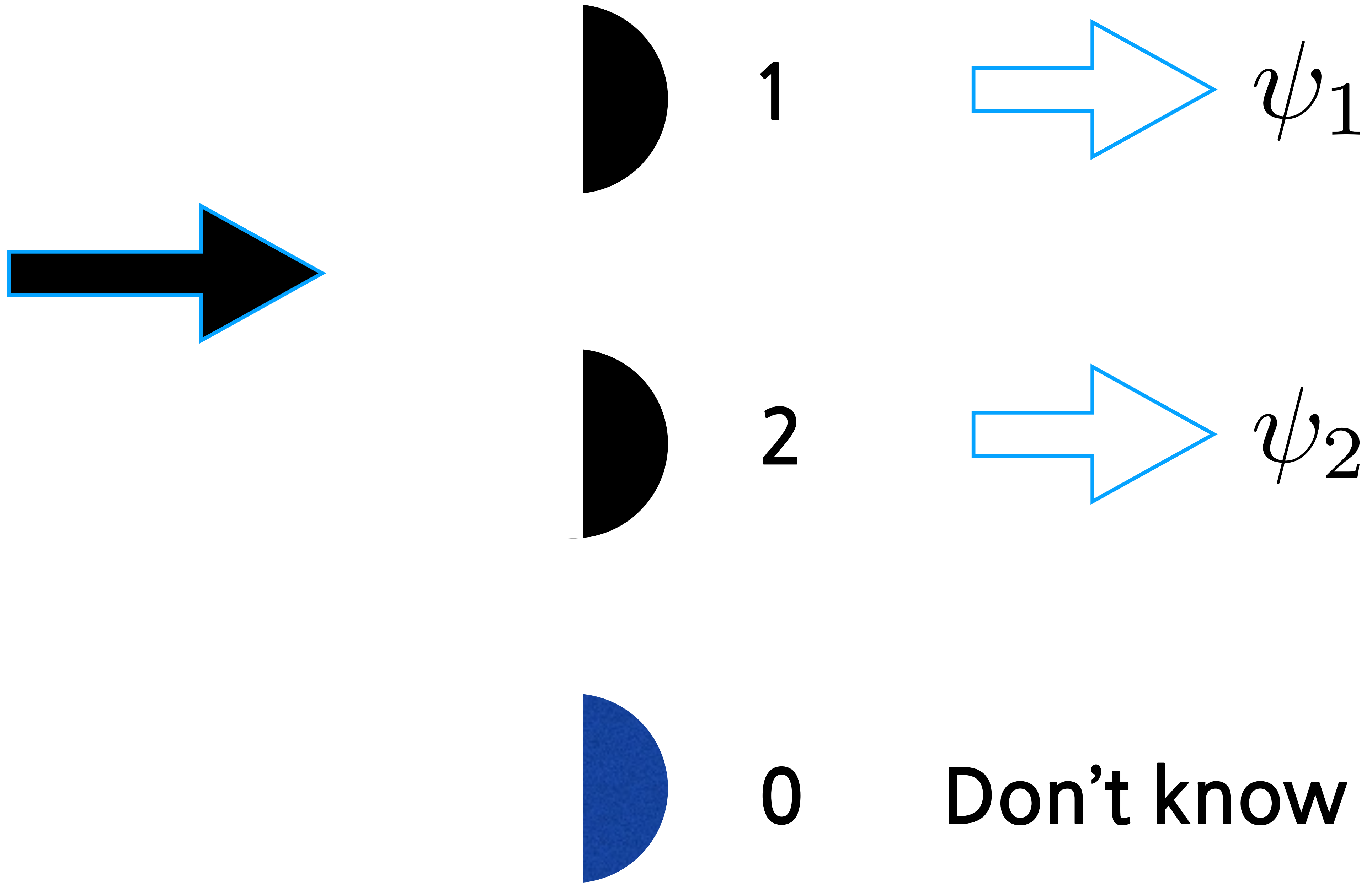
No detector is perfect





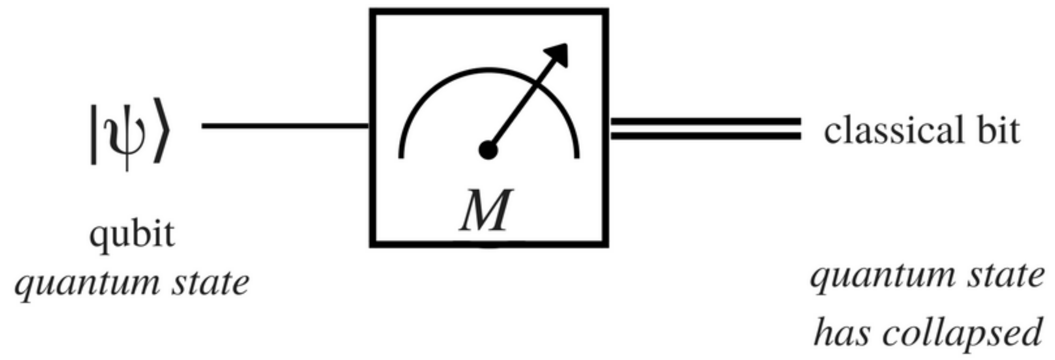
Maximum Confidence Discrimination





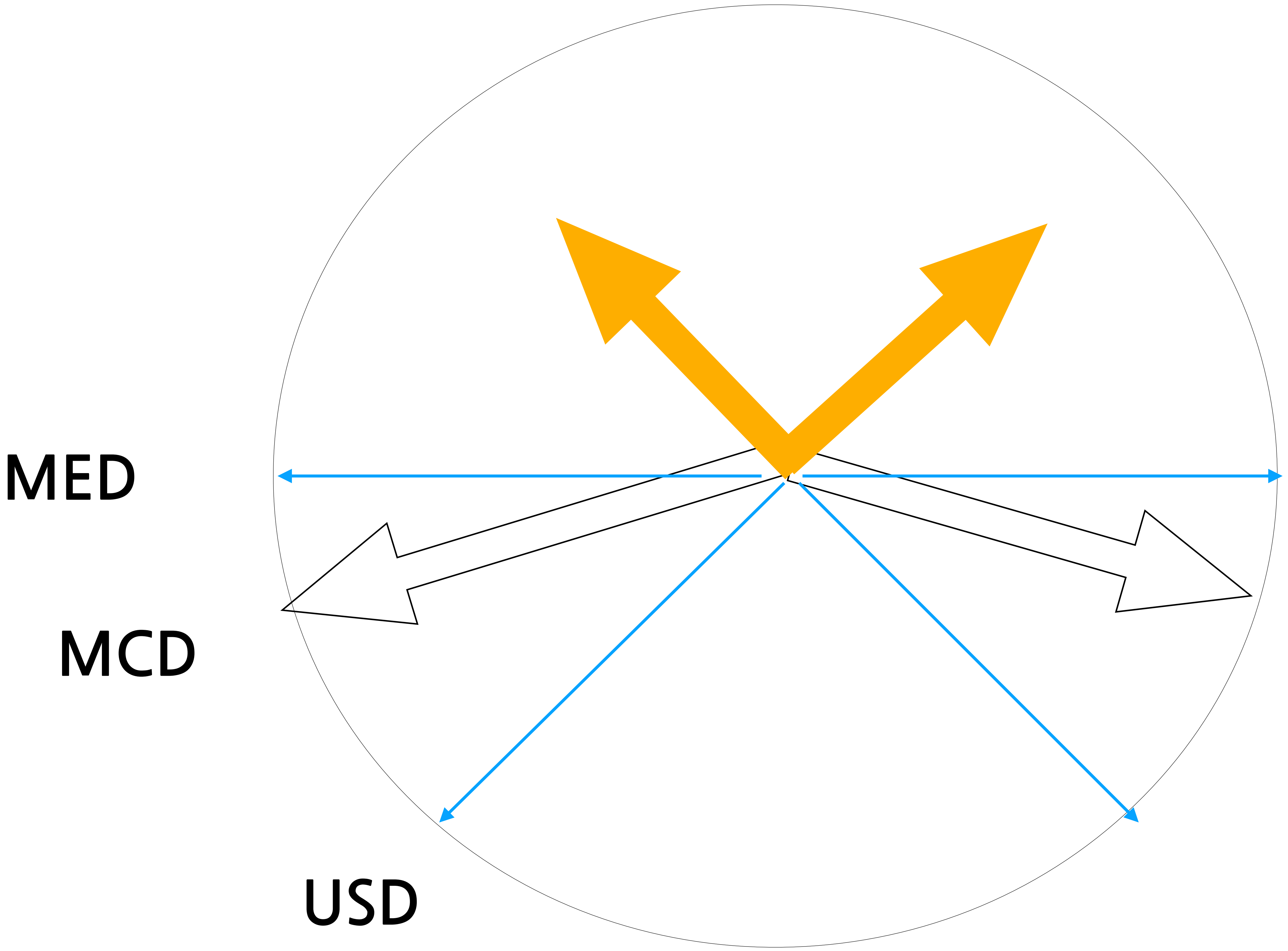
$$C_1 = \max_{M_1} \frac{\mathrm{tr}[\psi_1 M_1]}{\mathrm{tr}[\rho M_1]} = \max_{M_1} \frac{p(\psi_1|M_1)}{p(M_1)}$$

$$M_0 = I - M_1 - M_2$$



Retrodictive: given an outcome, one makes a guess about a preparation

Quantum State Discrimination



Unambiguous Discrimination

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Contextual Advantages and Certification for Maximum-Confidence Discrimination

Kieran Flatt[1], Hanwool Lee[1], Carles Roch I Carceller[2], Jonatan Bohr Brask[2], and Joonwoo Bae[1,*]

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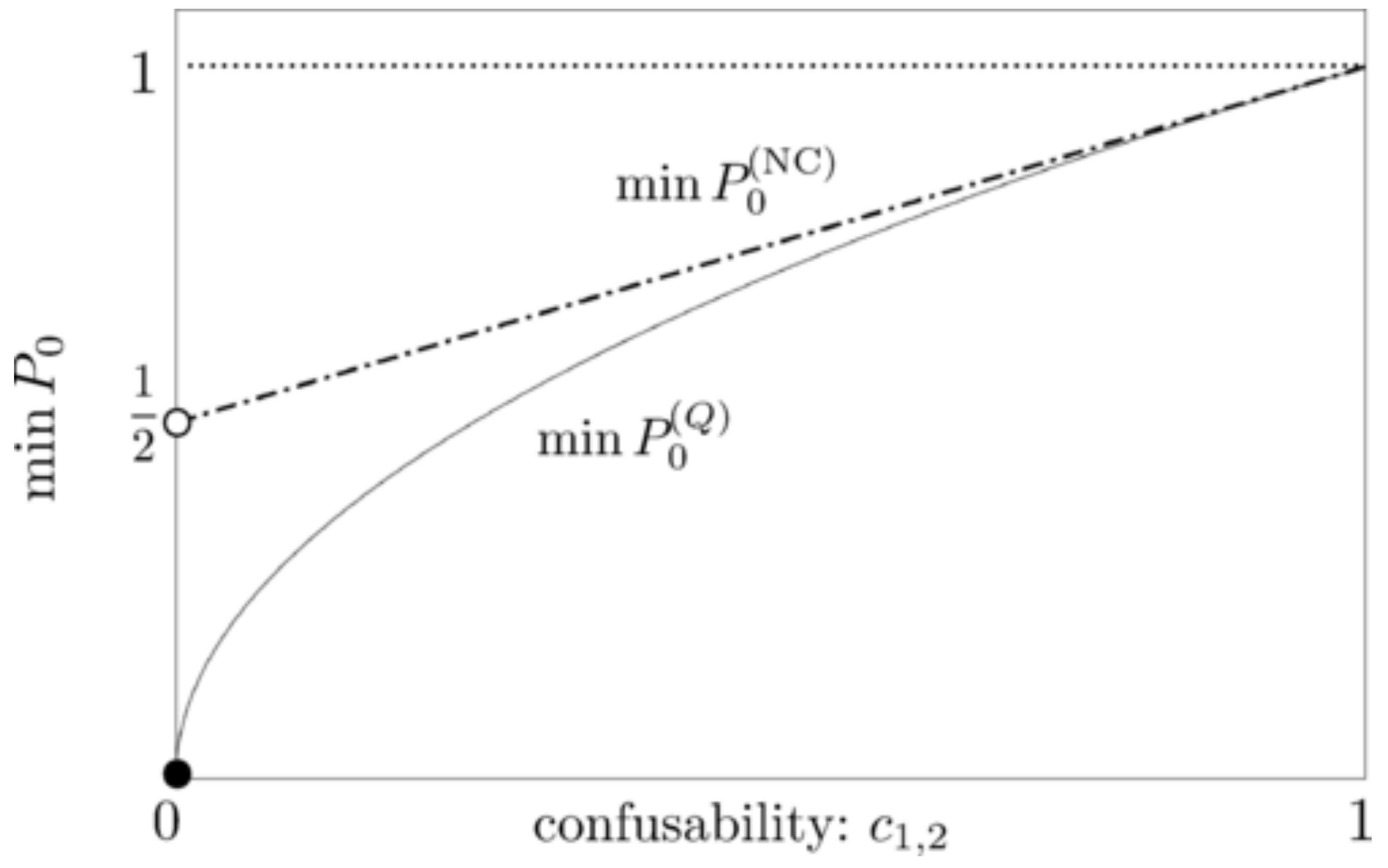
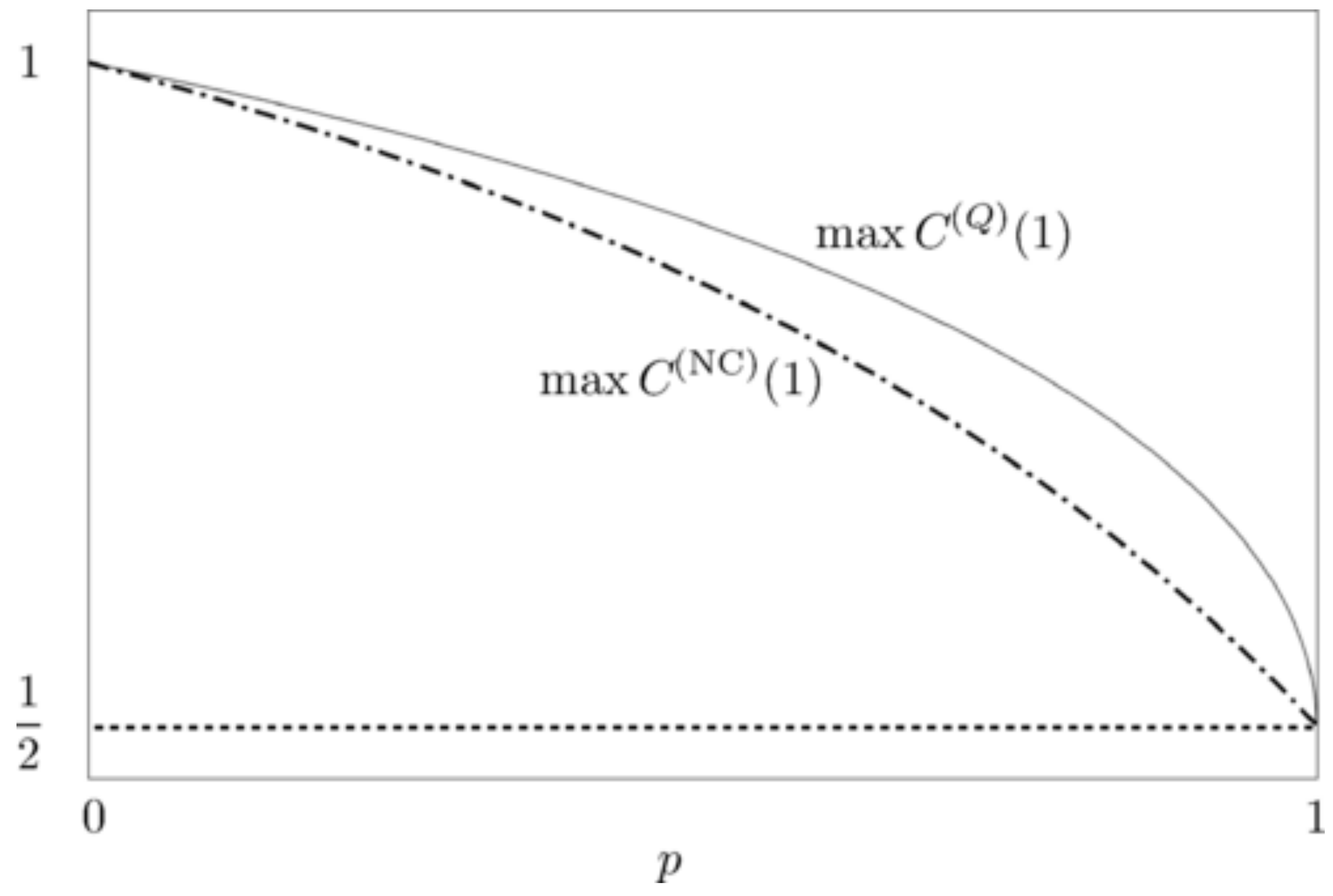
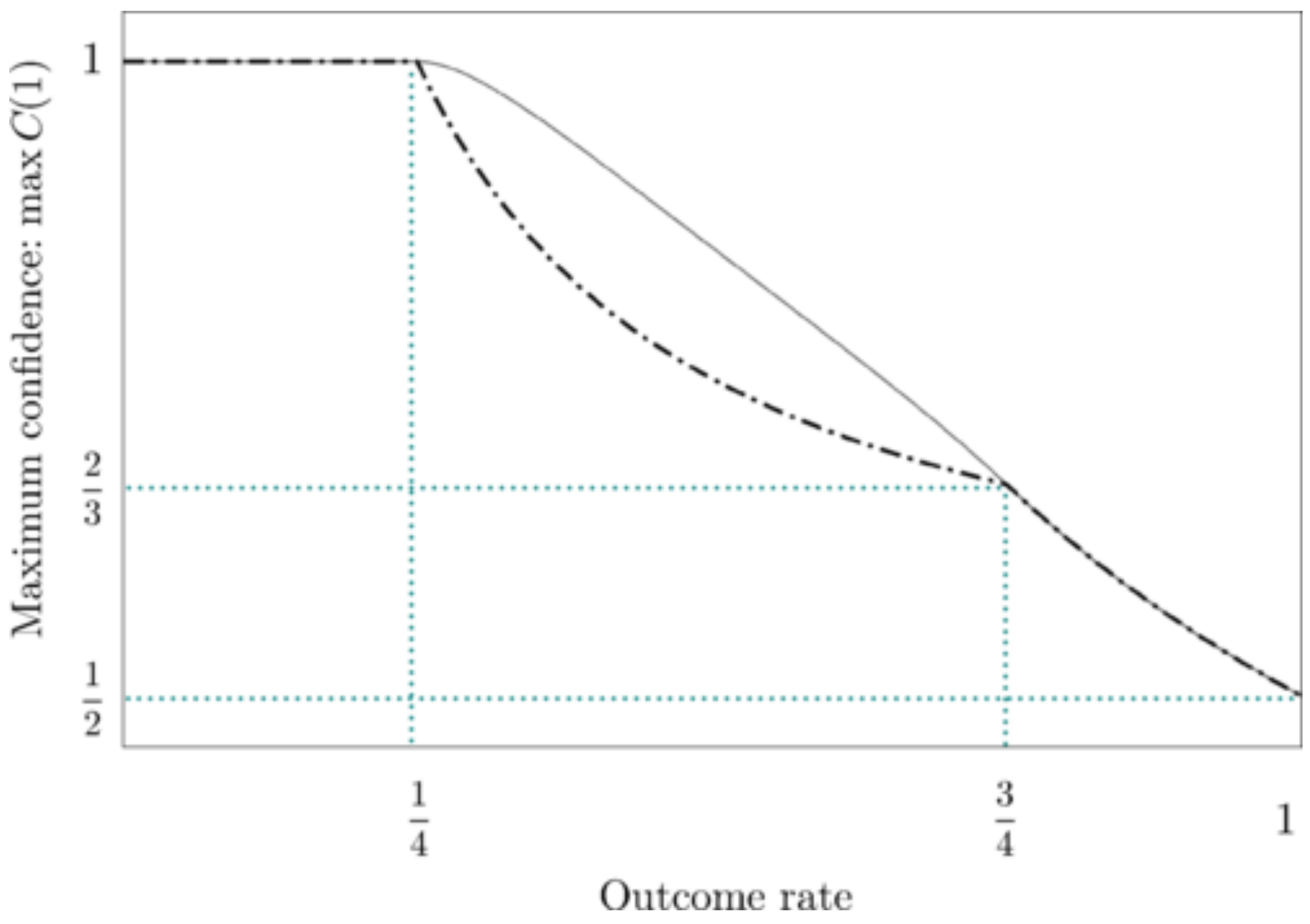
PRX Quantum **3**, 030337 – **Published 13 September, 2022**

DOI: https://doi.org/10.1103/PRXQuantum.3.030337



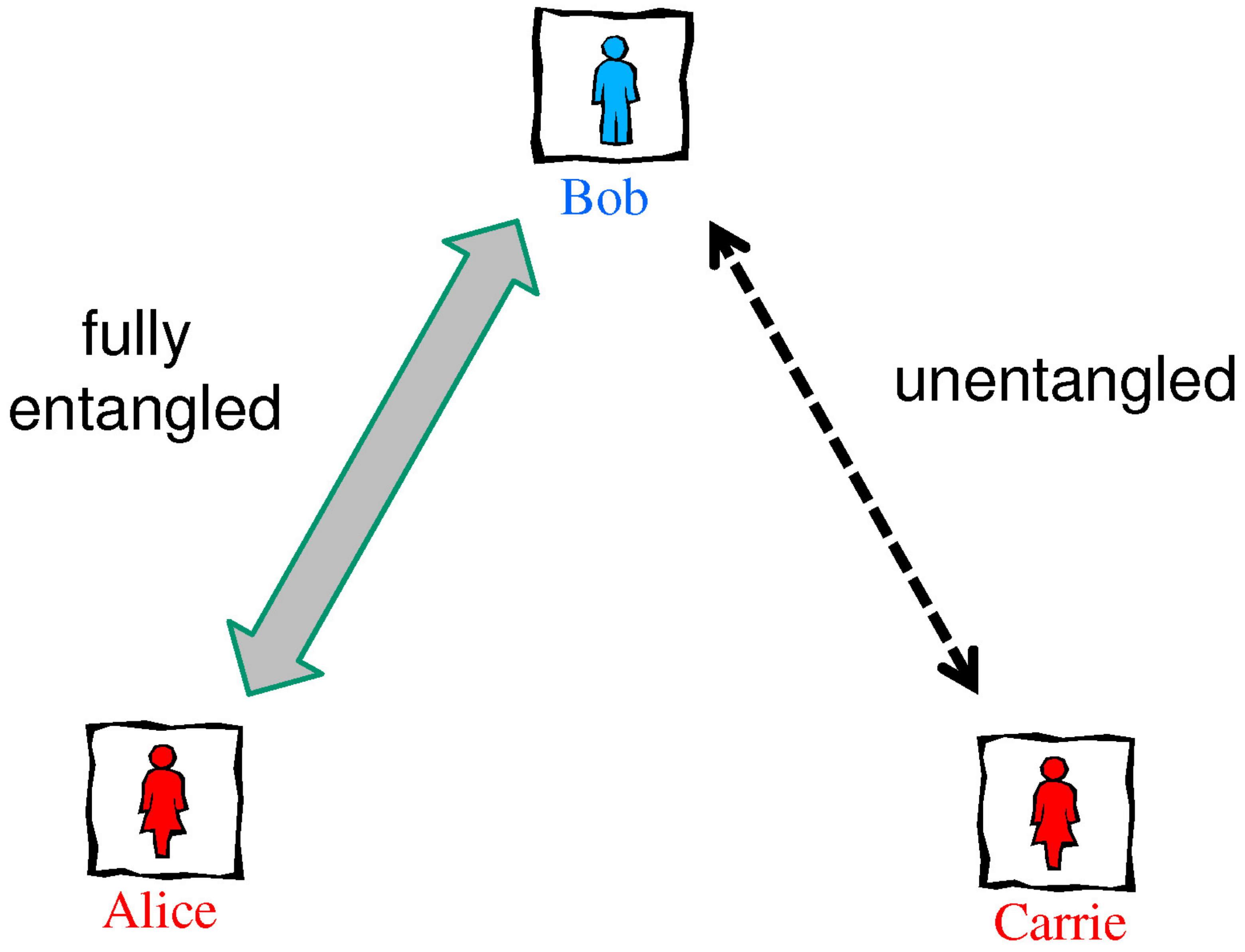
Maximum Confidence Discrimination





Sequantum Quantum Scenario

Monogamy is *frustrating*!



Asymptotic Quantum Cloning Is State Estimation

Joonwoo Bae and Antonio Acín

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Phys. Rev. Lett. **97**, 030402 – **Published 19 July, 2006**

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Quantum Information Becomes Classical When Distributed to Many Users

Giulio Chiribella* and Giacomo Mauro D'Ariano†

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Phys. Rev. Lett. **97**, 250503 – **Published 21 December, 2006**

DOI: https://doi.org/10.1103/PhysRevLett.97.250503

Monogamy Polygamy

General properties of nonsignaling theories

Ll. Masanes[1], A. Acin[2], and N. Gisin[3]

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Phys. Rev. A **73**, 012112 – **Published 30 January, 2006**

DOI: https://doi.org/10.1103/PhysRevA.73.012112

Proposition. All non-signaling theories having Bell violations contain monogamy of correlations.

Proposition. All non-signaling theories having Bell violations contain a no-cloning theorem.

1. Entanglement is monogamous; quantum states cannot be copied // quantum states cannot be discriminated.
2. Nonlocality is monogamous; nonlocal correlations cannot be copied ~ randomness

State Cloning vs. State Discrimination: One is possible, so is the other, and vice versa.

$$\psi_1 \to \psi_1^{\otimes N}, \quad \psi_2 \to \psi_2^{\otimes N} \qquad\qquad \mathrm{tr}[\psi_1^{\otimes N}\psi_2^{\otimes N}] \to 0$$

Sequential nonlocality

New Journal of Physics
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Deutsche Physikalische Gesellschaft DPG | IOP Institute of Physics

Nonlocality in sequential correlation scenarios

Rodrigo Gallego[1], Lars Erik Würflinger[2], Rafael Chaves[3], Antonio Acín[2,4] and Miguel Navascués[5]

Multiple Observers Can Share the Nonlocality of Half of an Entangled Pair by Using Optimal Weak Measurements

Ralph Silva[1,*], Nicolas Gisin[2], Yelena Guryanova[1], and Sandu Popescu[1]

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Arbitrarily Many Independent Observers Can Share the Nonlocality of a Single Maximally Entangled Qubit Pair

Peter J. Brown[1,2,*] and Roger Colbeck[2,†]

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Phys. Rev. Lett. **125**, 090401 – **Published 24 August, 2020**

DOI: https://doi.org/10.1103/PhysRevLett.125.090401

Sequential state discrimination (USD)

Extracting Information from a Qubit by Multiple Observers: Toward a Theory of Sequential State Discrimination

Janos Bergou[1], Edgar Feldman[2], and Mark Hillery[1]

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Phys. Rev. Lett. **111**, 100501 – **Published 3 September, 2013**

DOI: https://doi.org/10.1103/PhysRevLett.111.100501

Sequential measurements on qubits by multiple observers: Joint Best Guess strategy

Publisher: IEEE Cite This PDF

Dov Fields ; János A. Bergou ; Árpád Varga **All Authors**

Multiple Observers Can Share the Nonlocality of Half of an Entangled Pair by Using Optimal Weak Measurements

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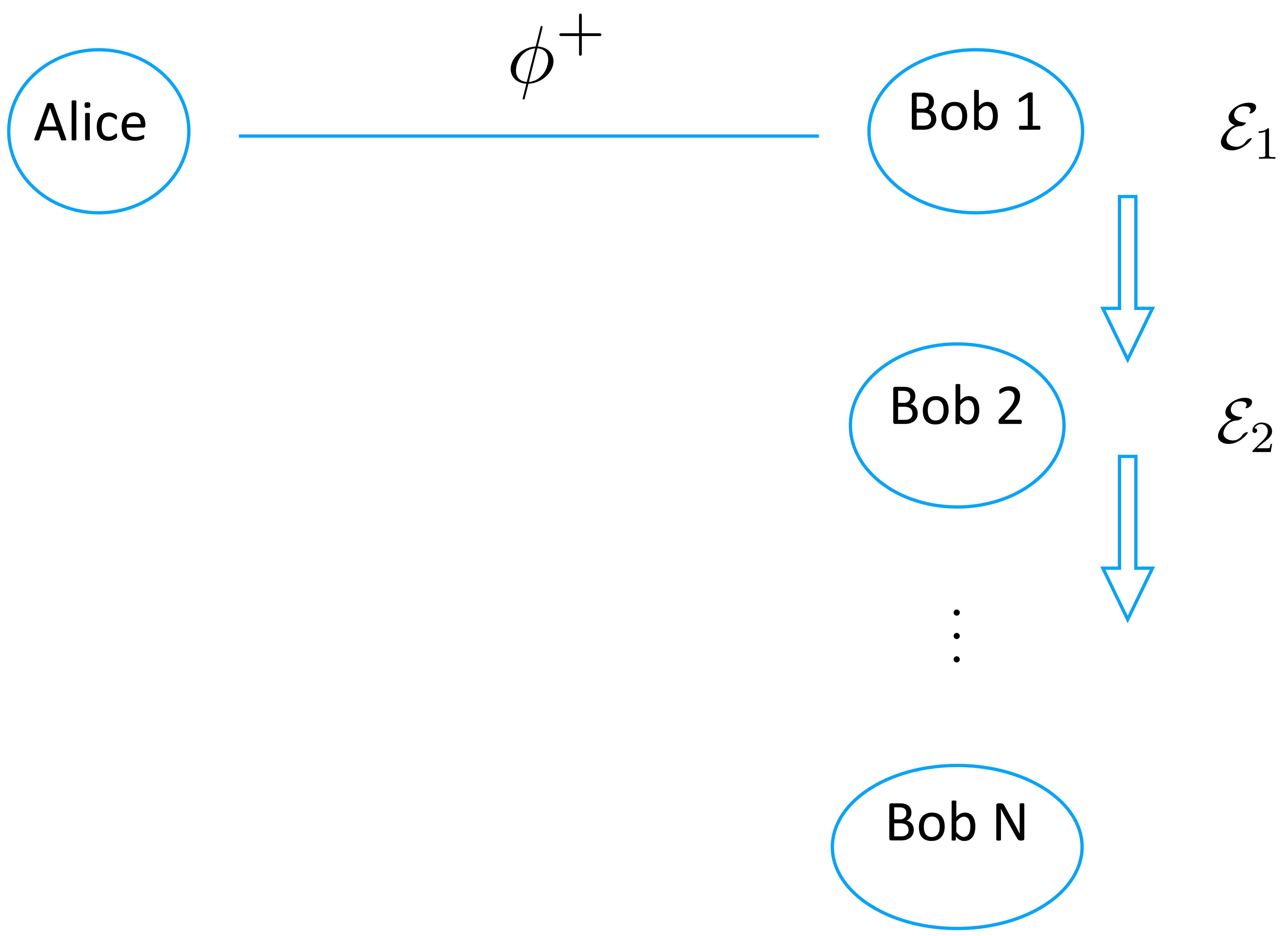
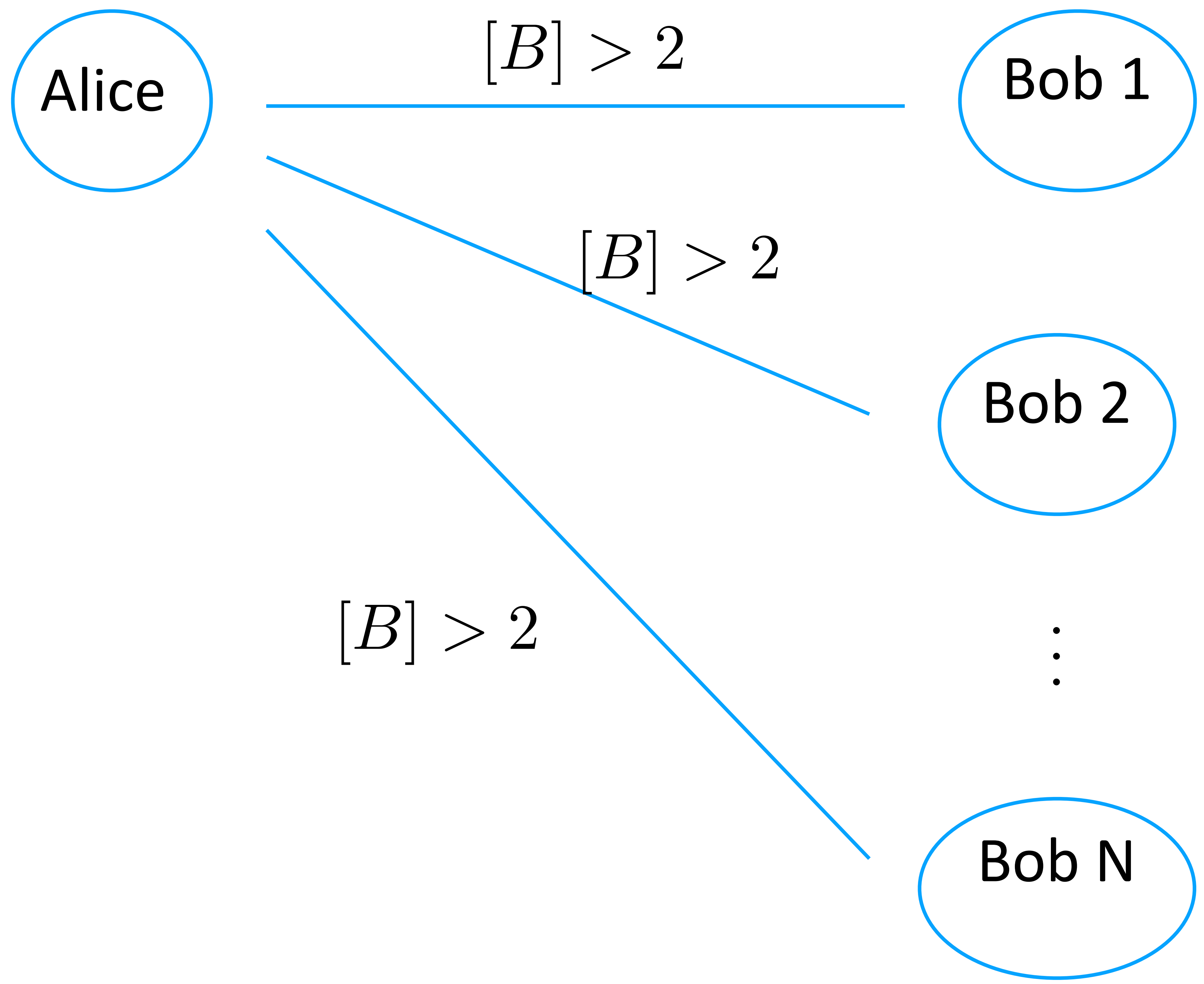
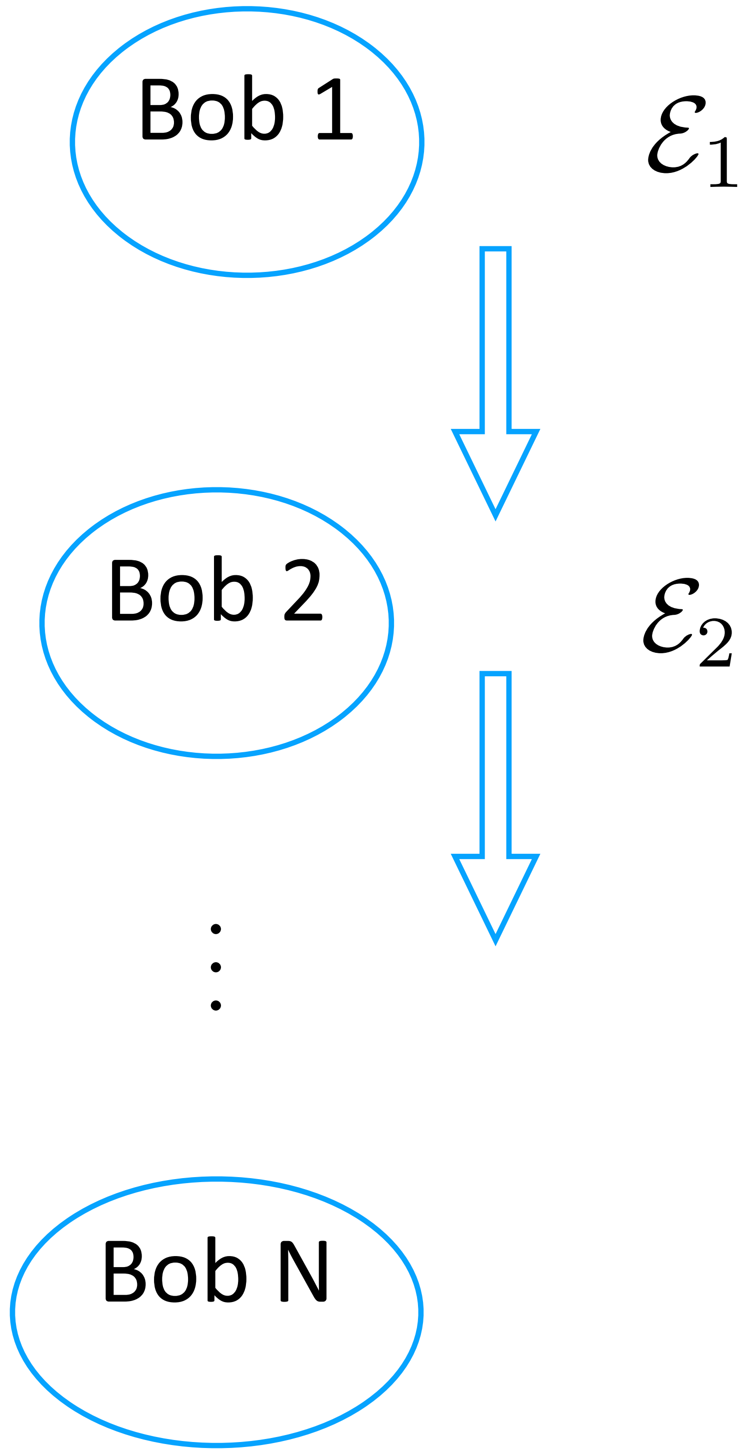
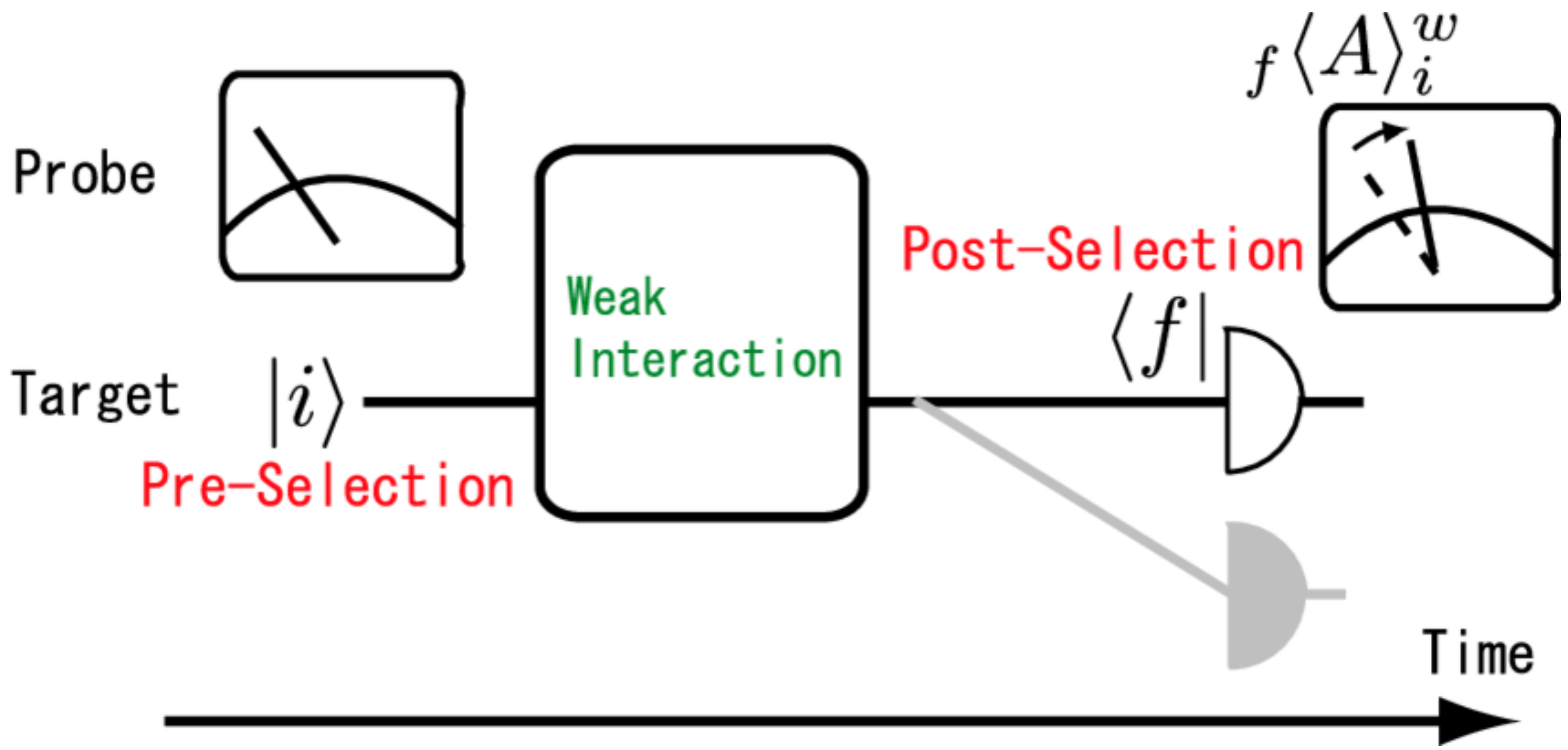


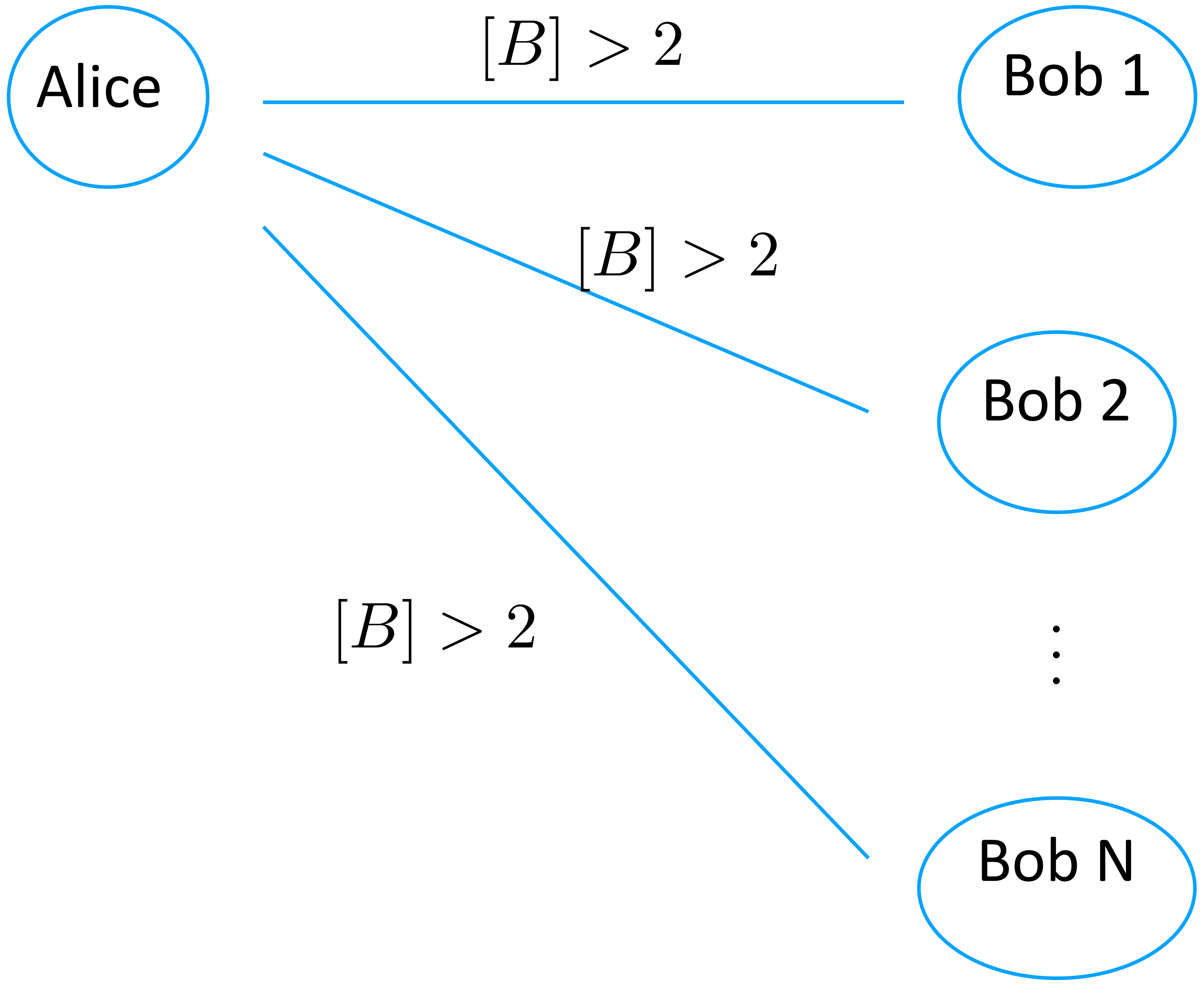


What makes a Bell scenario sequential? Weak measurements





Tradeoff between Bell violations and the number of parties



$$[B] = 2 + \epsilon_i$$

$$N = N(\epsilon_1, \cdots, \epsilon_N) < \infty$$

Extracting Information from a Qubit by Multiple Observers: Toward a Theory of Sequential State Discrimination

Janos Bergou[1], Edgar Feldman[2], and Mark Hillery[1]

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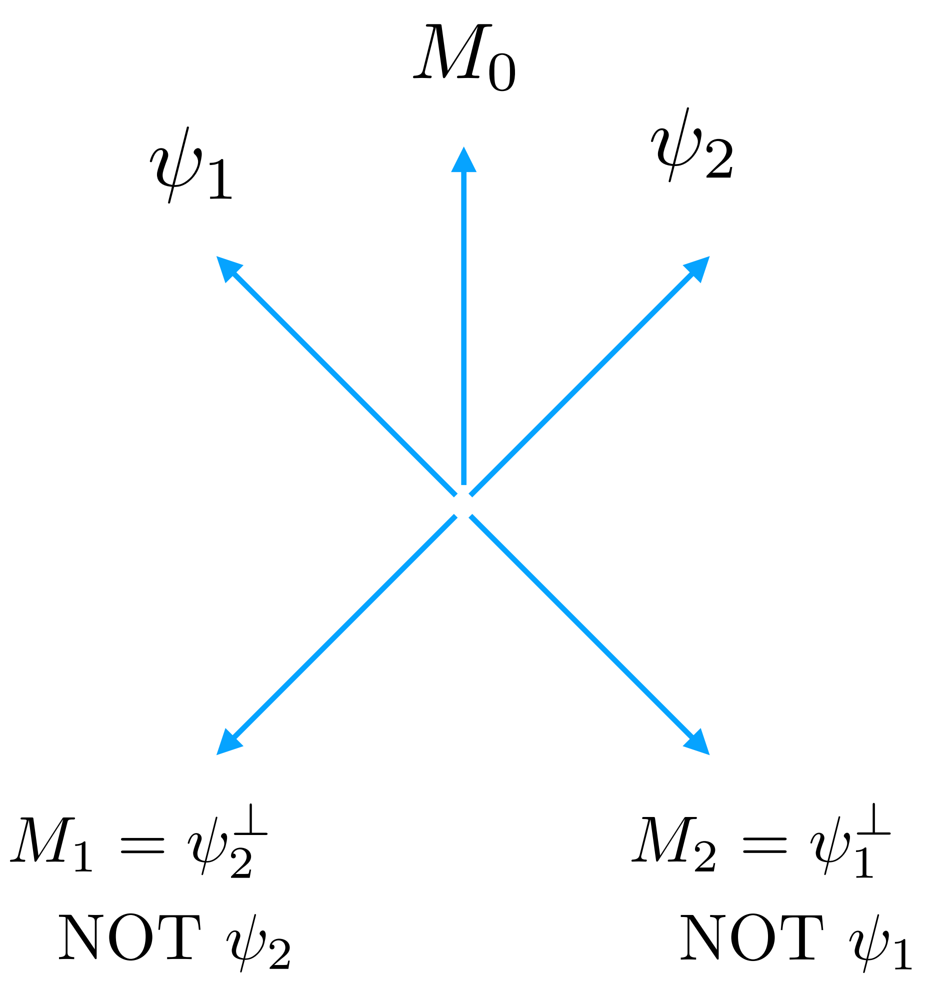
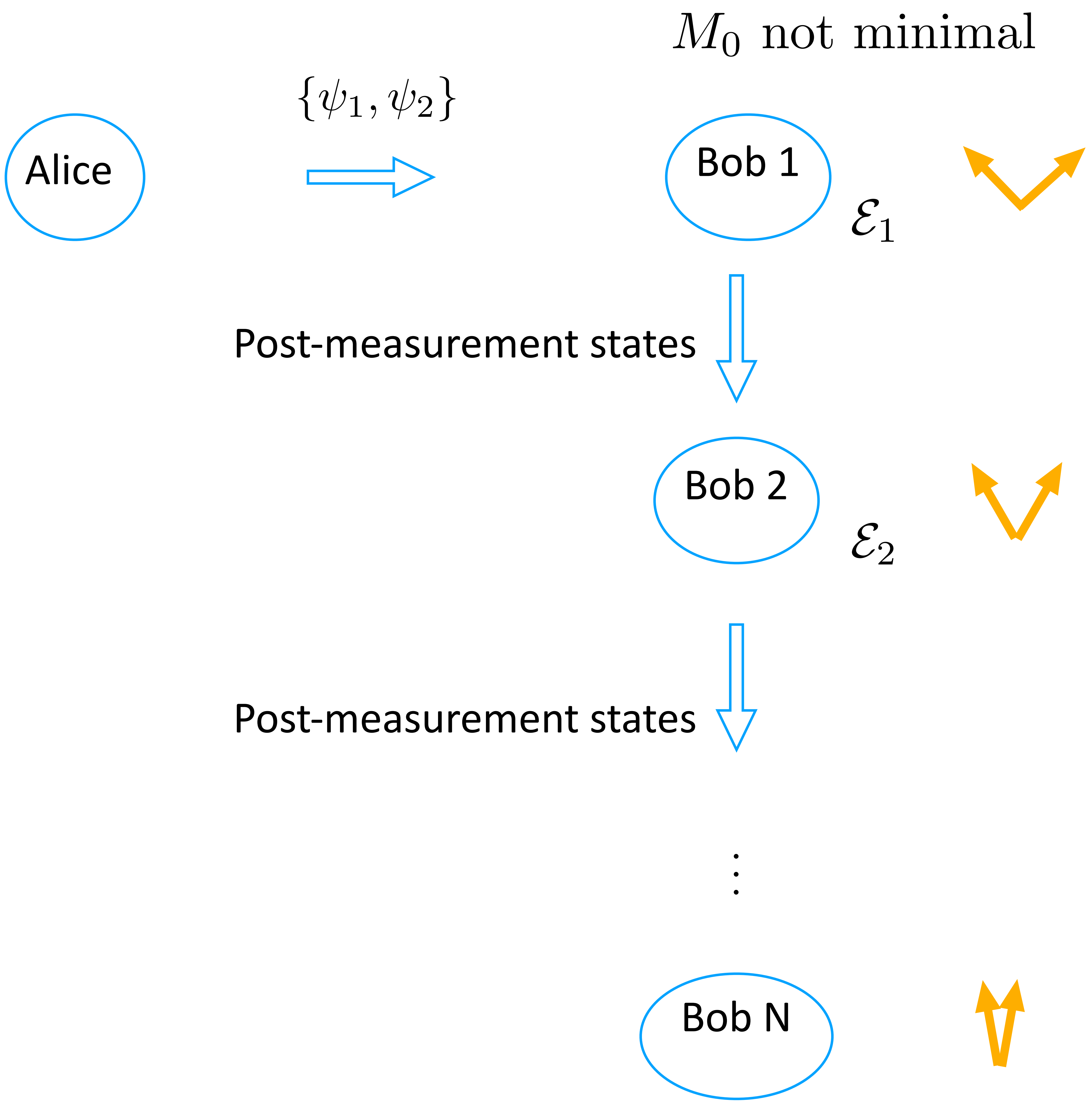
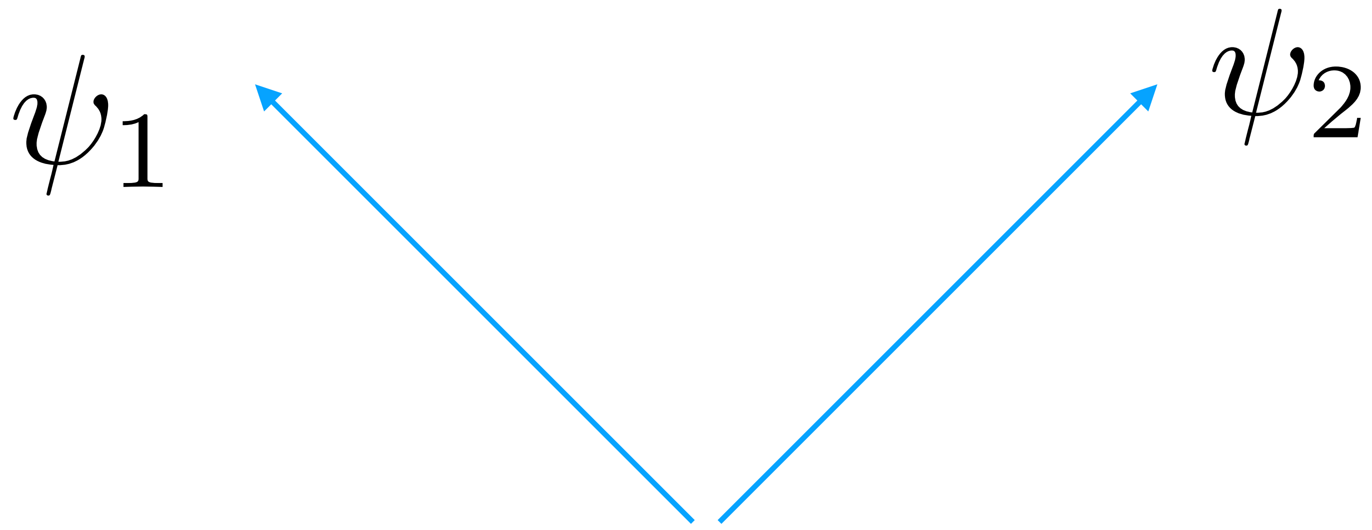


$\mathrm{tr}[\psi_2 M_1] = 0 \qquad \mathrm{tr}[\psi_1 M_2] = 0$



State-measurement disturbance

Can Maximum Confidence Discrimination Be Sequential?



Confidence

$$C_1 = \max_{M_1} \frac{\mathrm{tr}[\psi_1 M_1]}{\mathrm{tr}[\rho M_1]} = \max_{M_1} \frac{p(\psi_1|M_1)}{p(M_1)}$$

Sequential Quantum Maximum Confidence Discrimination

Hanwool Lee, Kieran Flatt, Joonwoo Bae

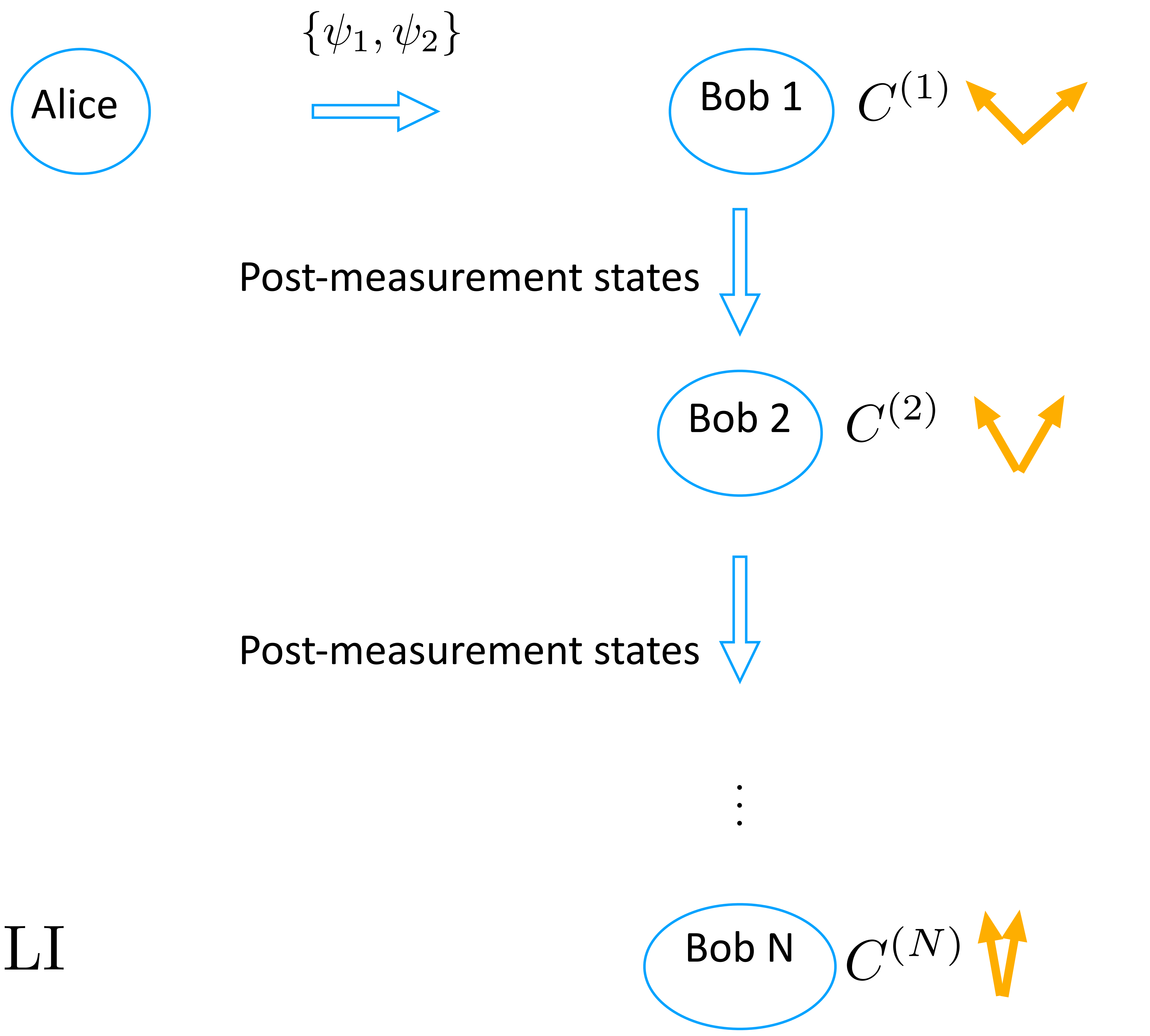
Confidence

$$\max_{M_1} \frac{\mathrm{tr}[\psi_1 M_1]}{\mathrm{tr}[\rho M_1]} = \max_{M_1} \frac{p(\psi_1|M_1)}{p(M_1)}$$

Result 1.

$$C^{(1)} = C^{(2)} = \cdots = C^{(N)} \quad \text{iff states are LI}$$

Remark. Bobs do not apply weak measurements.



State-measurement disturbance

Sequential Quantum Maximum Confidence Discrimination

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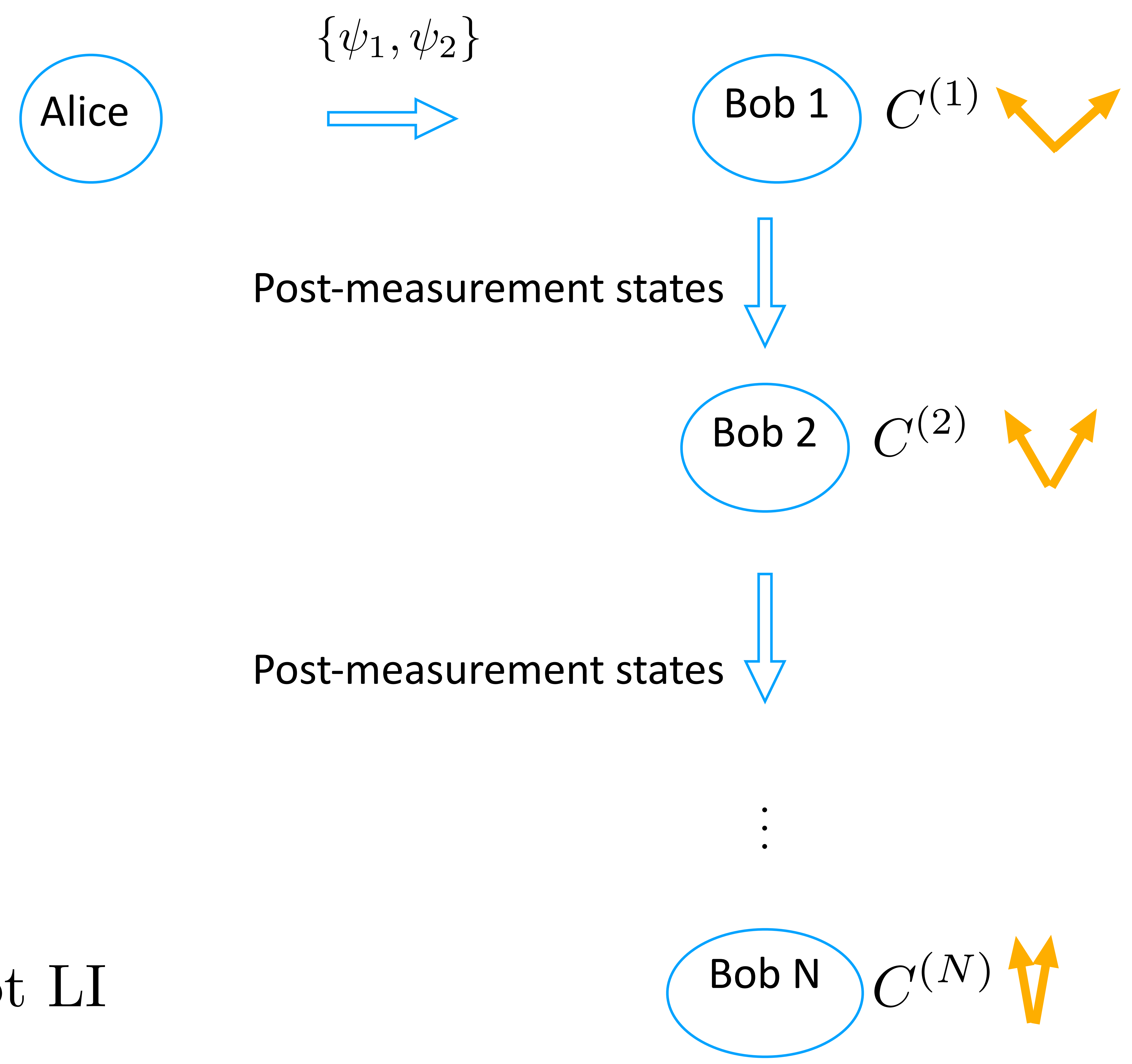


State-measurement disturbance

Confidence

$$\max_{M_1} \frac{\mathrm{tr}[\psi_1 M_1]}{\mathrm{tr}[\rho M_1]} = \max_{M_1} \frac{p(\psi_1|M_1)}{p(M_1)}$$

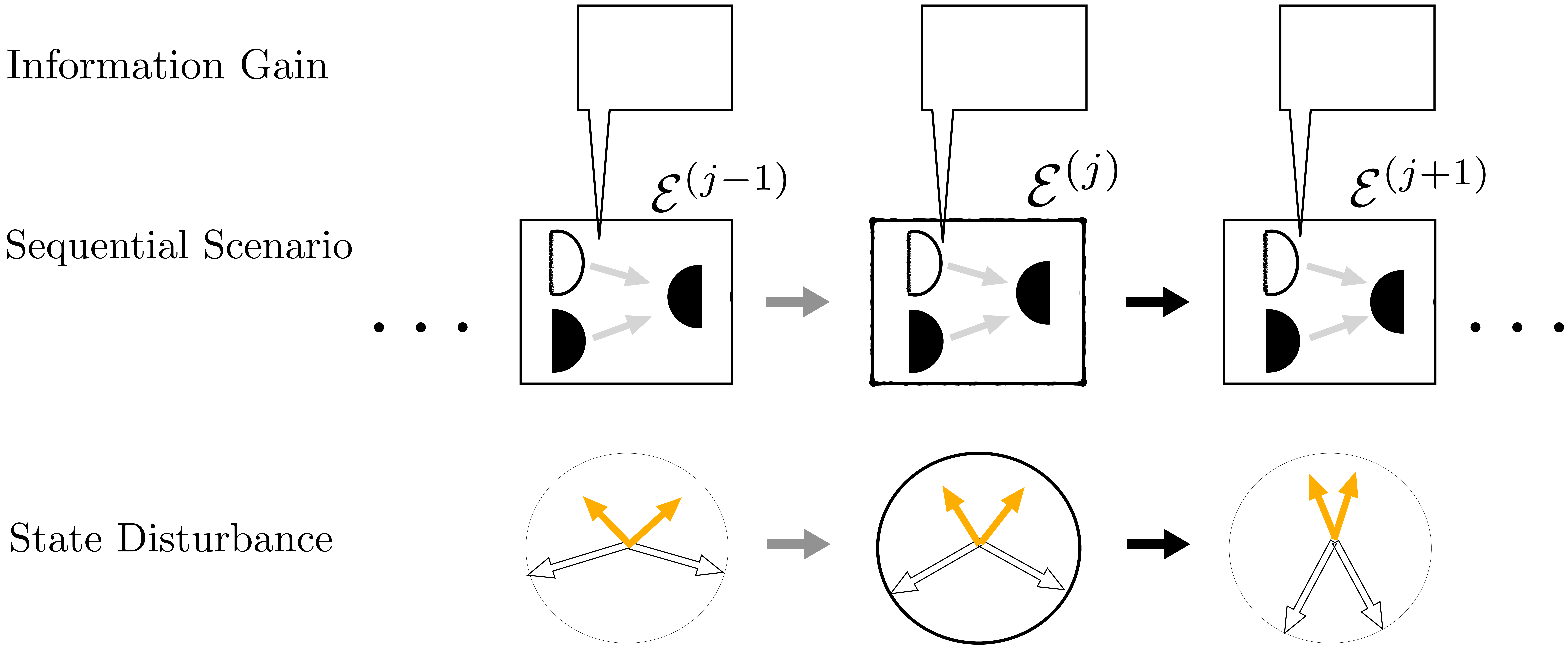
Result 2.

$$C^{(1)} > C^{(2)} > \cdots > C^{(N)} \quad \text{iff states are not LI}$$

Remark. Bobs apply weak measurements.

Sequential Quantum Maximum Confidence Discrimination

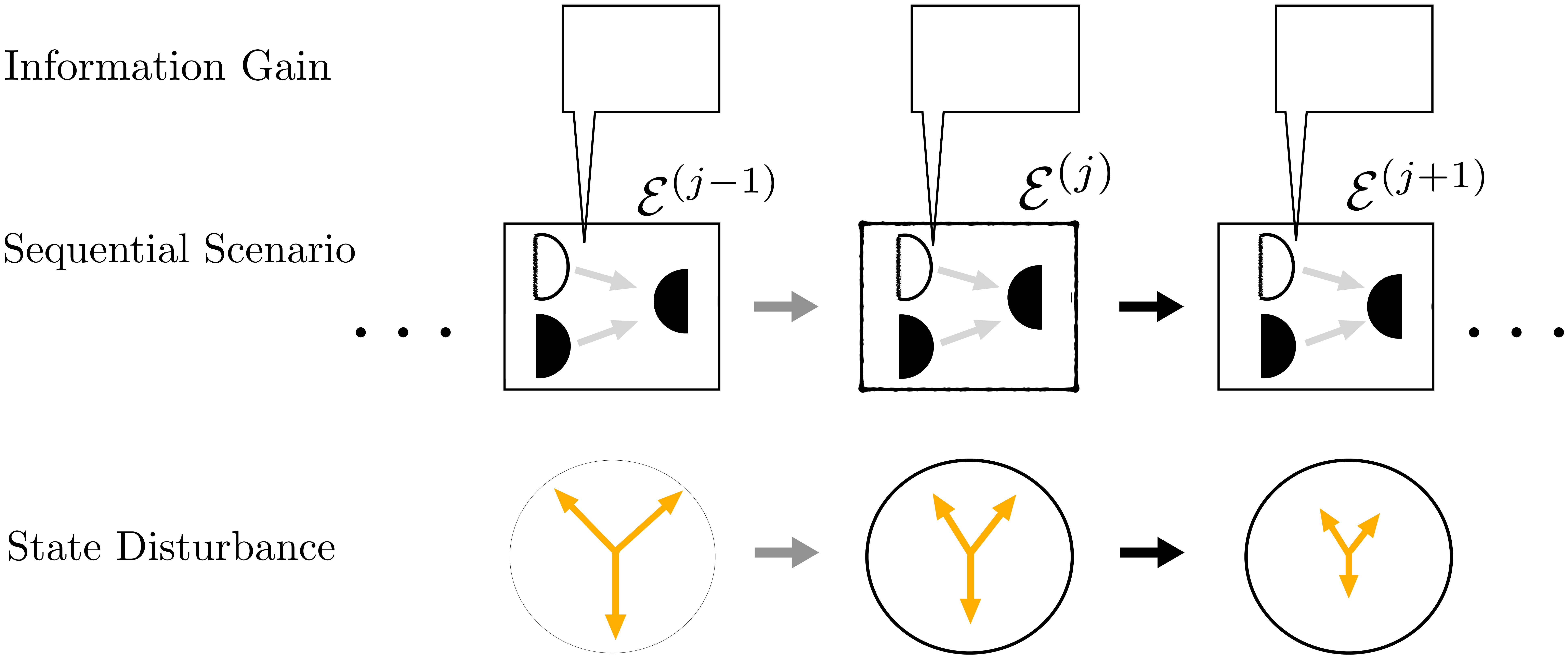
Hanwool Lee, Kieran Flatt, Joonwoo Bae



$$R < \left(\log \frac{\widetilde{s}^{(1)}}{1-\delta}\right) \frac{1}{(\log \eta_0)}$$

Sequential Quantum Maximum Confidence Discrimination

Hanwool Lee, Kieran Flatt, Joonwoo Bae



$$R < 1 + \frac{\log(nC_{th} - 1)}{\log\left((1+\eta_0)/2\right)}.$$

Introduction: Quantum State Discrimination (single-shot)

Motivation

- Sequential quantum information processing: Bell nonlocality, State discrimination
- Monogamy: Entanglement, Nonlocality, …

Sequential Quantum Maximum Confidence Discrimination (Sequential MCD)

- Main result 1: sequential quantum MCD for linearly independent states
- Main result 2: sequential quantum MCD for linearly dependent states

Future Directions

- Sequential discrimination for non-LI states: less distinguishability from purify vs geometry
- Construction of Multipartite Quantum States by Sequential Operations: Network
- Sequential tasks based on quantum state discrimination