

Matrix Quantum Black Hole Horizon

Geometry, Hawking Radiation, Membrane Paradigm, deSitter space and Quantum Chaos

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BH matrix QM: 2406.01466, 2406.12704
Hawking radiation: 2603.12199
Membrane paradigm: 2607.10561, 2607.28746
Spectral chaos: 2609.15286
de Sitter space: 2608.21178
Quantum stability: 2608.16374

Outline

- 1 Overview (2)
- 2 I. Microscopic Horizon = fuzzy sphere + partons (3)
- 3 II. Spectral Chaos (14)
- 4 IIIa. Hawking Radiation (2)
- 5 IIIb. Membrane Paradigm (4)
- 6 IIIc. Emergence of de Sitter space (2)
- 7 Summary

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Fundamental problems of quantum black hole

Black hole provides an ideal window to quantum gravity!

- Berkenstein-Hawking entropy
- Temperature and thermalization
- Black Hole chaos: spectral correlations, scrambling etc
- Typicality
- Information
- Bulk
 - Emergence of GR
 - Membrane paradigm
 - BH interior
 - IR screening of cosmological constant
- ...

Despite significant progress in AdS/CFT, BH microstates are needed for a direct fundamental understanding of these quantum properties of BH.

Proposal: Matrix Model of Black Hole

We want:

(Proposal: 2406.01466)

- Matrix quantum mechanics (NCG; BFSS; Sekino-Susskind 08)
- Degree of freedom effectively locks to horizon (infinite redshift)

Proposal: large N quantum mechanics of 3d quantum space

$$L = \text{tr} \left[\frac{1}{2a_0^2 M_P} \dot{X}^{a2} + \frac{M_P}{N^2} ([X^a, X^b]^2 + 4X^{a2}) + i\dot{\psi}^\dagger \psi - \frac{a_2 M_P}{N^2} \psi^\dagger \sigma^a X^a \psi \right]$$

where $a = 1, 2, 3$, $X_{mn}^a, \psi_{mn}, \psi_{mn}^\dagger$ $N \times N$ traceless matrices.

1. M_P mass scale, a_0, a_2 numerical coefficients.
2. M_P will be fixed in terms of G , a 's will be partially fixed by matching physics.
3. We have a negative mass term and a vector fermion. Both crucially important!

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NCG of Fuzzy Sphere

- EOM admits bosonic ($\psi = 0$) static solution given by $N \times N$ matrices obeying

$$[X^a, X^b] = i\epsilon^{abc} X^c, \quad \sum_a X^{a2} = \frac{N^2 - 1}{4} \mathbf{1},$$

- These are simply spin $J = (N - 1)/2$ reps. of $SU(2)$. It describe a fuzzy sphere.
- Introduce dimensional coordinates $Y^a = 2l_P X^a$, the fuzzy sphere has radius $R = Nl_P$ at large N .

Fermionic parton states

- The fermionic states of the model can be obtained by quantizing

$$H_F = a_2 \frac{M_P}{N^2} \text{Tr} \psi^\dagger \sigma^a X^a \psi$$

over the fuzzy sphere bkgd.

- In diagonalized form, we get from $2N^2$ oscillators

$$\psi_{mp\alpha} \rightarrow \begin{pmatrix} u_m^{(p)} \\ d_m^{(p)\dagger} \end{pmatrix} \quad m = 1, \dots, N; \quad p = 1, \dots, N_f = N; \quad \alpha = 1, 2.$$

and

$$H_F = \frac{a_2 M_P}{2N} \sum_{p,m=1}^N (u_m^{p\dagger} u_m^p + d_m^{p\dagger} d_m^p) = \frac{a_2 M_P}{2N} (r + s), \quad r + s = 0, \dots, 2N^2,$$

($\sim$ parton states of Susskind, link states of loop quantum gravity)

- For a half filled Fermi sea ($r + s = N^2$), energy of the fermi system reproduces the Schwarzschild mass-radius relation

$$E = \frac{R}{2G}$$

if $G \sim M_P^2$.

Microstate counting

- Note that $\psi \rightarrow e^{i\alpha} \psi$ is a global symmetry. This implies that $r - s$ is a conserved quantity.
- In particular, the ensemble of states with $r = s = N^2/2$ has a ground state degeneracy of

$$\Omega = 2^{2N^2}$$

in the leading large N limit.

- This reproduces the Bekenstein-Hawking entropy

$$S = 2N^2 = \frac{A}{4G},$$

if $l_P \sim 1/M_P$.

Note that fluctuation is stable quantum mechanically despite the negative mass term in the Lagrangian
(Quantum stability: 2408.16374)

Note that a **thermal temperature** is obtained immediately from the mass and entropy relations:

$$T = \partial E / \partial S = T_H = 1/4\pi R.$$

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Black-hole chaos: from scrambling to random matrices

String theory/holography

- **Sekino & Susskind (2008): fast scrambling**
Black holes as fastest scramblers:

$$t_{\text{scr}} \sim \frac{\beta}{2\pi} \log S_{\text{BH}}.$$

- **Shenker & Stanford (2014): butterfly effect**
Near-horizon shock waves give exponential sensitivity to early perturbations and the gravitational origin of OTOC growth.
- **Maldacena, Shenker & Stanford (2016): chaos bound**

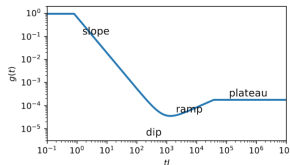
$$\lambda_L \leq \frac{2\pi}{\beta}$$

Black holes are not only fast scramblers:
their fine-grained spectra are expected to show universal quantum-chaotic correlations.

Cotler et al. (2017): Black Holes and Random Matrices

SYK, used as a simple black-hole model, exhibits the RMT spectral-form-factor structure

$$g(t; \beta) = \frac{\langle Z(\beta + it)Z(\beta - it) \rangle}{\langle Z(\beta) \rangle^2}.$$



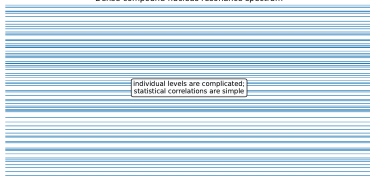
Slope: model-dependent decay / black-hole relaxation.

Ramp: approximately linear RMT regime from spectral rigidity.

Plateau: discreteness of the finite-entropy spectrum.

Wigner's original idea: forget the detailed nucleus

Dense compound-nucleus resonance spectrum



- A highly excited compound nucleus has an enormous number of strongly mixed configurations. Computing every microscopic level is not the useful question.
- Wigner's idea was to replace the detailed Hamiltonian by an **ensemble of matrices constrained only by symmetry** and ask for statistical properties of the spectrum.
- For unfolded nearest-neighbor spacing s ,

$$P_{\text{Poisson}}(s) = e^{-s}, \quad P_{\text{GOE}}(s) \simeq \frac{\pi}{2} s e^{-\pi s^2/4}.$$

The universal signal is level repulsion: $P(0) = 0$.

Why RMT is a diagnostic of quantum chaos

RMT does *not* claim that the microscopic Hamiltonian is literally a Gaussian random matrix. It claims that after unfolding, sufficiently fine spectral correlations become insensitive to microscopic details:

microscopic dynamics + symmetry $\longrightarrow$ universal spectral correlations.

Useful diagnostics span increasing spectral range:

- adjacent-gap ratio $r_n = \min(\delta_n, \delta_{n+1}) / \max(\delta_n, \delta_{n+1})$;
- nearest-neighbor distribution $P(s)$;
- connected spectral form factor $K_c(t)$: long-range correlations.

Dyson's threefold way

class	antiunitary symmetry	T^2	matrices	β
GOE	present	+1	real symmetric	1
GUE	absent	–	complex Hermitian	2
GSE	present	–1	quaternionic self-dual	4

At small spacing,

$$P(s) \propto s^\beta.$$

Hence the repulsion exponent itself records the antiunitary symmetry.

The question

(Chaos: 2609.15286)

A black hole should be a strongly chaotic quantum system.

Can this chaos be derived from the microscopic horizon geometry?

- Main idea: **quantum fluctuations of the fuzzy sphere** lift a highly degenerate horizon spectrum and **generate universal random-matrix correlations**.
- Main results:
 - ① Spectral chaos signatures clearly displayed
 - ② Quantum geometry $\longrightarrow$ ensemble of random Hamiltonian $\longrightarrow$ Dyson universality
 - ③ Quantum geometry selects the Dyson class.

Microscopic origin of the ensemble

- The round horizon is a fuzzy sphere,

$$X_a^{(0)} = J_a,$$

and a fundamental horizon parton couples through $H_p \sim \sigma_a X_a$.

- The round bkgd gives two highly degenerate one-parton bands.
- Geometric fluctuation δX_a are labelled by mode number (L, K, A)

$$X_a = X_a^{(0)} + \delta X_a, \quad \delta X_a = \sum_{LKQ} q_{LKA} B_a^{LKA}$$

induces a perturbation

$$\delta H_p = \sigma_a \delta X_a.$$

- Physical $K = L \pm 1$ fluctuations lift and mix the degeneracies.

- Each fluctuation amplitude q_{LKA} is an SHO:

$$L_{LK,A} = \frac{1}{2a_0^2 M_P} \left[\dot{q}_{LK,A}^2 - \Omega_0^2 \Lambda_{LK}^{\text{eff}}(N) q_{LK,A}^2 \right], \quad \Omega_0 = \frac{2a_0 M_P}{N}, \quad \Lambda_{LK}^{\text{eff}} > 0$$

- Fuzzy sphere is at its ground state

$$|0_{\text{geom}}\rangle = \int d\mathbf{q} \Psi_0(\mathbf{q}) |\mathbf{q}\rangle.$$

- Therefore fluctuation amplitudes are distributed by the **probability of the quantum-geometric vacuum**

$$P_0(\mathbf{q}) = |\Psi_0(\mathbf{q})|^2 \sim \prod_{LKA} \exp\left(-\frac{2\sqrt{\Lambda^{\text{eff}}}}{a_0 N} q_{LKA}^2\right),$$

- Thus we obtain a structured Gaussian ensembles of

$$\{\delta H_P(\mathbf{q})\}_{q_{LKA} \sim \mathcal{N}(0, \sigma_{LK}^2)}$$

- Compared with SYK

$$\{\delta H_{\text{SYK}}(\mathbf{J})\}_{J_{ijkl} \sim \mathcal{N}(0,1)}$$

- the horizon ensemble is derived, **not postulated**
- the horizon ensemble is highly structured, **not fully Gaussian**

A structured ensemble – not a Gaussian matrix put in by hand

After projection to a one-parton branch,

$$\mathcal{H}_s = \sum_{L,K=L\pm 1,A} w_{LK}^{(s)}(N) \xi_{LK,A} M_{KA}^{(s)}, \quad \xi_{LK,A} \sim \mathcal{N}(0,1).$$

The ingredients are microscopic:

- $M_{KA}^{(s)}$: fuzzy-sphere tensor operators fixed by rotational symmetry;
- $w_{LK}^{(s)}(N)$: branch- and rank-dependent weights fixed by the finite- N fluctuation spectrum;
- no $U(d)$ -invariant Gaussian covariance is assumed.

The nontrivial question is therefore:

Does this highly structured geometric ensemble flow to RMT spectral universality?

Geometry selects GOE, GUE, or GSE

For the full physical fluctuation ensemble the relevant antiunitary symmetry is broken:

$$\text{full ensemble} \longrightarrow \text{GUE.}$$

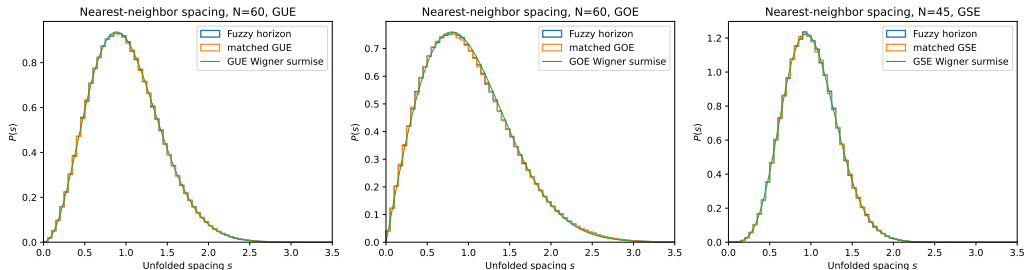
Restricting to the time-reversal-even sector (even K) restores the antiunitary symmetry:

$$N \text{ even : GOE},$$

$$N \text{ odd : GSE after Kramers-pair reduction}.$$

Thus all three Dyson classes occur in the *same microscopic construction*. What changes is the action of the antiunitary symmetry on the quantum geometry.

Nearest-neighbor spacings: all three Dyson classes

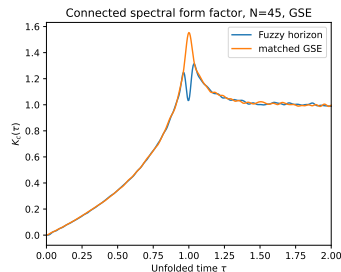
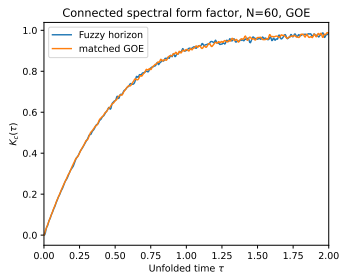
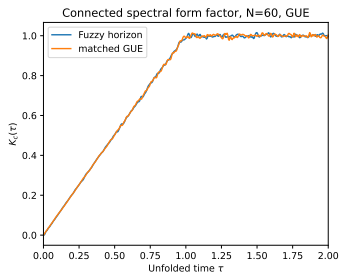


Representative spectra: full $N = 60$ (GUE), even- K $N = 60$ (GOE), and Kramers-reduced even- K $N = 45$ (GSE).

Long-range correlations: connected spectral form factor

Define

$$Z(t) = \text{Tr} e^{-iHt}, \quad K(t) = \langle |Z(t)|^2 \rangle, \quad K_c = K - K_{\text{disc}}.$$



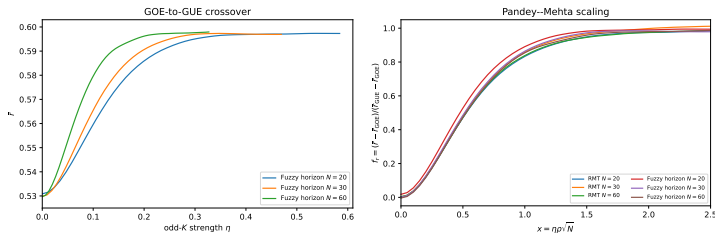
The horizon data track the matched GUE, GOE and GSE form factors through the universal ramp and late-time behavior. This is substantially stronger than nearest-neighbor level repulsion alone.

A stronger test: continuous move btw symmetry classes

- Endpoint agreement with GOE and GUE is not enough. Break the antiunitary symmetry continuously:

$$\mathcal{H}_s(\eta) = \mathcal{H}_{s,e} + \eta \mathcal{H}_{s,o}.$$

- For even N , $\eta = 0$: GOE, $\eta > 0$: time reversal broken, $\eta \rightarrow \text{large}$: GUE.



- The bare η is model dependent. The covariance of the microscopic even/odd sectors determines ρ , giving the universal finite-size coordinate

$$x = \eta \rho^{1/2} \sqrt{N}.$$

What is random – and what is not?

- The randomness here is **quantum-geometric**, not thermal.
- The geometric modes are far above the Hawking temperature, so their thermal occupation is exponentially suppressed. The distribution sampled in the chaos calculation is the ground-state Born distribution of the fluctuating horizon geometry.
- For a fixed geometric realization $\mathbf{q}$, $H_p(\mathbf{q})$ is one definite Hamiltonian. The ensemble describes the unobserved quantum geometry, just as Wigner's construction statistically describes unresolved microscopic nuclear complexity.

RMT universality emerges although the microscopic covariance is highly nonuniform.

Summary of results

quantum horizon geometry $\implies$ random-matrix spectral universality

Microscopic result	Universal consequence
vacuum fuzzy-sphere fluctuations	structured conditional ensemble
antiunitary geometry	GOE / GUE / GSE
level statistics	Wigner–Dyson repulsion
connected SFF	universal ramp
$t_{\text{Th}}/t_H \sim N^{-1}$	geometric onset scale
controlled symmetry breaking	Pandey–Mehta crossover

**The chaos is not inserted into the model;
it emerges from quantum geometry.**

What have we learned about the horizon?

- 1 **Spectral chaos is microscopic.** It already appears in the one-parton sector.
- 2 **Entropy and chaos involve the same horizon parton states.** Quantum geometry lifts the degeneracy of the states underlying the entropy counting.
- 3 **The Dyson class is geometric.** Antiunitary symmetry of the horizon realization selects GOE/GUE/GSE.
- 4 **Universality is strong.** Local spacings, long-range SFF correlations, and the GOE–GUE crossover all agree with matched RMT.
- 5 **The onset is fast on the one-parton scale.** $t_{\text{Th}}^{\text{geom}} \sim 1/\Gamma_N$.

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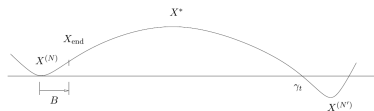
Monopole channel and barrier potential

(Tunneling: 2603.12199)

- We have a collection of fuzzy sphere vacua

$$E = \frac{bNM_P}{2}$$

of different sizes (rank N). Decay by tunneling from N to a small N' is possible.



- However since there is an excess of Fermi states in the Fermi sea of $X^{(N)}$, it cannot decay to the small fuzzy sphere with a smaller Fermi sea unless the decay path has the right number of fermi zero modes to saturate the excess fermi states: $\Delta = N^2 - (N - n)^2 = 2nN$

- Turns out the difference of two fuzzy spheres

$$A_a^{(k)} := X_a^{(N)} - X_a^{(N-n)},$$

describes a Dirac monopole on S_N^2 with precisely $2nN$ zero modes .

- Therefore the tunneling path is uniquely determined to be the interpolating path

$$X_a = X_a^{(N)} - \gamma(X_a^{(N)} - X_a^{(N')}), \quad 0 \leq \gamma \leq 1.$$

- Coleman textbook computation reproduces the tunneling rate of Page

$$\Gamma \sim \frac{1}{N^3} \sim \frac{\hbar}{G^2 M^3}$$

- Remark: $a_0 = \frac{\pi}{3\hbar}$ confirmed in two indep calculations provides a nontrivial check of the model.

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Classical EM membrane

(Membrane: 2607.10561, 2607.28476)

- BH membrane paradigm (Damour, Throne, Price, Macdonald (1982))
horizon properties of BH is reproduced with a fictitious membrane at the stretched horizon with certain EM (electromagnetic) or EnM (energy momentum) properties
- EM membrane: BH horizon is a Ohmic conductor with a surface conductivity of $\sigma_0 = 1/4\pi$

$$j_a = \frac{1}{4\pi} E_a, \quad a = \text{tangential direction}$$

- This is simply the statement of **ingoing BC** at the horizon

$$(\partial_t - \partial_{r_*})A_a = 0, \quad A_a = A_a(t + r_*)$$

- Note that the current is fictitious and carries no independent physical information.

Quantum BH membrane paradigm

- In our model, the parton states u, d not just accounts microscopically for the BH entropy.
- Turns out fuzzy sphere S_N has a Berry monopole B -field

$$A_{\bar{z}} = i \frac{N-1}{2} \frac{z}{1+|z|^2}.$$

and the partons are just it's lowest landau level (LLL) state.

- When there is a worldvolume (wv) E -field, the charged partons acquires a Ohmic response as well as a (chiral) Hall response

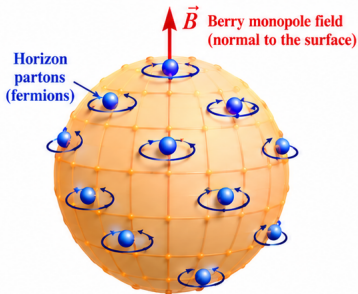
$$j_a^{(H)} = \sigma_H \epsilon_{ab} E_b, \quad a, b = \theta, \varphi$$

The Hall conductance $\sigma_H \propto Q$, $Q := q(r-s)$ is the charge of BH

Membrane paradigm from horizon partons

(1) Horizon partons in a Berry monopole field

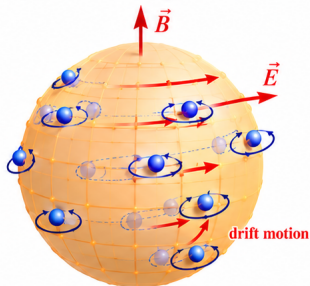
Fermionic horizon partons on a fuzzy sphere



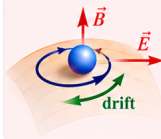
- The partons move in the lowest Landau level of a Berry monopole with charge $J = (N - 1)/2$.
- Each parton executes a circular motion around the local normal direction (like a charged particle in a magnetic field).
- The radius of the orbit is of order the Planck scale.

(2) Apply a tangential electric field

A slowly varying external gauge field induces a tangential electric field E_A on the horizon.



Guiding-center drift



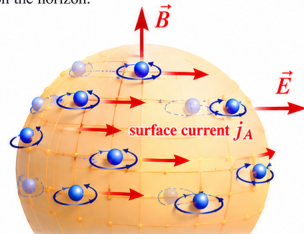
The tangential electric field causes a transverse drift

$$\vec{v}_d \propto \vec{E} \times \vec{B},$$

which gives a net motion of the partons along the horizon.

(3) Membrane response: horizon current

The drifting partons generate a surface current on the horizon.



Linear response (membrane paradigm)

$$j_A(\omega) = \sigma_{xx}(\omega)E_A + \sigma_H(\omega)\epsilon_{AB}E^B + i\omega D E_A + \dots$$

- σ_{xx} : Ohmic (dissipative) conductivity
- σ_H : Hall conductivity (parity-odd)
- D : polarization/susceptibility (reactive)

Classical value (low frequency)

$$\sigma_0 = \frac{1}{4\pi}, \quad \tau_d \sim R, \quad \Gamma_d \sim 1/R.$$

Reflective horizon

- A nontrivial **locking mechanism** of matrix model:

$$\mathbf{x}_a = \begin{pmatrix} X_a^H & W_a \\ W_a^\dagger & X_a^E \end{pmatrix},$$

The **link** W_a has the kinetic term: $(D_\mu W_a)^2 \sim (A_\mu - B_\mu)^2 W^2$.

- A **heavy condensate** $|W| \sim N$ occuring at the stretch horizon effectively **locks the wv gauge field B with the external Maxwell A** .
- This turns the wv partons transport into an EM BC: $\partial_u = \partial_t - \partial_{r^*}$,

$$\partial_u A_\pm = R_\pm \partial_v A_\pm, \quad R_\pm(\omega) = \frac{i}{2}(D\omega \pm \sigma_H), \quad D \sim l_P,$$

- As a result, we predict that **BH is reflective**, with a **characteristic frequency dependence** and a **chiral response**.
- A fundamental echo is predicted (see also Cardoso etal 2016)

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A quantum tadpole of the horizon

(dS: 2608.21178)

- The fluctuation spectrum contains an exceptional radial mode, the $(L, K) = (1, 0)$ mode,

$$X_a = X_a^{(0)} + q \mathcal{Y}_a^{10} + \dots$$

- Unlike ordinary fluc. modes, this generate a quantum tadpole,

$$\left. \frac{\partial V_{\text{eff}}}{\partial q} \right|_{q=0} \neq 0.$$

This corresponds to a **negative horizon pressure**.

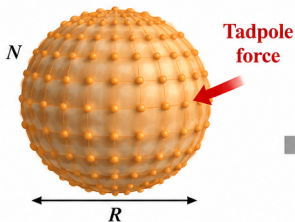
- Idea: invoke tadpole cancellation from the gravitational sector.

What gravitational interior can balance this microscopic pressure?

From the quantum tadpole to a de Sitter interior

(1) Quantum tadpole on the horizon

Fuzzy-sphere horizon



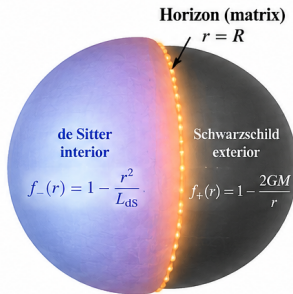
The (1,0) radial mode has a nonzero tadpole from quantum fluctuations

$$\left. \frac{\partial V_{\text{eff}}}{\partial q_{10}} \right|_{q_{10}=0} \neq 0.$$

Negative quantum pressure on the horizon.

(2) Junction condition

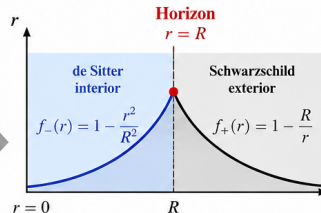
Match the horizon to a gravitational interior and exterior.



The quantum tadpole provides the microscopic stress at the junction. Solving the gravitational junction condition fixes the interior vacuum.

(3) Emergent de Sitter interior

The consistent solution is a de Sitter interior with the same horizon radius.



Interior metric (de Sitter):

$$ds_-^2 = -\left(1 - \frac{r^2}{R^2}\right) dt_-^2 + \frac{dr^2}{1 - r^2/R^2} + r^2 d\Omega_2^2.$$

The junction condition gives

$$L_{\text{ds}} = R.$$

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Summarizing, we have proposed a model of quantum black hole horizon. Same set of microstates explain:

- macroscopic properties,
- entropic counting of black hole,
- Hawking decay rate
- membrane paradigm
- emergence of dS
- quantum chaos

Work in progress:

- Typicality
- IR screening of dS

So far a model producing interesting BH horizon physics.

Emergence of gravity still to be seen.

Thank you!

Physics of the negative mass term

- ① The negative mass term implies a classical tachyonic mode at the quadratic level. However, it is stabilized at 1-loop.
- ② In fact, the whole fuzzy sphere is quantum mechanically stable for all its fluctuation mode. (Chu 2608.16374).
- ③ The negative mass term captures interesting physics associated with the infinite red shift of horizon:
 - a. **Hawking radiation:** It leads to a “logarithmic” $\log N$ barrier potential between fuzzy spheres, and reproduces the Page result for black hole decay rate from semiclassical Hawking radiation. (Chu 2603.12199)
 - b. **Membrane Paradigm:** It leads to a “locking mechanism” which explains the EM membrane paradigm (Chu 2607.10561)
 - c. **Tadpole cancellation and emergence of de Sitter space:** It leads to a tadpole for the $(1,0)$ fluctuation mode . Cancellation of which requires a de Sitter space as interior of the black hole. (Chu 2608.21178)

Physics of the non-commutator Yukawa term

Unlike the canonical BFSS commutator-type coupling $\text{Tr} \psi^\dagger \sigma_a [X_a, \psi]$ which is responsible for kinematical propagation, the coupling $\psi^\dagger \sigma_a X_a \psi \sim \psi^\dagger \sigma_a J_a \psi$ is not propagating

This is exactly what is needed:

- ① The filled Fermi sea determines the macroscopic energy, $E = E_{\text{bos}} + E_{\text{Fermi}}$, and its large- N behavior reproduces the required **black-hole mass-radius scaling**. (Chu 2406.01466).
- ② The coupling identifies these Fermi states with **LLL** of a background Berry B -field of the fuzzy sphere. This explains why these horizon partons are locked on the fuzzy sphere. (Chu 2607.10561)
- ③ The coupling determines a **zero mode** structure of the horizon partons, which turns out to fix a tunneling path for fuzzy spheres, and reproducing the decay rate of black hole. (Chu 2603.12199).
- ④ The coupling also give rises to **quantum chaos** of the horizon dynamics (Chu 2609.15286).